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OPTIMAL POLLUTION TAX UNDER INTERNATIONAL COMPETITION

Koji OKUGUCHI

Department of Economics, Tokyo Metropolitan University, Tokyo, Japan

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Abstract: International competition between home and foreign oligopolistic firms producing identical goods to export to a third country is formulated assuming that home country's firms are emitting pollutants and the optimal polluting tax rate is determined for the home country. The optimal pollution tax rate is then interpreted for some special cases to show whether it is equal to, falls short of ,or exceeds the marginal value of the domestic environmental damage. The equilibrium total supply of the good by the two countries is determined in the first place ,which greatly simplifies the mathematical analysis of determining the optimal pollution tax rate, which otherwise might become very complicated.

Key words: Pollution tax, international competition, social welfare, environmental damage.
JEL Classification Number: L13, F18, H23.

1. INTRODUCTION

The optimal pollution tax rate which maximizes social welfare has been analyzed by many economists for a single closed economy in relation to its market structure. If the market is perfectly competitive, the optimal pollution tax rate is equal to the marginal value of the environmental damage due to firms' productive activities, and in the case of monopoly, it falls short of the marginal value of the environmental damage (see Barnett, 1980, and Baumol and Oates, 1980). Under oligopoly no definite relationship exists between the optimal pollution tax rate and the marginal value of the environmental damage, as has been shown by Simpson (1995) for duopoly, and Okuguchi and Yamazaki (1994) and Okuguchi (2003) for general oligopoly. In international competition between two countries, say home and foreign countries, home country's firms may compete with foreign ones in at least one country, which may be home, foreign or a third country. International competition between two countries has been much analyzed especially from the view point of strategic trade policy (see Krugman, 1984, Dixit, 1984,

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E-mail: okuguchi351013@ybb.ne.jp

Brander and Spencer, 1985, Eaton and Grossman, 1986, for seminal papers). These works, however, have not taken into account the environmental degradation caused by productive activities in trading countries.The optimal pollution tax has been analyzed by some economists in the context of international competition between two trading countries in their own markets (see Daval and Hamilton, 2001; Copeland and Taylor, 2003, and a survey article by Requate, 2005).

In this paper we will formulate an international competition between home and foreign countries in a third country a la Okuguchi (2001), assuming that domestic firms emit pollutants which are subject to pollution taxation by the home country. In Section 2, we will describe our model of international competition and prove the existence of a unique Cournot equilibrium under general assumptions on the cost and demand functions. In Section 3, we will derive the formula for home country's optimal pollution tax rate which maximizes home country's social welfare net of the value of the environmental damage. We will then interpret the formula in relation to market structure to clarify whether the optimal pollution tax rate is equal to, falls short of,or exceeds the marginal value of the environmental damage in home country. Section 4 concludes.

2. MODEL

Consider two countries-say, home and foreign countries-whose firms producing identical goods to market only to a third country. Let there be n and n^* home and foreign firms, respectively, and assume that only home country imposes pollution tax at the rate of t on its firms' pollutants. Let x_i and x_j^* be i-th domestic firm's and j-th foreign firm's outputs, respectively, and $p = f(Y)$, $f' < 0$, be the inverse demand function, where p is the price of the good as a function of the total supply by home and foreign firms $Y \equiv X + X^*$, where $X \equiv \sum_{i=1}^{n} x_i$ is the home country's total supply and $X^* \equiv \sum_{j=1}^{n^*} x_j^*$ is the foreign country's total supply. Let $C_i(x_i, t)$ and $C_j^*(x_j^*)$ be cost functions of i-th domestic and j-th foreign firms, respectively. We assume following Simpson (1995) and Okuguchi (2003) that given domestic firms' outputs and prices of conventional factors of production like labor or capital, domestic firms choose their optimal amounts of pollution to minimize their total costs of production,therefore,a firm's amount of pollution depends not only on its output but also on the pollution tax rate.Alternatively, we may more restrictively assume that a firm's amount of pollution uniquely depends on its output. In the above cost function the prices of conventional factors of production are suppressed since they are implicitly assumed to remain constant.

By definition i-th domestic firm's profits π_i and j-th foreign firm's profits π_j^* are given as below:

$$\pi_i \equiv x_i p(Y) - C_i(x_i, t)\,. \tag{1.1}$$

$$\pi_j^* \equiv x_j^* p(Y) - C_j^*(x_j^*)\,. \tag{1.2}$$

In the following analysis, unless otherwise stated, suffixes i and j are understood to take the value from 1 to n and from 1 to n^*, respectively. We assume that all firms behave as Cournot oligopolists. Under this assumption, and given the value of t, the first order conditions for profit maximization for domestic and foreign firms are given as follows:

$$\frac{\partial \pi_i}{\partial x_i} = p(Y) + x_i p'(Y) - C_i'(x_i, t) = 0\,. \tag{2.1}$$

$$\frac{\partial \pi_j^*}{\partial x_j^*} = p(Y) + x_j^* p'(Y) - C_j^*(x_j^*) = 0\,. \tag{2.2}$$

We have used the notation $C_i' \equiv \frac{\partial C_i}{\partial x_i}$, and assumed interior maximum for all firms. Before we proceed further we impose the following assumptions on the cost and demand functions,which implies that any two firms' outputs in each country are strategic substitutes in the sense of Bulow *et al* (1985). These assumptions have been widely used in the existence and stability analysis of Cournot oligopoly equilibrium.

$$f' < C_i''\,. \tag{3.1}$$

$$f' < C_j^{*'} \tag{3.2}$$

$$f' + x_i f'' < 0\,. \tag{4.1}$$

$$f' + x_j^* f'' < 0 \tag{4.2}$$

We assume also that

$$C_{it} \equiv \frac{\partial C_i}{\partial t} > 0, \quad C_{it}' \equiv \frac{\partial^2 C_i}{\partial x_i \partial t} > 0\,. \tag{5}$$

It is easy to see that the second order conditions for profit maximization are satisfied under (3.1)–(4.2). Applying the implicit function theorem, we can solve (2.1) with respect to x_i as a function of Y and t, and (2.2) with respect to x_j^* as a function of the total supply of the good exported to the third country Y, respectively.

$$x_i \equiv \varphi_i(Y, t)\,, \tag{6.1}$$

where

$$\varphi_i' \equiv \frac{\partial \varphi_i}{\partial Y} = -\frac{p' + x_i p''}{p' - C_i''} < 0\,, \tag{6.2}$$

$$\varphi_{it}' \equiv \frac{\partial \varphi_i}{\partial t} = -\frac{C_{it}'}{p' - C_i''} < 0\,. \tag{6.3}$$

$$x_j^* \equiv \varphi_j^*(Y)\,, \tag{7.1}$$

where

$$\varphi_j^{*\prime} \equiv= \frac{d\varphi_j^*}{dY} = -\frac{p' + x_j^* p''}{p' - C_j^{*\prime\prime}} < 0\,. \tag{7.2}$$

By definition we have

$$X = \sum_{i=1}^{n} \varphi_i(Y, t) \equiv \varphi(Y, t)\,, \tag{8.1}$$

$$X^* = \sum_{j=1}^{n*} \varphi_j^*(Y) \equiv \varphi^*(Y)\,, \tag{8.2}$$

$$Y = X + X^* = \varphi(Y,t) + \varphi^*(Y) = \Psi(Y,t)\,. \tag{8.3}$$

Given the value of t, the equilibrium total supply of the good by the two countries is identical to the solution of one variable equation (8.3). The solution is

$$Y \equiv g(t)\,, \tag{9.1}$$

where

$$\frac{dg}{dt} = \frac{\varphi_t}{1-\varphi'-\varphi^{*\prime}} < 0\,, \tag{9.2}$$

and

$$\varphi' \equiv \frac{\partial\varphi}{\partial Y}, \quad \varphi_t \equiv \frac{\partial\varphi}{\partial t}\,.$$

We note from (6.2), (7.2) and (6.3) that $\varphi' < 0, \quad \varphi_t < 0, \varphi^{*\prime} < 0$. Inequality (9.2) states that the equilibrium total supply of the good decreases if the pollution tax rate in the home country increases. This fact is depicted in Figure 1. In the figure, if the pollution tax rate is t_1, the equilibrium total supply of the good, $Y_1 = g(t_1)$, corresponds to the intersection of 45 degree line through the origin and the downward-sloping curve for $\Psi(Y,t)$. If t increases to t_2, the curve for $\Psi(Y,t)$ shifts downwards, and the new equilibrium total supply becomes $Y_2 = g(t_2)$. If t changes,the equilibrium total output in each country changes as in (10.1) and (10.2) below.

$$\frac{dX}{dt} = \frac{dY}{dt} - \frac{dX^*}{dt} < 0 \tag{10.1}$$

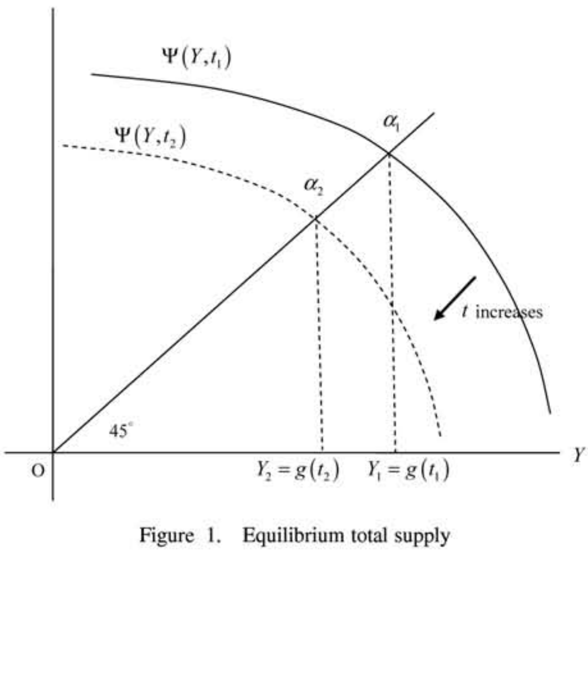

Figure 1. Equilibrium total supply

$$\frac{dX^*}{dt} = \varphi^{*\prime}\frac{dY}{dt} > 0\,. \tag{10.2}$$

Furthermore, we get

$$\frac{dx_i}{dt} = \varphi_i'\frac{dY}{dt} + \varphi_{it} \gtreqless 0\,. \tag{11.1}$$

$$\frac{dx_j^*}{dt} = \varphi_j^*\frac{dY}{dt} > 0\,. \tag{11.2}$$

In other words, the home country's total supply decreases,the foreign country's total supply increases, all foreign firms' outputs increase, and domestic firms' outputs may increase, decrease or do not change in the event of an increase in the pollution tax rate in the home country.

3. OPTIMAL POLLUTION TAX RATE

We are now in a position to determine the optimal pollution tax rate for the home country. By definition, the home country's total net social welfare W is the sum of domestic firms' profits plus the total pollution tax revenue minus the value of the environmental damage $D(E)$, that is,

$$W(t) \equiv \sum_{i=1}^{n}\{p(Y(t))x_i(t) - C_i(x_i(t), t)\} + tE(t) - D(E(t))\,, \tag{12}$$

where if e_i denotes i-th domestic firm's pollution, $E \equiv \sum_{i=1}^{n} e_i$ denotes the total pollution by all domestic firms. Note that the total cost function $C_i(x_i(t), t)$ includes i-th firm's expenditure for pollution tax and the consumers' surplus does not exist. The optimal pollution tax rate is the value of t which satisfies

$$\begin{aligned}\frac{dW}{dt} &= \sum_{i=1}^{n}\left\{p'x_i\frac{dY}{dt} + (p - C_i')\frac{dx_i}{dt} - C_{it}\right\} + E + t\frac{dE}{dt} - D'\frac{dE}{dt} \\ &= 0\,.\end{aligned} \tag{13}$$

If we take into account the first order condition for profit maximization (2.1) as well as $C_{it} = e_i$, which is the result of the duality theorem for the cost function, we obtain from (13) the following formula for the optimal pollution tax rate.

$$t = D' - \frac{p'\displaystyle\sum_{i=1}^{n} x_i\left(\frac{dX_{-i}}{dt} + \frac{dX^*}{dt}\right)}{\dfrac{dE}{dt}}\,, \tag{14}$$

where $X_{-i} \equiv \sum_{j\neq i}^{n} x_j$.

We have to consider three cases in order to make clear whether the optimal pollution tax rate shown above is equal to, falls short of, or exceeds the marginal value of the environmental damage in the home country.

Case 1: If perfect competition prevails in the third country, then $p = 0$ leading to

$$t = D' . \tag{15.1}$$

Case 2: If $n = 1$-monopoly in the home country-then $X_{-1} = 0$, $E = e_1$ and under normal condition $\frac{dE}{dt} < 0$, hence

$$t = D' - \frac{p' x_1 \dfrac{dX^*}{dt}}{\dfrac{dE}{dt}} < D' . \tag{15.2}$$

If home country's firm's pollution level uniquely depends only on its output level as a referee suggests,then we can easily prove that $\frac{dE}{dt} < 0$.

Case 3: In general when the sign of $\frac{dX_{-i}}{dt} + \frac{dX^*}{dt}$ is uniformly negative, zero or positive for all i, we can assert that

$$t \underset{<}{\overset{\geq}{=}} D' \quad \text{according as} \quad \frac{dX_{-i}}{dt} + \frac{dX^*}{dt} \underset{>}{\overset{\leq}{=}} 0 \quad \text{for all } i . \tag{15.3}$$

4. CONCLUSION

In this paper we have derived and given economic interpretations of the optimal pollution tax rate within a model of international competition between two countries in a third country assuming that oligopolistic firms in both countries export the identical goods only to a third country as well as assuming that only one country pollutes as a result of its productive activities. We have been able to derive our fundamental formula for the optimal pollution tax rate by determining the equilibrium total supply by the two countries in the first place. We have considered the firm's pollution as a factor of production similar to a conventional factor of production like labor or capital and taken into account the duality theorem for the cost function in order to derive the fundamental formula for the optimal pollution taxation. We have examined for some special cases whether the optimal pollution tax rate is equal to, fall shorts of,or exceeds the polluting country's marginal value of the environmental damage. Finally, we note that we have been able to derive the optimal pollution tax formula rather easily because of our identifying the unique Cournot equilibrium in our model of international competition in a third country as the unique solution of the one variable equation (8.3).

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