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THE AUTHORSHIP OF THE MARGINAL PRODUCTIVITY THEORY IN "THE OLD QUARREL"

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Abstract: We coordinate contributions of the economists in "the old quarrel" to marginal productivity theory in the light of the theory of long-run competitive equilibrium in the market economy. The exhaustion theorem is established if the total production function of the economy is homogeneous of degree one, even when individual production functions are not necessarily so. Wicksteed proved the exhaustion theorem in the theory of competitive producer. Walras tried to generalize it in the theory of competitive markets, and Barone revised Walras' theory. Pareto suggested the condition of producer's profit maximization compatible with the free competition equilibrium, but he had a negative view on the validity of the exhaustion theorem. Wicksell put forward the properties of production functions to prove the exhaustion theorem in Barone's theory of competitive markets.

Key words: exhaustion theorem, marginal productivity theory, long-run competitive equilibrium.
JEL Classification Number: B21, C62, D24, D41.

1. INTRODUCTION

The controversy on the exhaustion theorem in marginal productivity theory, known as "the old quarrel" by Jaffé (1964), played a significant role in the development of production and distribution theories. The exhaustion theorem, proposed by Wicksteed (1894/1992, 89) in his *Essay on the Coordination of the Laws of Distribution*, states that "if every factor of production draws a remuneration determined by its marginal efficiency or significance, the whole product will be exactly distributed." [1] On reading the *Essay*, Walras noticed that the theorem was immediately derived from the theory of free competition equilibrium of his own, and presumed that theory was more general than Wicksteed's. To verify his supposition, Walras involved Pareto and Barone in the

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[1] Wicksteed proposed the exhaustion theorem to support his earlier critique (Wicksteed, 1894/1992, 89) of the Marxian surplus theory of distribution (Steedman, 1992, 6–7).

controversy about the authorship of the exhaustion theorem in marginal productivity theory. Walras summarized his claim in the appendix in the 3rd edition of his *Éléments d'économie politique pure*. Stigler (1941) investigated the marginal productivity theories of Wicksteed, Marshall, Walras, Pareto, Barone, Wicksell and so on, and proposed his opinion about its authorship. Jaffé (1964) investigated the correspondences of Léon Walras about the old quarrel, and made clear that Barone's contribution was crucial to Walras' appendix.

To establish the authorship of marginal productivity theory, there are two difficulties to be pointed out. First, the old quarrel included the theoretical problems which had been open until the 1950s, namely the problem of existence of a competitive equilibrium in a market economy and the problem of duality. The economists involved in the old quarrel did not notice the significance of these problems, while the old quarrel was had within the framework of general equilibrium theory. Wicksteed proved the exhaustion theorem in the theory of competitive producers, where a price system is given. The exhaustion theorem is, however, meaningless if there is not a competitive equilibrium in the market economy. Walras tried to prove the exhaustion theorem in the theory of competitive markets, and claimed that Walras' exhaustion theorem was more general than Wicksteed's. But he did not realize that a certain assumption on the properties of production functions is necessary for his claim to be proved. Some economic theorists such as Schultz (1929) and Georgescu-Roegen (1935–36) state that Pareto also has the honor of sharing the authorship of marginal productivity theory with other economists, for the reason that Pareto's theory implies the exhaustion theorem. However, Pareto himself viewed the validity of the exhaustion theorem negatively because he did not know about the theory of duality, so that he could not show the theorem is provable in his theory. Thus, the historically inevitable ignorance of theoretical knowledge confused the matter, and therefore it is necessary for us to interpret the theories in the old quarrel in the light of modern microeconomic theory.

Second, however, there is no completed form of marginal productivity theory in the history of general equilibrium theory. The exhaustion theorem is meaningful only in a specific economic environment. For example, whether a commodity is a product or a production factor is *a priori* determined; there is no joint production; and production technology is expressed in terms of a differentiable production function, which is homogeneous of degree one. Such specifications of the economic environment disappeared as general equilibrium theory developed, because they were specific and unnecessary to prove the existence of a competitive equilibrium for a market economy. Thus, the participants in the old quarrel were unable to complete marginal productivity theory because of a lack of theoretical knowledge. Once the theoretical knowledge developed enough to complete the theory, the exhaustion theorem had lost the significance in the generalized economic environment general equilibrium theory assumes. This dilemma obscures the role that marginal productivity theory played in the history of general equilibrium theory. Hence we must first state what is an ideal form of marginal productivity theory if it were completed in the history of general equilibrium theory. Then, on the basis of it, we interpret the theories in the old quarrel and coordinate their contributions to marginal productivity theory.

This paper consists of five sections. We confirm what the economists involved in the old quarrel state about the exhaustion theorem, and reconstruct a consistent theory in which the exhaustion theorem is proved in the light of general equilibrium theory. We put forward an ideal form of the marginal productivity theory in Section 2, on the basis of which we will interpret Wicksteed (1894/1992) in Section 3, Walras (1874–77/1952), Barone (1895/1965), and Wicksell (1902/1958) in Section 4, and Pareto (1897/1964, 1909/1966) in Section 5. In Section 6, we establish the authorship of marginal productivity theory, and coordinate the contributions of economists involved in the old quarrel. Wicksteed's exhaustion theorem is valid if every individual production function is homogeneous of degree one. Walras' exhaustion theorem established by Barone is valid if the total production function of the economy is homogeneous of degree one, where any properties of individual production functions are acceptable on the condition mentioned above. Therefore, Walras' theorem is more general than Wicksteed's. However, marginal productivity theory should be based on the theory of individual producer. Pareto and Wicksell put forward the theory of individual producer compatible with Walras' theorem. In the last section, we make some remarks on the modern significance of the exhaustion theorem.

2. AN IDEAL FORM OF THE MARGINAL PRODUCTIVITY THEORY

Marginal productivity theories in the old quarrel were developed in the general equilibrium theory which would be crystallized into the theories of Arrow and Debreu (1954), Debreu (1959) and McKenzie (1959). We put forward an ideal form of the marginal productivity theory, namely, the general equilibrium theory in which the exhaustion theorem is established.

The exhaustion theorem in marginal productivity theory assumes certain specifications of the economic environment from the viewpoint of the later microeconomic theories such as Arrow and Debreu (1954), Debreu (1959) and McKenzie (1959). First, whether a commodity is a product or a production factor is not *a priori* determined, but it is determined properties of the economic environment such as a consumer preference and producer technology. Second, there is no joint product. This implies that production technology is expressed by an explicit production function $y = f(z_1, z_2, \ldots, z_n)$, where $y \in \boldsymbol{R}_+$ is a product and $(z_1, z_2, \ldots, z_n) \in \boldsymbol{R}^n_+$ are production factors. The first point makes this assumption meaningless, and the production function should be expressed by $f(y_1, y_2, \ldots, y_H) = 0$, where $(y_1, y_2, \ldots, y_H) \in \boldsymbol{R}^H$ is a production. Third, in order for the exhaustion theorem to be meaningful, it is necessary to prove the existence of a competitive equilibrium in a market economy. In marginal productivity theory, production technology is expressed by a differentiable production function satisfying the laws of decreasing marginal productivity and of constant returns to scale. However, to prove the existence of an equilibrium, it is sufficient to assume continuity, convexity, and several adequate conditions of consumption sets, preference orderings, and production sets, but it is not necessary to assume differentiability and other specific properties of utility and production functions (Debreu, 1959).

We will preserve, however, the assumption of differentiability of production functions, because it is the historical hypothesis which characterizes not only the marginal productivity theories in the old quarrel but also all the theories based on the marginal analysis especially in the period from the marginal revolution in the 1870s through the proof of existence of a competitive equilibrium in a market economy in the 1950s. Then, the exhaustion of products is expressed by $\sum_{h=1}^{H} \frac{\partial f}{\partial y_h}(y^*) y_h^* = 0$ at an equilibrium production $y^* = (y_1^*, y_2^*, \dots, y_H^*)$.

We show that in the competitive equilibrium of a market economy, the exhaustion theorem is established if the total production function of the economy is homogeneous of degree one, while the production function of every individual producer may not always be homogeneous of degree one. Consider the Arrow-Debreu model consisting of I consumers and J producers with H commodities, where the equilibrium conditions of consumer, producer, market, and zero profit, which is Walrasian condition of free competition equilibrium (Walras, 1874–77/1952), are satisfied. For for every $j \in \{1, 2, \dots, J\}$, let $y_j = (y_{j1}, y_{j2}, \dots, y_{jH}) \in \boldsymbol{R}^H$ be the production of producer j, and $f_j(y_j) \leqq 0$ be a production function, which satisfies $f_j(0) \leqq 0$. The total production function is defined as $f(y) \leqq 0$ such that $f(y) = f(y_1, y_2, \dots, y_J)$, where $y = \sum_{j=1}^{J} y_j$. The equilibrium condition of producer j is described by:

$$y_j^* \text{ maximizes } p^* \cdot y_j \text{ subject to } f_j(y_j) \leqq 0\,, \quad \text{for every } j \in \{1, 2, \dots, J\}\,.$$

The zero profit equilibrium is described by

$$p^* \cdot y^* = 0 \quad \text{where } y_h^* = \sum_{j=1}^{J} y_{jh}^* \quad \text{for every } h \in \{1, 2, \dots, H\}\,.$$

By the producer's equilibrium, the price of commodity h is equal to product of the marginal productivity of the commodity $\frac{\partial f_j}{\partial y_{jh}}(y_j^*)$ with a Lagrangean multiplier $\lambda_j > 0$, that is $p_h^* = \lambda_j \frac{\partial f_j}{\partial y_{jh}}(y_j^*)$ for every commodity $h \in \{1, 2, \dots, H\}$. In a competitive situation, the following theorem is valid (Debreu, 1959, 3.4 (1)).

THEOREM 1. *Given a price system p^*, y^* maximizes $p^* \cdot y$ subject to $f(y) \leqq 0$, if and only if y_j^* maximizes $p^* \cdot y_j$ subject to $f_j(y_j) \leqq 0$, for every $j \in \{1, 2, \dots, J\}$.*

Theorem 1 states that all the individual producers maximize their profits subject to their production functions if and only if the total production maximizes the total profit subject to the total production function. The producer's equilibrium condition and theorem 1 imply that the marginal principle is also valid for the equilibrium total production. That is,

$$\text{(MPT1)} \qquad p_h^* = \lambda_j \frac{\partial f_j}{\partial y_{jh}}(y_j^*) = \lambda \frac{\partial f}{\partial y_h}(y^*)$$
$$\text{for every } j \in \{1, 2, \dots, J\} \quad \text{for every } h \in \{1, 2, \dots, H\}\,.$$

This result, together with the zero profit condition, implies that

$$\text{(MPT2)} \qquad \sum_{h=1}^{H}\sum_{j=1}^{J}\frac{\partial f_j}{\partial y_{jh}}(y_j^*)y_{jh}^*$$
$$= \sum_{h=1}^{H}\frac{\partial f}{\partial y_h}(y^*)y_h^* = 0 \quad \text{at an equilibrium production } y^*,$$

which is valid by Euler's theorem if the total production function of the economy is homogeneous of degree one.[2] It would be the theory of McKenzie (1959) where the production function is differentiable. We suppose it is an ideal form of the marginal productivity theory.

If the exhaustion theorem is established for the equilibrium total production, then it is also valid for every individual producer. In general equilibrium theory, the exhaustion theorem is established for the total equilibrium production (Debreu, 1959, 5.7 (1), 88, n.1). Therefore, applying Euler's theorem, if f is homogeneous of degree one, the exhaustion theorem is established, and it is not necessary for an individual production function to be homogeneous of degree one. Any individual production function is acceptable provided f is homogeneous of degree one.

However, any microeconomic theory should describe an economy consisting of many consumers and many producers with many commodities, where consumers are characterized by their preference pre-orderings and producers by their production technologies. Therefore, marginal productivity theory must assume a certain property of individual production functions so that the total production function of the economy is homogeneous of degree one. A complete description of the marginal productivity theory is given by Osana (1987). Osana (1987, 10) "introduced a concept of long-run competitive equilibrium for the Arrow-Debreu model in which the number of producers is taken into explicit account, and investigated its relationship to a competitive equilibrium for the McKenzie model of the long-run economy." Osana (1987, 1–2) defines a period as short-run (rep. long-run) if there is at least one (rep. there is no) production factor, for example entrepreneurship, whose input level is fixed within the period. The Arrow-Debreu model is considered as a short-run model, because the number of producers, namely the input of entrepreneurship, is given in the model. The McKenzie model is considered as a long-run model, because the input of entrepreneurship is variable and constant returns to scale prevail as a result of entry and exit of producers in the model. He shows that every long-run competitive equilibrium for the Arrow-Debreu model is a competitive equilibrium for the McKenzie model of the long-run economy, and that there is a long-run competitive equilibrium for the Arrow-Debreu model for every competitive equilibrium in the McKenzie model of the long-run economy. Suppose a producer whose production function is concave and exhibits constant returns to

[2] Euler's theorem states:

THEOREM 2. *Let* $f(x_1, \dots, x_n)$ *be differentiable. If* f *is homogeneous of degree* k, *then* $k\,f(x_1, \dots, x_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(x)x_i$ *for any* $(x_1, \dots, x_n) \in \boldsymbol{R}^n$.

scale up to a certain level and decreasing returns to scale at the larger end of the scale. The total production function of the industry will show constant returns to scale when a sufficient number of producers enter the industry. This concept of long-run competitive equilibrium for the Arrow-Debreu model is the same as competitive equilibrium for the McKenzie model of a long-run economy where the allotment of the equilibrium production to producers is given.

3. WICKSTEED'S MARGINAL PRODUCTIVITY THEORY

In the classical theories of production and distribution before Wicksteed, the product was supposed to be produced from the three factors of production of capital, labor, and land. The role of each production factor was explained by distinct laws of distribution. Wicksteed (1894/1992) put forward an idea to coordinate those laws of distribution into the law of marginal productivity in terms of the exhaustion theorem, and extended the theorem to a theory of competitive producer.

3.1. Significance of the Coordination of the Laws of Distribution

The classical theory of distribution was founded by Ricardo (1817/1953).[3] Ricardo supposes an economy consisting of three classes of economic agents, namely, the capitalist-producer, the laborer, and the land-owner, with the four commodities of product, capital, labor, and land. The capitalist-producer owns the resource of capital, the laborer that of labor, and the land-owner that of land. In the original theory of distribution, the output of product, denoted by $y \in \boldsymbol{R}_+$, is produced from the input $z_1 \in \boldsymbol{R}_+$ of labor employed with the capital (capital + labor). The production technology is described by the production function $y = f(z_1)$, which is differentiable and satisfies the law of decreasing marginal productivity. The capitalist-producer employs labor z_1^* with capital to maximize the product $y = f(z_1)$. In Ricardo's model, it is assumed that the quantity of capital is given and the wage rate is determined exogenously at a certain level w^* of the natural wage rate, and therefore the available quantity ω_1 of labor employed with capital is given. It follows that the capitalist-producer chooses the input z_1^* of labor so as to maximize the product $y = f(z_1)$ under the resource constraint $z_1 \leqq \omega_1$.[4] The rate of returns to capital + labor then is determined by their marginal productivity $\frac{df}{dz_1}(z_1^*)$, and their returns are equal to $\frac{df}{dz_1}(z_1^*)z_1^*$. Since the wage is $w^*z_1^*$, the profit is equal to $\frac{df}{dz_1}(z_1^*)z_1^* - w^*z_1^*$, and the rent is determined as the residue $f(z_1^*) - \frac{df}{dz_1}(z_1^*)z_1^*$. Thus, in the classical theory of distribution, the share to each production factor is explained by distinct laws of distribution. We should note that in the classical theory of distribution, the values or prices of production factors are determined, the output of product and the

[3] See Cannan (1893/1967) about the history of the classical theories of production and distribution. Pasinetti (1960) formulates a mathematical model of the Ricardian theory of distribution.

[4] In this sense we can omit the production factor of capital. Jevons (1871/1957, ch. 6) put forward an interpretation of this theory as the theory of rent. The coordination of the laws of distribution by Wicksteed (1894/1992, 51) is clearly based on his interpretation. Our interpretation is the same with that of Pasinetti (1960) without gold.

inputs of the production factors are determined, and therefore the shares to the production factors are determined. The laws of distribution to production factors are different from one another, but over all products the distributions are exactly exhausted to all the production factors explicitly or implicitly enumerated in the model.

Wicksteed reduced the Ricardian theory to a simpler theory by supposing the capital + labor to be a composite production factor, and the profit + wage rate the price of capital + labor, which we can call them simply labor and wage respectively. We can consider the classical theory of distribution to assume an economy consisting of a producer with commodities of product, labor, and land. In the theory of wage, the output y of product is produced from the input z_1 of labor. The technology is described by the production function $y = f(z_1)$, which is differentiable and satisfies the law of decreasing marginal productivity. Assuming that the quantity $\omega_1 \in \boldsymbol{R}_+$ of labor is given, the producer chooses his/her input z_1^* of labor so as to maximize his/her output y of product subject to his/her production function $y = f(z_1)$ and the resource constraint $z_1 \leqq \omega_1$. The rate of wage is then determined as the productive contribution (shadow price) of labor and equal to its marginal productivity $\frac{df}{dz_1}(z_1^*)$, so that the wage is equal to $\frac{df}{dz_1}(z_1^*)z_1^*$ and the rent is equal to the residue $f(z_1^*) - \frac{df}{dz_1}(z_1^*)z_1^*$. On the other hand, in the theory of rent,[5] the output y of product is produced from the input $z_2 \in \boldsymbol{R}_+$ of land. The technology is described by the production function $y = g(z_2)$, which is differentiable and satisfies the law of decreasing marginal productivity. Assuming that the quantity $\omega_2 \in \boldsymbol{R}_+$ of land is given, the producer chooses his/her input z_2^* of land so as to maximize his/her output y of product subject to his/her production function $y = g(z_2)$ and the resource constraint $z_2 \leqq \omega_2$. The rent of a unit of land is determined as a productive contribution (shadow price) of land and equal to its marginal productivity $\frac{dg}{dz_1}(z_2^*)$, and so the rent is equal to $\frac{dg}{dz_1}(z_2^*)z_2^*$, and the wage is equal to the residue $g(z_2^*) - \frac{dg}{dz_1}(z_2^*)z_2^*$. Thus, in the classical theories of distribution, the value of a production factor is determined by its marginal productivity with given quantity of its resource on the one hand, and the share to the other is determined by the residue.

Wicksteed realized that the classical theory of distribution presupposed not only explicitly enumerated production factors, but also implicit production factors. He had the idea that if all the production factors were enumerated in the model then the production function became homogeneous of degree one, and therefore the laws of distribution would be coordinated into the law of marginal productivity in terms of the exhaustion theorem.[6] Suppose the product y is produced from the inputs (z_1, z_2) of labor and land. Technology is described by the differentiable production function

[5] Wicksteed (1894/1992, 66) refers to the residual theory of wages of Francis Amasa Walker.

[6] "Each factor of production may be scheduled in its own unit, and when this has been done the enumeration of the factors of production may be regarded as complete. With this understanding it is obvious that a proportional increase in all the factors of production will cause a proportional increase of the product." (Wicksteed, 1894/1992, 84). In a production model with one product and one production factor, the law the decreasing marginal productivity implies the law of decreasing returns to scale. Wicksteed shows that the law of returns to scale is also necessary for a producer's optimization problem to have a solution.

$y = F(z_1, z_2)$, and satisfies the law of decreasing marginal productivity for every production factor and the law of constant returns to scale, namely, homogeneity of degree one. Assuming the quantities (ω_1, ω_2) of labor and land is given, the producer chooses his/her inputs (z_1^*, z_2^*) of labor and land so as to maximize his/her output of product subject to his/her production function $y = F(z_1, z_2)$ and resource constraint $(z_1, z_2) \leqq (\omega_1, \omega_2)$. The wage rate and the rent of a unit land are determined as productive contributions of production factors to the product.[7] The wage rate is equal to the marginal productivity $\frac{\partial F}{\partial z_1}(z_1^*, z_2^*)$, and the rent of a unit land to the marginal productivity $\frac{\partial F}{\partial z_2}(z_1^*, z_2^*)$. The output of product is then exhausted to the wage and the rent, that is, $F(z_1^*, z_2^*) = \frac{\partial F}{\partial z_1}(z_1^*, z_2^*)z_1^* + \frac{\partial F}{\partial z_2}(z_1^*, z_2^*)z_2^*$. This is the significance of the coordination of the laws of distribution. Note that in this model, the values and inputs of the production factors are determined to prove the exhaustion theorem. In this sense, it is a theory of distribution. Thus, marginal productivity theory is said to be qualified as a complete form provided that it successfully establishes the following three basic assertions: (1) all the production factors are rewarded in accord with their marginal products, (2) the total product is exactly exhausted, (3) the prices (values) and the inputs of production factors are determined.

Wicksteed's theory of distribution assumes the specific economic model in which the exhaustion theorem is established. First, in the exhaustion theorem, the share to every production factor is explained in terms of the law of marginal productivity. This does not mean that Wicksteed's theory is a development or an integration of the classical theories of distribution. The classical theory of distribution is a special case of Wicksteed's theory where the input of a certain production factor is fixed. They are just different specifications, for example short-run and long-run, of the same model of distribution. Second, Wicksteed's theory of distribution is only self-contained if there is only one product in the macro economy, supposed to be a kind of index such as GDP or social welfare. If there are more than two products, then it is impossible to determine an allocation of resources among different products without determining the values or prices of products. Thus, the assumption such that there is only one product is essential to the classical theory of distribution.

3.2. *Wicksteed's Exhaustion Theorem in a Producer Theory*

The classical theory of distribution cannot be applied to an economy consisting of many products and many production factors, and so Wicksteed extended it to a theory of competitive producers. He assumes a producer choosing a production so as to maximize his/her profit under the constraint of his/her production function for a given price system (Wicksteed, 1894/1992, 86–88). He considers the production function to be homogeneous of degree one, because all the commodities are traded in their markets (Wicksteed, 1894/1992, 83–86). Denote the output of product by $y \in \boldsymbol{R}_+$, the inputs of n production factors by $z = (z_1, z_2, \ldots, z_n) \in \boldsymbol{R}^n$, the price of product by $p \in \boldsymbol{R}_+$, and

[7] As Makowski and Ostroy (1992) suggest, the values of the production factors are considered to be determined as Shapley values.

the prices of production factors by $q = (q_1, q_2, \ldots, q_n) \in \boldsymbol{R}^n$. A producer is characterized by the differentiable production function denoted by $y = f(z)$, which satisfies the law of decreasing marginal productivity for every production factor, and is homogeneous of degree one and therefore concave. Given a price system $(p^*, q^*) \in \boldsymbol{R}^{n+1}$, the producer chooses his/her production $(y, z) \in \boldsymbol{R}^{n+1}$ so as to maximize his/her profit $p^*y - q^* \cdot z$. Since $p^*y - q^* \cdot z$ and f are concave functions, a producer equilibrium (y^*, z^*) exists by the Kuhn-Tucker theorem (Sundaram, 1996). It follows that:

(MPT1) $$q_h^* = p^* \frac{\partial f}{\partial z_h}(z^*), \quad \text{for every } h \in \{1, 2, \ldots, n\}.$$

By Euler's theorem,

(MPT2) $$y^* = \sum_{h=1}^{n} \frac{\partial f}{\partial z_h}(z^*) z_h^* \quad \text{at an equilibrium production } (y^*, z^*).$$

Wicksteed (1894/1992, 89) states that "under ordinary conditions of competitive industry, it is sensibly or approximately true that if every factor of production draws a remuneration determined by its marginal efficiency or significance, the whole product will be exactly distributed."

Wicksteed proved the exhaustion theorem in the theory of competitive producer. This fact characterizes the implication of his theorem. First, as Flux (1894) pointed out, the exhaustion theorem consists of two theorems, namely, (MPT1) and Euler's theorem. It means that Wicksteed's net contribution is just to apply Euler's theorem to the theory of competitive producers. Second, when a production function is homogeneous of degree one, the output of product maximizing profit may be 0 or infinite for an arbitrarily given price system. If so, the first theorem (MPT1) is not proved, because the equilibrium production is a corner solution. This problem should be solved in the theory of competitive markets as is shown by McKenzie (1959). Third, note that a producer theory assumes a price system to be determined in the competitive equilibrium of a market economy. Wicksteed's exhaustion theorem merely characterizes the producer's choice criterion. The demand functions for production factors are derived from (MPT1), but the price determination is not. In order for marginal productivity theory to be a distribution theory, it must be a theory which determines the prices and inputs of all the production factors. In this sense, Wicksteed's exhaustion theorem is not self-contained without a theory of a competitive market, or unless it presupposes that theory.

4. WALRAS AND THE OLD QUARREL OVER THE EXHAUSTION THEOREM

Walras constructed a general equilibrium theory with the support of his colleagues (Jaffé, 1964, 1965), but his theory had several defects in the producer theory and the proof of existence of a competitive equilibrium of a market economy. The old quarrel on the exhaustion theorem played a significant role in revising Walras' producer theory and completing his theory of competitive markets.[8]

[8] See Jaffé (1964) about the detail of the old quarrel.

Walras' general equilibrium theory consists of the theories of consumers, producers, and market equilibrium. He had an idea of a general equilibrium system from the textbook on mechanics *Éléments de statique* published in 1803 by Louis Poinsot (Jaffé, 1965, L. 1483). He consulted with Piccard on the problem of maximizing the consumer's utility, and with Amstein on that of minimizing the producer's cost. In 1872 Piccard answered Walras by explaining a solution in terms of elementary graphical presentation (Jaffé, 1965, L. 211). Walras understood Piccard's explanation and utilized it in the first edition of his *Éléments*. In 1877 Amstein answered Walras by explaining a solution using a Lagrangian multiplier method (Jaffé, 1965, L. 364). Amstein's explanation was, however, too technical for Walras to understand. Walras did not utilize it in the first edition of his *Éléments*, instead assuming a production technology of constant input-output coefficients. In 1894 Pareto (1894/1982) solved the problem of determining the coefficients of production. Then, having discussed the problem with Pareto, Walras tried again to construct a producer theory in which the coefficients of production are determined, and hence studied Wicksteed's *Essay*.

In his theory, Walras assumes the condition of free competition equilibrium, where a producer makes neither profit nor loss, $p^*y^* = q^* \cdot z^*$ holds at the equilibrium $((y^*, z^*), (p^*, q^*)) \in \boldsymbol{R}^{2n+2}$. Walras immediately noticed that the exhaustion theorem could be derived from the conditions of producer equilibrium and free competition equilibrium. Walras supposed that the first proposition of the exhaustion theorem (MPT1) was derived from producer's cost minimization under the constraint of production technology, and the second proposition (MPT2) was derived from (MPT1) and the condition of free competition equilibrium.

From the theoretical point of view, two points must be noted. First, as Pareto (1901–02/1955) suggested, (MPT1) is derived not from the cost minimization by producers, but also from profit maximization. In this sense, Walras' demonstration of (MPT1) was not sufficient. Walras' producer theory was revised by Barone (1895/1965) in the review of Wicksteed's *Essay*. Second, Walras' reasoning is apparently valid, but it does not make sense if a free competition equilibrium does not exist. Walras did not suggest any condition for the existence of producer's equilibrium and of competitive equilibrium of a market economy. Walras presumed that he had proved the existence of competitive equilibrium of a market economy by confirming that the number of demand-supply equilibrium equations is equal to that of relative prices as unknowns. Among his contemporaries, including mathematicians and scientists, his reasoning that a system of equations with the same number of equations as there are unknowns can be solved was common, and the problem of existence of the market equilibrium was an open question. The existence of market equilibrium is proved from the properties of the economic environment characterizing the consumer's preference and the producer's technology. It is necessary to assume not only decreasing marginal productivity for every production factor, but also, for example, the linear homogeneity of the production function. Walras did not refer to any properties of the production function. Instead, he presumed that his exhaustion theorem was more general than Wicksteed's because it does not assume any properties of production functions.

Since Walras was not familiar enough with mathematics to solve the optimization problem, he could not prove his claim by himself. He first consulted Pareto about his idea, but Pareto was not so interested in it and suggested that Walras read Barone's review of Wicksteed's *Essay*. Barone (1895/1965) had written this review to revise Walras' producer theory by taking account of Pareto's advice, and tried to prove the exhaustion theorem in the theory of a competitive market and to show that Walras' exhaustion theorem was more general than Wicksteed's. He submitted the review to *Economic Journal*, but the editor, Edgeworth, rejected it. Walras was gradually getting angry at this, and criticized the English theory of distribution in the third edition of his *Éléments* Walras (1896/1954), which is rewritten as the 36^{th} lesson of the definitive edition Walras (1874–77/1952). Eventually, Barone abandoned his support of Walras and co-authorship with him (Jaffé, 1964, 90–91). There was no response from the English economists.[9]

Based on the producer's profit maximization constrained by the production function, Wicksteed proved the exhaustion theorem from the properties of the production function. Barone proved it under the condition that the price of product is equal to the marginal cost and the average cost at the free competition equilibrium, and did not refer to any properties of production functions. Wicksell's response seems to address the point of the old quarrel. Wicksell (1902/1958) later states,

> My opinion was confirmed by the fact that I had already arrived at the same result independently of Wicksteed. Moreover, the criticism of Wicksteed which Walras put forward in the third edition of his *Éléments d'économie politique pure*, after consulting the Italian economist Enrico Barone, seemed to me *a priori* incorrect, since Walras assumes it to be a self-evident fact that the cost price and the sales price of the goods must be the same under free competition; ···
>
> I have found on further reflection, however, that on this point I did Walras — or rather his collaborator, Barone — an *injustice*, and that the law of marginal productivity actually has a far greater field of application theoretically than either Wicksteed or I had hitherto imagined.

Wicksell (1902/1958) showed that the exhaustion theorem is valid if the production function is homogeneous of degree one and exhibits first increasing and then decreasing returns to scale, has and hence has a U-shaped average cost curve. This supports Walras and Barone's claim that they proved the exhaustion theorem within the more general framework of general equilibrium theory.

As we have seen in Section 2, if the total production function of the economy is homogeneous of degree one, the exhaustion theorem is established for the equilibrium total production (Debreu, 1959, 5.7 (1), 88, n.1), and therefore, it is also valid for every individual producer. Any individual production function is acceptable provided the

[9] In English distribution theory, the laws of returns to scale were the key concept for their theories to be coherent. However, Walras and Barone never mentioned to them. Moreover, Edgeworth (1904, 181–83) disagreed with the assumption that the production function is homogeneous of degree one. These seem to be reasons why English economists were silent for Walras' critics.

total production function of the economy is homogeneous of degree one. This is the implication of Walras' exhaustion theorem, and supports his claim. Having explained Barone's reasoning, Walras states that "M. Barone deduced this proposition with logical rigour from my theory of economic equilibrium. Mr. Wicksteed, however, fell short of establishing it for the more general case and would have been better inspired if he had not made such efforts to appear ignorant of the works of his predecessors" (Walras, 1896/1954, 495).

However, in order to prove the exhaustion theorem constructively based on the optimization behavior of every individual producer, it is necessary to assume a certain property of individual production functions so that the total production function of the economy is homogeneous of degree one. Wicksell (1902/1958) assumed an individual production function exhibiting increasing returns to scale at the smaller end of the scale and decreasing returns at the larger end. Strictly speaking, Wicksell's condition does not satisfy the concavity for production functions, which is a sufficient condition for producer's optimization. Osana (1987) reconstructs Walras' marginal productivity theory consistently by assuming that production functions exhibit constant returns to scale at the smaller end of the scale and decreasing returns at the larger end. It has been emphasized that the linear homogeneity is not necessary for exhaustion in long-run competitive equilibrium, if the condition such that the price of product = the marginal cost = the average cost at an equilibrium production is assumed. It is valid for individual production functions, but it is necessary to assume the linear homogeneity of total production function of the economy (Osana, 1987, 10).

5. PARETO'S OBJECTION IN THE LIGHT OF DUALITY THEORY

Pareto generalized the concept of utility functions in general equilibrium theory from being separable and additive to being ordinal. He also generalized the concept of production technology from the constant input-output coefficients of production to the production function from which the coefficients of productions are determined. Pareto (1897/1964, 717, n.2) knew Euler's theorem and proved the exhaustion theorem under the condition that the production function is homogeneous of degree one. However, Pareto (1909/1966, 631–39) assumed a production function where some of the factors of production are variable and others are fixed (Pareto, 1909/1966, 636), and therefore the production function cannot be homogeneous of degree one.

Pareto assumed a free competition equilibrium, defining the concept by the conditions of market equilibrium, no profit, and efficient production. With regard to the allotment of the output to individual producers, Pareto states:

> The question of the division of quantities among the enterprises remains to be examined (V, 78). If an enterprise produces q_z of Z , and increases its production by δq_z , the cost of production of Z , will vary by a certain amount, which we must set equal to zero if the enterprise wants to have a minimum cost of production. Thus we will have the equation

$$0 = \frac{\partial a_z}{\partial q_z} + p_b \frac{\partial b_z}{\partial q_z} + \cdots . \tag{126}$$

There will be other similar equations, one for each enterprise, and they will determine the division of production. (Pareto, 1909/1966, §107)

Equation (126), together with equation (D) in §83, (121) in §103, and (123) in §104, means that every individual producer chooses his/her production so as to minimize his/her average cost. It follows that the price is equal to the marginal cost and the average cost at an equilibrium production. Pareto did not however put forward any properties of individual production functions compatible with the free competition equilibrium.[10]

Given a product output $\bar{y}$, consider the problem of cost minimization subject to a production function $\bar{y} = f^S(z_1, \ldots, \bar{z_h}, \ldots, z_n)$ with a fixed production factor h, and the problem of cost minimization subject to a production function $\bar{y} = f(z_1, \ldots, z_n)$ without a fixed production factor. We then have one cost function with a fixed production factor h, $c^S(q_1, \ldots, q_n, \bar{y}, \bar{z_h})$, and one without a fixed factor, $c(q_1, \ldots, q_n, \bar{y})$. We can consider $f(\cdot)$ to be the production function of the whole industry, and $f^S(\cdot)$ to be that of an individual producer. We can further consider $c(\cdot)$ to be the cost function of the entire industry, and $c^S(\cdot)$ to be that of the individual producer. The envelope theorem characterizes the relationship between these cost functions.[11] As in Walras' free competition equilibrium, the exhaustion theorem may be established and the production function of the industry $f(\cdot)$ is homogeneous of degree one. Then, by duality theory, the average cost curve of the industry AC is horizontal, and the average cost curve of an individual producer AC^S is tangential to it. Even if a producer has a fixed production factor, the producer can choose the input of all the variable production factors to minimize average cost. Otherwise, a producer can do nothing but exit the market. Thus, even if an individual production function $f^S(\cdot)$ has a fixed factor and is not homogeneous of degree one, the exhaustion theorem can be established for every individual producer in action if the production function $f(\cdot)$ of the industry is homogeneous of degree one. Thus, as long as we assume a free competition equilibrium and there exists a competitive equilibrium in the market economy, Pareto's theory implies the exhaustion theorem.

In modern microeconomics, the envelope theorem is considered to characterize the relationship between short-run cost functions of individual producers and the long-run cost function of the industry. Since the production function without a fixed production

[10] Schultz (1929, 1932) and Hicks (1932b,a) had a disagreement about Pareto's objection to marginal productivity theory, and Georgescu-Roegen (1935–36) put forward an interpretation of Pareto's theory to resolve it. His interpretation is essentially the same as Wicksell's idea.

[11] The envelope theorem states:

THEOREM 3. *Let μ^S be the Lagrangian multiplier of the cost minimization problem with a fixed production factor h, and μ the Lagrangian multiplier of the cost minimization problem without a fixed production factor. For $y = \bar{y} > 0$, if $c^S(q_1, \ldots, q_n, \bar{y}, \bar{z_h}) = c(q_1, \ldots, q_n, \bar{y})$, then* $\mu^S = \frac{\partial c^S(q_1, \ldots, q_n, \bar{y}, \bar{z_h})}{\partial y} = \frac{\partial c(q_1, \ldots, q_n, \bar{y})}{\partial y} = \mu$.

factor should be interpreted as long-run, the exhaustion theorem should be considered to be established for a long-run free competition equilibrium (Makowski (1980), Mas-Colell et al. (1995, 670–73)). As we have shown in section 2, a complete description of the marginal productivity theory is given by Osana (1987).

6. AUTHORSHIP OF MARGINAL PRODUCTIVITY THEORY

In previous sections, we interpreted the theories of Wicksteed, Walras, Barone, Pareto, and Wicksell on the basis of our interpretation of marginal productivity theory in section 2, and can now establish the authorship of the exhaustion theorem in marginal productivity theory and coordinate their contributions to it.

Wicksteed put forward and proved the exhaustion theorem in the simplified classical theory of distribution, in which the values and the inputs of production factors are determined. He then extended it to the theory of competitive producers, where a price system is given. However, when the production function is homogeneous of degree one, a producer's equilibrium may be incompatible with an arbitrarily given price system. In order for the exhaustion theorem to be meaningful, it is necessary to prove it in the theory of competitive markets, and it follows that Wicksteed's theory is incomplete. In spite of this defect, Wicksteed alone has a claim on the first priority of the authorship of the exhaustion theorem. The other economists in the old quarrel have gradually contributed to extending and generalizing Wicksteed's exhaustion theorem towards a theory of long-run competitive or perfectly competitive markets.

Walras constructed the framework of general equilibrium theory and defined a notion of free competition equilibrium, but some of the details of his theory contained defects. Pareto revised the defects of Walras' theory of competitive producers, and Barone put forward the exhaustion theorem based on the condition of free competition equilibrium, namely that the product price is equal to the marginal cost and the average cost. We cannot say that Walras and Barone completed marginal productivity theory, because they did not realize that a certain property of production functions was necessary for the exhaustion theorem to be compatible with the theory of competitive markets.

We can also interpret Walras' exhaustion theorem in the McKenzie model (McKenzie, 1959). Walras' exhaustion theorem can be established if the total production function of the economy is homogeneous of degree one, while any individual production functions compatible with this are acceptable. In this sense, Walras' marginal productivity theory is more general than Wicksteed's. It should be noted that the theoretical results that clarified the implication of Walras' exhaustion theorem were unknown to economic theorists until the 1950s, when Debreu (1959) and McKenzie (1959) proved it in a general economic environment. Debreu (1959, 88, n.1) notes that the existence theorem (1) of 5.7 in Debreu (1959, 83–84) was modified according to a suggestion from H. Uzawa, a mimeograph (Uzawa, 1956) and private correspondence, replacing Arrow and Debreu (1954)'s assumption that "every Y_j is convex" with "Y is convex", where Y_j is the production set of producer j and Y that of the economy.

Marginal productivity theory should be based on the theory of individual producers compatible with the the free competition equilibrium, and should assume properties of individual production functions compatible with the total production of the economy. Pareto put forward the condition of producer's profit maximization compatible with the free competition equilibrium, without showing what property of technology was compatible with that equilibrium. Wicksell put forward a concrete property of individual production functions necessary for the exhaustion theorem to be proved in a theory of competitive market. Wicksell's condition does not satisfy concavity of production functions, and therefore it is insufficient to prove the existence of producer equilibrium.

Pareto was not convinced of the validity of exhaustion theorem, because he assumed that some production factors are variable and others are fixed, and therefore the individual production functions cannot be homogeneous of degree one. As long as we assume a free competition equilibrium, the exhaustion theorem can be established under certain adequate conditions. This was not proved until duality theory (Shephard, 1953) provided the envelope theorem and le Châtelier principle in the 1950s.

Thus, no economist in the old quarrel has the right to claim sole authorship of marginal productivity theory, although all of them have contributed to it. We can interpret them as developing the framework for marginal productivity theory by extending and generalizing the economic environment. It was the objective of later microeconomic theory to define a long-run competitive equilibrium in the Arrow-Debreu economy, and to characterize the relationship between a long-run competitive equilibrium in the Arrow-Debreu economy and a competitive equilibrium in McKenzie long-run economy (Osana, 1987).

7. CONCLUDING REMARKS

Stigler (1941, 320) points out that "Wicksteed's solution is the preferable one, in the writer's opinion, because – at the level of analysis to which it is appropriate – it is informative, yet based on simpler assumptions." Jaffé (1964, 102) concludes that "the quarrel was, therefore, not over the theory of Distribution, but over the theory of Production." We add that the old quarrel was also over the theory of competitive markets. The essence of the exhaustion theorem exists not in the marginal law of distribution but in the compatibility of the producer's optimization and the characterization of the long-run competitive equilibrium (Osana, 1987). Makowski (1980) and Ostroy (1980) consider the no-surplus condition as a characterization of perfectly competitive equilibrium. In the light of modern microeconomic theory, the solution given by Walras, Barone and Pareto is certainly more general and self-contained than Wicksteed's.

Distribution theory is considered to be a theory in which the prices (or values) and inputs of production factors are determined. There can be several alternatives to marginal productivity theory other than the classical theory of distribution or a theory of competitive markets (Debreu, 1959; McKenzie, 1959). The exhaustion theorem is established in the Robinson Crusoe model of Wieser (1889/1893), and in the theory of Shapley values (Makowski and Ostroy, 1992). It can also be proved in a Nash equilibrium of non-cooperative games.

As Stigler (1941, 320) further points out, some economists presupposed the production technology that makes the exhaustion theorem obvious. Walras (1874–77/1952) and Wieser (1889/1893) assumed the fixed input-output coefficients of production, which is now called a Leontief System. The Leontief system is characterized by the non-substitution theorem. Let $(y_1, y_2, \ldots, y_H, l)$ be the total production, where l is an input of labor, and $a_1, a_2, \ldots, a_H$ are constants, and suppose that individual production functions are differentiable. According to Samuelson (1951), the non-substitution theorem is as follows:

THEOREM 4. *If the production technology satisfies the following conditions,* (1) *Labor is the sole primary production factor which is not produced from the other commodities,* (2) *There is no joint production,* (3) *Every individual production function is homogeneous of degree one, then the production structure is* $a_1y_1 + a_2y_2 + \cdots + a_Hy_H = l$.

If the non-substitution theorem is valid in an economic environment where there are several primary production factors, then the production structure is the same as that of Walras and Wieser. This seems to suggest that the production technology presumed by Walras and Wieser is very close to that supposed by the exhaustion theorem.

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