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## THE FIXED FACTOR PROPORTIONS MODEL OF PRODUCTION AND TRADE

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**Abstract:** This paper reviews the model of production and trade with fixed input coefficients and more than a single factor of production, a model that may be underappreciated. This “fixed factor proportions” model is a bridge between the constant cost model with one input and the factor proportions model with two inputs. The direction of trade is determined by differences in technology or differences in factor abundance. With the same number of factors and products, the factor proportions theorems are identical. The paper also presents assumptions sufficient for tractable comparative static results with different numbers of factors and products.

**Key words:** factor proportions, production, trade, fixed inputs.

**JEL Classification Number:** F11.

Complete specialization and a linear production frontier characterize the classical constant cost trade model based on the writings of Ricardo (1817). With the single factor of production and fixed unit inputs, opportunity cost or technology determines the direction of trade. Chipman (1965), Morishima (1989), Maneschi (1992), and Ruffin (2002) extend and clarify the constant cost model.

Partial specialization and a concave production frontier characterize the factor proportions model of Heckscher and Ohlin (Flam and Flanders, 1991). The factor proportions model has cost minimization between inputs in neoclassical production functions. The direction of trade is determined by differences in factor endowments. Samuelson (1953), Chipman (1966), and Jones and Scheinkman (1977) formalize and extend the factor proportions model.

This paper reviews the “fixed factor proportions” model that combines the fixed unit inputs of the constant cost model with the multiple inputs of the factor proportions model. This FFP model has appeared in the literature but may be underappreciated. The

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direction of trade is determined by differences in technology or factor abundance. Jones (1973) develops the FFP model with two factors, two goods, and inequality employment constraints.

The major advantage of the FFP model is the simpler derivation of the factor proportions trade theorems due to the lack of substitution. Algebraic comparative static results are identical to the factor proportions model except that price changes have no output effects. Input substitution leads to output adjustments in the factor proportions model. The FFP model is an immediate run when factor prices adjust to price changes while output adjustments require changes in inputs and a longer time to adjust.

The FFP model is introduced in the first section followed by a section that reconsiders opportunity cost and comparative advantage. The algebraic FFP model is presented in the third section. The fourth section develops assumptions that allow solutions of “uneven” FFP models with different numbers of factors and goods. The model with more products than factors assumes arbitrarily small markup pricing for one product. The model with more factors than products assumes arbitrarily small substitution between any pair of inputs. A final section compares the FFP model to Ruffin’s (1988) Ricardian factor endowment model with fixed unit inputs, each factor producing independently, and trade occurring between different factors residing in different countries.

#### 1. THE FIXED FACTOR PROPORTIONS FFP MODEL

Consider the FFP model in Figure 1 with Leontief right angled isoquants for factors  $v_1$  and  $v_2$  and outputs  $x_1$  and  $x_2$ . The unit value isoquants in Figure 1 represent one unit of numeraire at  $x_j = 1/p_j$ . Scale outputs to  $p_j = 1$ , and competitive pricing and factor mobility imply the single isocost line  $p_j = c_j = 1 = a_{1j}w_1 + a_{2j}w_2$ .

Full employment determines outputs  $x_j$  along expansion paths with product 1 using factor 1 intensively,  $a_{11}/a_{21} > a_{12}/a_{22}$ . Diversified production requires an endowment point  $E = (v_1, v_2)$  inside the production cone under the condition  $a_{11}/a_{21} > v_1/v_2 > a_{12}/a_{22}$ .

Changes in  $E$  inside the cone alter outputs in a linear fashion according to factor intensity. An increase in the endowment of factor  $v_1$  raises  $x_1$  and lowers  $x_2$  as both factors leave industry 2 and outputs adjust along the Rybczynski (1955) line identical to the factor proportions adjustment developed by Kemp (1964). Increased  $v_1$  leads toward specialization in product 1 and beyond the expansion path becomes redundant in the factor proportions problem of Eckaus (1955). The Heckscher-Ohlin theorem follows for two countries with different endowments as each country exports the product using its abundant factor intensively.

Local and global factor price equalization also follow in the FFP model. A changing endowment point inside the production cone in Figure 1 with prices constant has no effect on factor prices, identical to the factor proportions model.

In spite of the lack of substitution, Stolper-Samuelson (1941) effects of price changes on factor prices in the FFP model are also identical to the factor proportions model. Price changes shift the unit isocost line and factor prices adjust. The slope of the isocost

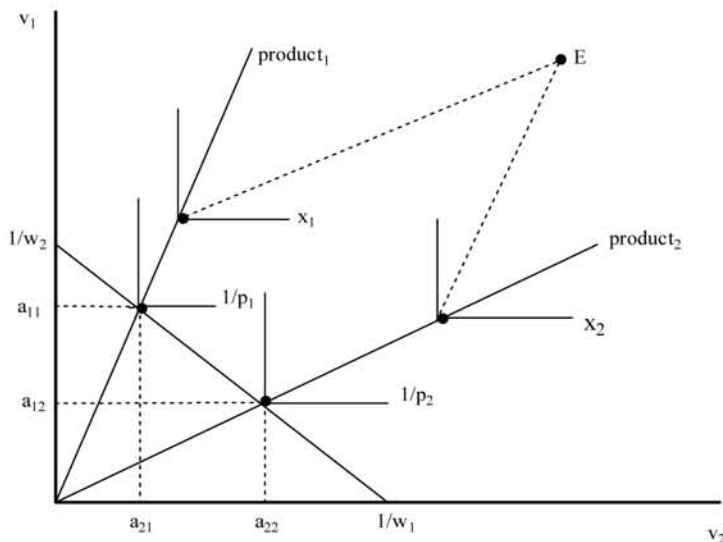


Figure 1. Production in the FFP Model.

line is the relative factor price  $w_2/w_1$ . An increase in the price of product 1 shifts the unit value isoquant  $1/p_1$  toward the origin representing less physical output. The result is a lower slope  $w_2/w_1$  in an adjustment that is algebraically identical to the factor proportions model with convex isoquants.

Two countries that differ only in endowments in the FFP model export the product that uses their abundant factor intensively, the Heckscher-Ohlin theorem. Two countries with identical endowments but different technologies export according to their technological advantage or factor intensity.

Figure 2 illustrates the equilibrium of production and trade. The autarky relative price  $p_2/p_1$  is determined by the marginal rate of substitution of the indifference curve passing through production point A. The country imports product 2 if its world relative price is lower at the terms of trade  $p_2^*/p_1^*$ . Home consumption with free trade at point  $C_1$  has higher utility than autarky consumption at point A.

A tariff does not alter production but utility maximizing consumers face tariff distorted prices. The relative price of imports rises to  $p_2^*(1+t)/p_1^*$  in Figure 2 where  $t$  is the tariff rate. There is decreased consumption of the imported product along the terms of trade line to point  $C_2$  where the marginal rate of substitution equals the distorted domestic relative price. The tariff lowers both utility and real income in autarky prices. It may seem odd but the output distortions of the tariff in the factor proportions model imply larger net losses than in the FFP model where at least production is not affected by the tariff. In contrast, the associated partial equilibrium deadweight loss of a tariff is

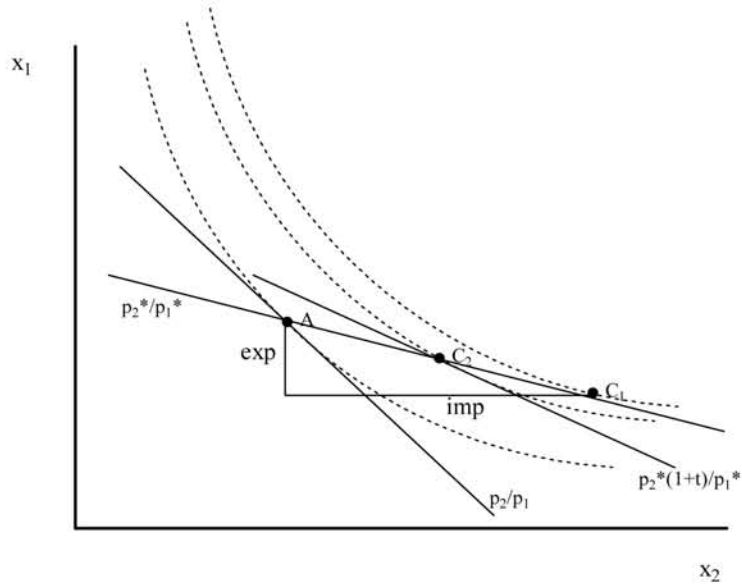


Figure 2. Trade in the FFP Model.

larger in the FFP model due to the perfectly inelastic domestic supply and the lack of an offsetting gain in producer surplus.

## 2. OPPORTUNITY COST IN THE FFP MODEL

In the constant cost model, low opportunity cost of the single input is equivalent to comparative advantage and leads to exports. In the FFP model with two inputs, however, low opportunity cost of a factor implies imports.

To define factor intensity, full employment of the two inputs  $v_i = a_{i1}x_1 + a_{i2}x_2$  implies output levels  $x_1 = (a_{22}v_1 - a_{12}v_2)/b$  and  $x_2 = (a_{11}v_2 - a_{21}v_1)/b$  where  $b \equiv a_{11}a_{22} - a_{12}a_{21}$ . The factor intensity ranking

$$a_{11}/a_{21} > a_{12}/a_{22} \quad (1)$$

implies factor 1 (2) is intensive in product 1 (2) and  $b > 0$ . Focus on technology differences between countries and suppose there are identical endowments as in Figure 3 with expansion paths for each country spanning the identical endowment point  $E \equiv (v_1, v_2) = E^* \equiv (v_1^*, v_2^*)$ . With no loss of generality rescale factor 1 to  $a_{11} = 1$  and product 1 to  $a_{11}^* = 1$ . Similarly rescale factor 2 and product 2 to  $a_{22} = a_{22}^* = 1$ .

Suppose product 1 uses factor 1 intensively and the foreign country has an intensity bias in factor 1,

$$a_{11}^*/a_{21}^* > a_{11}/a_{21} > a_{12}^*/a_{22}^* > a_{12}/a_{22}. \quad (2)$$

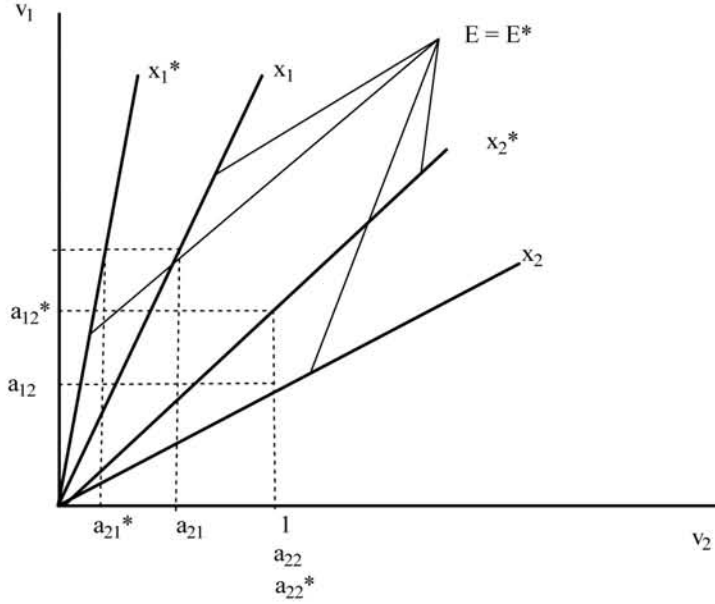


Figure 3. Technology Induced Trade in the FFP Model.

The other possibility is considered below. Rescaling implies  $1 > a_{12}^* > a_{12}$  and  $1 > a_{21} > a_{21}^*$ . The home output ratio  $x_1/x_2 = (1 - a_{12})/(1 - a_{21})$  must be higher than the foreign output ratio  $x_1^*/x_2^* = (1 - a_{12}^*)/(1 - a_{21}^*)$ . Given identical homothetic preferences in the two countries, the home country exports product 1 although both factors have higher opportunity costs for that product,  $a_{i1}/a_{i2} > a_{i1}^*/a_{i2}^*$ .

Exports are associated with higher opportunity cost for a factor, exactly the opposite from the single input model. The reason is that higher factor intensity consumes more of the factor and implies lower relative output. As pointed out in communication by Yutaka Horiba, the direction of trade with identical factor intensities would be determined by factor intensities of the other product.

One production cone may also span the other making the output ratio between countries ambiguous. Given the rescaling suppose the home cone spans the foreign cone,  $1 > a_{12} > a_{12}^*$  and  $1 > a_{21}^* > a_{21}$ . Output ratios and exports would then depend on degrees of factor intensity. A country would more likely export a product using a factor intensively if there were less intensity of that factor in the other product. In the limiting case there is no trade at all as when  $(a_{12}a_{21}a_{12}^*a_{21}^*) = (.7 .4 .8 .6)$  implying  $x_1/x_2 = x_1^*/x_2^*$ .

## 3. THE ALGEBRAIC FFP MODEL

The comparative static model clarifies adjustments in the FFP model and establishes the foundation for higher dimensional models. Competitive pricing of product  $j$  implies  $p_j = a_{1j}w_1 + a_{2j}w_2$  and exogenous world prices  $p_j$  imply the static solutions  $w_1 = (a_{22}p_1 - a_{21}p_2)/b$  and  $w_2 = (a_{11}p_2 - a_{12}p_1)/b$ . Positive factor prices require factor intensity span the relative price,  $a_{11}/a_{21} \geq p_1/p_2 \geq a_{12}/a_{22}$ .

Differentiate the full employment and competitive pricing conditions to find  $dv_i = a_{i1}dx_1 + a_{i2}dx_2$  and  $dp_j = a_{1j}dw_1 + a_{2j}dw_2$ . Combine these four equations into the comparative static system

$$\begin{pmatrix} 0 & 0 & a_{11} & a_{12} \\ 0 & 0 & a_{21} & a_{22} \\ a_{11} & a_{21} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \end{pmatrix} \begin{pmatrix} dw_1 \\ dw_2 \\ dx_1 \\ dx_2 \end{pmatrix} = \begin{pmatrix} dv_1 \\ dv_2 \\ dp_1 \\ dp_2 \end{pmatrix}. \quad (3)$$

The determinant  $\Delta = b^2$  of this block recursive matrix is positive. Cramer's rule leads to partial derivative solutions for endogenous  $dw_i$  and  $dx_j$  with respect to exogenous  $dv_i$  and  $dp_j$ .

Endowment changes do not affect factor prices,  $\delta w_i/\delta v_k = 0$ , the factor price equalization result. Any substitution between inputs would enter the upper left quadrant of the matrix but would be cancelled by zeros in the lower right quadrant in the cofactors of  $\delta w_i/\delta v_k$  terms.

Price changes do not affect outputs  $\delta x_j/\delta p_m = 0$  due to the absence of substitution. Outputs cannot change given their fixed input mix and full employment. An arbitrarily small degree of substitution in the upper left quadrant of the system matrix would, however, lead to output adjustments.

The other partial derivatives in (3) are reciprocal,

$$\begin{aligned} \delta w_1/\delta p_1 &= \delta x_1/\delta v_1 = a_{22}/b > 0 \\ \delta w_1/\delta p_2 &= \delta x_2/\delta v_1 = -a_{21}/b < 0 \\ \delta w_2/\delta p_1 &= \delta x_1/\delta v_2 = -a_{12}/b < 0 \\ \delta w_2/\delta p_2 &= \delta x_2/\delta v_2 = a_{11}/b > 0. \end{aligned} \quad (4)$$

These terms are identical to the factor proportions model with any degree of substitution consistent with the point of Thompson (1995) that factor intensity plays a more critical role than substitution in the general equilibrium production adjustment process.

Price changes affect factor prices but have no effects on outputs. A higher  $p_1$  increases demand for its intensive factor 1 and raises  $w_1$ . Factor price adjustments are magnified effects of price changes identical to Jones (1965). With substitution factor 1 would be bid into industry 1 and its output would expand. Output adjustments to price changes in the factor proportions model are independent of factor price adjustments.

Endowment changes lead to output adjustments but not factor price adjustments. Firms hire inputs in their ratio with one industry contracting as the other expands in

a linear Rybczynski adjustment. An increase in the endowment of factor 1 causes industry 1 to absorb all of that additional input and attract both factors from the other industry. Output changes for  $dv_1 = 1$  are  $dx_2 = -a_{21}/b$  and  $dx_1 = a_{22}/b$  implying the Rybczynski line slope  $dx_1/dx_2 = -a_{22}/a_{21}$ .

Adjustment to a changing endowment must involve temporary factor price changes that induce factor movements between industries. An increase in  $v_1$  raises the marginal productivity and return to factor 2 in industry 1 explaining its movement to industry 1. The return to factor 1 is temporarily higher in industry 1 with its marginal productivity stimulated by the incoming factor 2. Both factor prices return to exactly their original levels as output adjustment absorbs the endowment change.

Higher dimensional even FFP models with the same number of factors and products are straightforward applications of the model in (3). Factor intensity becomes difficult to interpret in the usual manner in the high dimensional model with three or more factors and products. For analysis of the  $3 \times 3$  concepts see Thompson ( ).

#### 4. UNEVEN FFP MODELS

If there are more products than factors, the FFP model is under-determined as in the  $1 \times 2$  Ricardian model with only labor input. Any output combination is consistent with full employment along the linear production frontier. There is no solution to the underdetermined production adjustment model due to its zero determinant. In the  $2 \times 3$  model of an expanded comparative static model similar to (3) the null matrix in the lower right hand corner dominates the system matrix.

One way to close the model is to allow markup pricing as a function of output  $\mu_j(x_j)$  as in Thompson (2003) leading to the pricing condition  $p_j = a_{1j}w_1 + a_{2j}w_2 + \mu_j(x_j)$ . Euler's theorem with constant returns implies the competitive pricing condition  $p_j = a_{1j}w_1 + a_{2j}w_2$ . Markup pricing reflected by the  $\mu_j(x_j)$  term must involve variable returns or a distortion in either a factor market or the product market. Differentiate to find  $dp_j = a_{1j}dw_1 + a_{2j}dw_2 + \mu'_j dx_j$  where  $\mu'_j$  is the derivative of  $\mu_j(x_j)$ . A  $\mu'_j$  term for any product  $j$  in the lower right quadrant of the system similar to (3) leads to a nonzero determinant and comparative static solutions. The degree of markup pricing can be arbitrarily small  $\mu'_j \rightarrow 0$ , and  $\mu'_j$  can be constant.

As an example consider linear inverse demand in sector 1,  $p_1 = a - bx_1$ . The firm is a price searcher, introducing a distorted product market. Given cost minimization, average cost and marginal cost  $a_{11}w_1 + a_{21}w_2$  equal to marginal revenue  $p_1 - 2bx_1$ . Differentiating, the pricing condition is  $a_{11}dw_1 + a_{21}dw_2 = dp_1 - 2bdx_1$ . Assume competitive pricing in the other two sectors. Factor price equalization holds and the  $\partial w/\partial p$  results depend on factor intensity.

When there are more factors than products, the FFP model is over-determined given fixed unit inputs and arbitrary endowments. As an example, the  $2 \times 1$  model expansion path  $a_{11}/a_{21}$  might not match the endowment point  $v_1/v_2$ . Any substitution, however, would lead to tractable results as substitution terms enter the upper left quadrant of the system matrix in (3).



Table 1. A  $3 \times 3$  Model of Alabama

	$\theta_{iA}$	$\theta_{iS}$	$\theta_{iM}$
<i>K</i>	.661	.401	.428
<i>L</i>	.186	.579	.338
<i>E</i>	.152	.020	.234
	$\partial p_A$	$\partial p_S$	$\partial p_M$
$\partial r$	2.47	0.15	-1.62
$\partial w$	-1.64	1.72	0.92
$\partial e$	-2.15	-2.75	5.90

Table 2. An Aggregated  $2 \times 3$  Model

	$\theta_{iA}$	$\theta_{iS}$	$\theta_{iM}$
<i>L</i>	.186	.579	.338
<i>Q</i>	.814	.421	.662
	$\partial p_A$	$\partial p_S$	$\partial p_M$
$\partial q$	0.05	-1.37	2.32
$\partial w$	-0.04	2.72	-1.67

Substitution terms are output weighted adjustments in unit inputs with respect to factor prices,  $s_{ik} \equiv \sum_j x_j \delta a_{ij} / \delta w_k$ . Suppose there is substitution between factors 1 and 2 with  $s_{12} = s_{21} = s$  in the  $3 \times 2$  model of Thompson (1985). With homogeneity and scaling, own substitution terms can be written  $s_{11} = s_{22} = -s$ . In the comparative static  $\delta w / \delta p$  and symmetric  $\delta x / \delta v$  results, the substitution term  $s$  is factored out of the cofactors. Signs of these Stolper-Samuelson and Rybczynski results are independent of substitution. In the limit with arbitrarily small substitution as  $s \rightarrow 0$ , the comparative static  $\delta x / \delta p$  production possibility terms become large while  $\delta w / \delta v$  terms approach zero. Any degree of substitution between any pair of inputs leads to tractable results in the model with more factors than products.

As an example of models with different numbers of factors and products, start with the data on the Alabama economy for capital *K*, labor *L*, and energy *E* inputs in agriculture *A*, services *S*, and manufactures *M* in Table 1. Data is from the standard sources in the US Departments of Commerce and Energy. Table 1 presents the factor share data and the comparative static elasticities of this  $3 \times 3$  model. Capital has a positive link to agriculture, labor to services, and energy to manufacturing.

Table 2 is a related  $2 \times 3$  model with capital and energy aggregated to input *Q* and uniform markup pricing  $\mu'$  across industries. The  $\delta w / \delta p$  partial derivatives elasticities are identical for any uniform degree of markup pricing. Price in the large labor intensive service sector has the largest wage effect. Price in the small agriculture sector has little impact on factor prices. A higher price of manufactures lowers the wage. As a general

Table 3. An Aggregated  $3 \times 2$  Model

	$\theta_{iS}$	$\theta_{iR}$
$K$	.401	.430
$L$	.579	.336
$E$	.020	.233
	$\partial p_S$	$\partial p_R$
$\partial r$	0.40	0.60
$\partial w$	1.55	-0.55
$\partial e$	-2.97	3.97

warning for aggregating data, there is bias in the wage elasticities even though labor is not involved in the aggregation.

Table 3 presents a derived  $3 \times 2$  model with manufacturing and agriculture aggregated to sector  $R$  and uniform cross price substitution. Comparing labor shares across sectors  $\theta_{LS}/\theta_{LR} = (L_S/L_R)(R_R/R_S)$  where  $L_j$  is labor employed and  $R_j$  the revenue in sector  $j$  given equal wages in the two sectors. Comparing these ratios of factor shares, it follows that  $L_S/L_R = 1.72 > K_S/K_R = 0.93 > E_S/E_R = 0.86$  since the relative revenues  $R_R/R_S$  cancel. The service sector is labor intensive while the rest of the economy  $R$  is energy intensive, with capital in the middle and closer to energy intensity. The intensity link between labor and services is clear in the comparative static results, as is the link between the aggregate sector  $R$  and energy. As the degree of substitution gets arbitrarily small  $s \rightarrow 0$  and the  $\delta x/\delta p$  elasticities become smaller.

## 5. THE RICARDIAN FACTOR ENDOWMENT MODEL

The Ricardian factor endowment RFE model of Ruffin (1988) also integrates concepts of constant cost and factor proportion models but inputs produce outputs independently. Opportunity cost or comparative advantage between industries is critical rather than between countries. Industries employ the factor with a lower opportunity cost and employ the other factor only if demand exceeds ability to produce with the lower opportunity cost input.

Assume product 1 use factor 1 intensively,  $a_{11}/a_{12} > a_{21}/a_{22}$  as in (1). Factor 1 has a lower opportunity cost in product 2. The production frontier model has two flat regions that connect at a hinge with each factor employed in its comparative advantage industry. Production point A in Figure 4 has specialized outputs  $x_1 = v_2/a_{21}$  and  $x_2 = v_1/a_{12}$ . The output of  $x_1$  would increase by  $v_1/a_{11}$  moving from point A to complete specialization, and the absolute value of the slope of that segment is  $a_{12}/a_{11}$ . Similarly the lower section has slope  $a_{22}/a_{21}$ .

Autarky production would take place at point A for a range of preferences that determine the domestic relative price  $p_2/p_1$  under the restriction  $a_{22}/a_{21} > p_2/p_1 > a_{12}/a_{11}$ . With preferences biased toward  $x_1$  the relative price would fall as far as

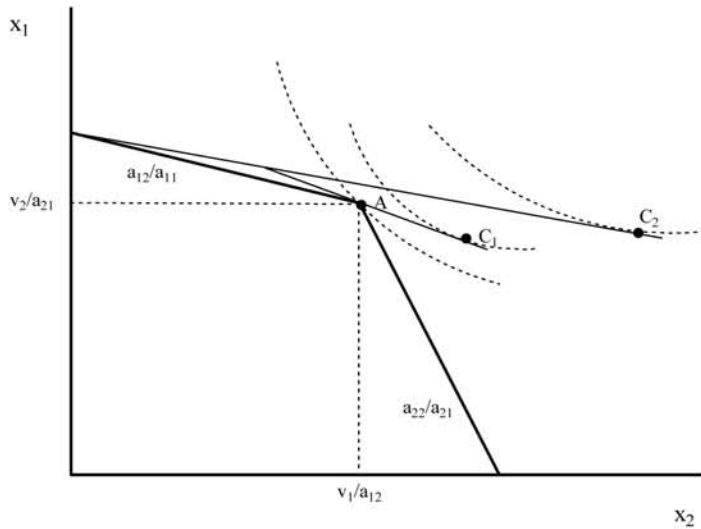


Figure 4. The Missing Link Model.

$p_2/p_1 = a_{12}/a_{11}$  along the upper section of the production frontier. Prices are lower with lower opportunity cost inputs,  $p_1 = w_2 a_{21} < w_1 a_{11}$  and  $p_2 = w_1 a_{12} < w_2 a_{22}$  implying the relative factor price is limited by technology according to  $a_{22}/a_{12} > w_2/w_1 > a_{21}/a_{11}$ .

A move from autarky to free trade does not necessarily alter production. Let  $p_2/p_1$  be the domestic relative price and suppose  $p_2/p_1 > p_2^*/p_1^* > a_{12}/a_{11}$  with a higher exogenous international relative price of  $x_1$  below the opportunity cost of factor 1. The economy exports product 1 trading to point  $C_1$  with higher utility than at point A but there is no change in production.

Factor prices depend on prices. The price of factor 1 is tied to production of the import competing product,  $w_1 = p_2/a_{12}$ . Free trade lowers  $p_2$  to  $p_2^*$  and  $w_1$  falls. With unspecialized trade  $w_1$  would fall while  $w_2$  rises from  $p_1/a_{21}$  to  $p_1^*/a_{21}$ . Prices of the same factor converge across countries but stop short of factor price equalization.

Specialization occurs if the world relative price of product 1 is higher than the opportunity cost of factor 1,  $p_2^*/p_1^* < a_{12}/a_{11}$ . Factor 1 is then more valuable in industry 1 and  $x_1 = v_2/a_{21} + v_1/a_{11}$ . Trade moves consumption to the utility maximization at point  $C_2$ .

Changes in factor endowments shift the hinge point and affect factor prices. An increase in factor 1 increases  $x_2$  but  $x_1$  is unchanged, lowering autarky  $p_2/p_1$  and  $w_1/w_2$  given homothetic demand. If technology is identical between two economies that have identical preferences, endowment differences lead to trade. If  $v_1 > v_1^*$  and  $v_2 < v_2^*$  the two countries could trade with no change in production. This foreign production point

A\* would be northwest of home point A in Figure 4 with the home country exporting product 2 based on its abundant factor 1.

Factor owners effectively trade with each other, the country with a relative abundance exporting that lower opportunity cost product. As in the FFP model, comparative advantage takes on a different meaning than in the single input Ricardian model.

Ruffin (1992) develops the related two country model in which differences in technology, endowments, or preferences determine trade. Two countries with identical endowments and preferences might trade based on opportunity costs between factors within each country as well as opportunity cost of factors across countries. Endowment differences can also lead to trade as in the factor proportions model.

## 6. CONCLUSION

The present fixed factor proportions model provides a link between classical and factor proportions models of production and trade, stressing the importance of factor intensity relative to input substitution. The fixed factor proportions model can be applied to the immediate time period before firms are able to adjust cost minimizing inputs to altered factor prices. Low opportunity cost of an input predicts imports rather than exports as in the single input classical model. Models with more products than factors can assume an arbitrarily small degree of markup pricing, and models with more factors than products can assume an arbitrarily small degree of substitution between any pair of inputs. Regarding applications, the fixed factor proportions model can be simulated with data for factor shares and industry shares alone without estimates of input substitution, and insignificant substitution coefficients would suggest the fixed factor proportions model.

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