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ENDOGENOUS NETWORK EFFECT, QUALITY CHOICE, AND MONOPOLY: A NOTE

Tsuyoshi TOSHIMITSU

School of Economics, Kwansei Gakuin University, Nishinomiya, Japan

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Abstract: We internalize network effects, which are assumed to be exogenously given in prior literature (Lambertini and Orsini, 2001, 2003; Toshimitsu, 2007). To do so, we propose the argument *network function* as one of the firm's strategies that is accompanied by the properties of goods and services in network industries. That is, the *network function* stands for a maneuver of operation that works on all customers and its improvement equally increases their utilities. Employing a vertically differentiated product model with endogenous network effects, we show the implications of a monopolist's choice of *network function* and quality level for social welfare. Compared with the social optimum, the monopolist has an incentive to undersupply a product attached to a less efficient *network function* and to either the over- or under-provision of quality.

Key words: Network industry, Network effect, Quality, Vertically differentiated product, Monopoly.
JEL Classification Number: D42, L12, L15.

1. INTRODUCTION

As digital technology progresses, we observe the remarkable growth of Information and Communications Technology (ICT) industries such as telecommunications, computer hardware and software, cable TV, and broadcasting. Many researchers have already analyzed the market for goods and services that generate network externalities (Katz and Shapiro, 1994; Economides, 1996; Shy, 2002; Koski and Kretschmer, 2004). Network externalities are commonly defined as a general property whereby the utility of each consumer increases with an increase in the total number of consumers purchasing either the same brand or a compatible brand. These studies usually distinguish between direct (communications) network effects and indirect (systems) network effects. In the first case, the utility of an individual consumer increases when there are others with whom to communicate. In the second case, utility depends on the availability of complementary goods, which, in turn, depends on the number of potential buyers.

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E-mail: ttsutomu@kwansei.ac.jp

Accordingly, other users generate a positive effect from particular users. For example, Doganoglu and Crzybowski (2007) find that direct effects have influenced the evolution of the mobile telephony industry in Germany. Furthermore, Clements and Ohashi (2005) empirically analyze indirect network effects in the case of video games in the United States.

In the development of the literature discussing industrial policies and public interventions, e.g., Spence (1975) and Sheshinski (1976), some studies have examined the welfare implications of the provision of quality in the cases of monopoly and duopoly with network effects (Lambertini and Orsini, 2001, 2003, 2005; Shy, 2002; Sappington, 2005; Toshimitsu, 2007). In these studies, the authors have assumed that the weight of network effects associated with a consumer's utility is exogenously given. For example, Lambertini and Orsini (2001, 2003) show that the monopolist provides a higher level of quality than the social optimum, and they prove that an increase in the weight of network effects reduces the level of quality. On the other hand, Toshimitsu (2007) presents that, if the cost function of quality is not related to quantity, there is an underprovision of quality in the monopoly equilibrium compared with the social optimum, and that an increase in the weight of network effects increases the level of quality.

It has been assumed in some prior studies that network effects associated with goods and services are exogenously given (Lambertini and Orsini, 2001, 2003; Toshimitsu, 2007). However, it is not inappropriate to assume that the network effect and the quality grade accompanied by characteristics of a product are significant strategies for firms, particularly, in ICT industries. In this paper, in order to internalize network effects, we assume that the characteristics of a product are composed of quality and network effects, which affect utilities of customers. Therefore, we assume that a firm can decide the weight of network effects as well as the quality grade of a product as its strategies.

We let the weight of network effects be a firm's strategic variable. The evaluation of quality depends on the preference of individual consumers, whereas the weight of network effects is the same for all customers. Hereafter, we call the weight of network effects the argument *network function*. That is, with regard to a communications network effect, an improving network function increases the magnitude of network effects. Thus, as the number of consumers increases, the improvement in network function further increases the utilities of all consumers. For example, we suppose that a firm enriches the network function to make mobile phone functions more convenient for all subscribers at the expense of some costs such as network function-improving investments. In addition, we must consider the level of quality that the firm chooses. Therefore, accounting for the endogenous network effects, we address the implication of the monopolist's choice of the degree of network function as well as the level of quality for social welfare. That is, compared with the social optimum, the monopolist undersupplies a product attached to a lower degree of network function. Furthermore, whether the monopolist chooses over- or under-provision of quality depends upon the nature of the cost function of quality.

The remainder of the paper is structured as follows. Section 2 presents the model. In Section 3, the monopoly equilibrium and the social optimum are compared, and we analyze the welfare implications in the case where both the degree of network function and the level of quality are endogenously decided by the monopolist. Then, Section 4 considers whether the results shown in Section 3 depend on the nature of the cost function. Finally, Section 5 summarizes the results and presents a few remaining issues.

2. THE MODEL

Based on the models of Lambertini and Orsini (2001, 2003), we introduce network effects into a utility function presented in a model of vertical product differentiation (Mussa and Rosen, 1978). That is, there is a continuum of consumers, in $\theta \in [\underline{\theta}, 1+\underline{\theta}]$, $\underline{\theta} \geq 0$. To simplify, consumers are assumed to be uniformly distributed with a density of one in the market. The utility function of consumer θ is given by $u(q, x; \theta) = \theta q + N(x)$, where quality q is modeled as a one-dimensional variable, i.e., $q \in [0, \infty)$ and quantity x represents the proportion of consumers purchasing a product in the market, i.e., $x \in [0, 1]$. Furthermore, $N(x)$ expresses network effects: $N(0) = 0$ and $N'(x) \geq 0$. The magnitude of network effects is assumed to be the same for all customers, although the valuation of quality grade differs between individual customers.

Here, we internalize network effects as a maneuver of the firm's strategies. For example, Swann (2002) generally considers the functional forms of network effects. However, to simplify, the network effect is given by $N(x) = \beta x$, where β is modeled as a one-dimensional variable, i.e., $\beta \in [0, \infty)$, and stands for the marginal efficiency of network effects. We assume that a monopolist or a social planner chooses the magnitude of marginal efficiency of network effects at the expense of some costs, e.g., network function-improving investments. Hereafter, we let the marginal efficiency of network effects, β, be defined as the degree of network function.

A consumer purchases at most either one unit of the product or none. Hence, the net surplus of consumer θ can be expressed as $v = \max\{\theta q + \beta x - p\ ,\ 0\}$. In this case, the index of the marginal consumer who has the same net surplus from purchasing one unit of the product or none is given by $\theta_m = \frac{p-\beta x}{q}$. Therefore, we derive the quantity demanded for the product as follows:

Case (a) $\quad x = \dfrac{q(1+\underline{\theta}) - p}{q - \beta}, \quad \text{if} \quad 1+\underline{\theta} > \theta_m > \underline{\theta}$,

Case (b) $\quad x = 1$, if $\underline{\theta} \geq \theta_m$, and

Case (c) $\quad x = 0$, if $\theta_m \geq 1 + \underline{\theta}$.

Case (a) illustrates partial market coverage in which there are some consumers not purchasing the product. Case (b) shows full market coverage in which all consumers purchase the product. To simplify, let $\underline{\theta}$ to be zero. Hence, the case of partial market coverage holds if and only if $q > p > \beta$. However, if $q > \beta \geq p$, then the case of full market coverage holds. In Case (c), there are no customers in the market. Thus, if $\theta_m \geq 1 \Leftrightarrow p \geq q > \beta$, then it holds that $x = 0$. In what follows, we deal mainly with the case of partial market coverage. Therefore, the corresponding inverse demand function is given by:

$$p = p(q, x, \beta) = q - (q - \beta)x\,. \tag{1}$$

Next, we assume that the total cost of producing x units composed of quality q and network function β is given by:

$$C = C(q, x, \beta) = c(q)x + f(\beta)\,, \quad c' > 0,\ c'' > 0,\ f' > 0,\ f'' > 0\,. \tag{2}$$

Equation (2) implies that the cost of upgrading quality is positively related to quantity, whereas the cost of improving the degree of network function is unrelated to quality and quantity. For example, on the one hand, the monopolist needs to use expensive parts, such as super large-scale integrated circuits, per output to improve the quality grade of pictures and sounds on a mobile phone. On the other hand, the monopolist must undertake investments in facilities to extend the network system that connects to all customers.

Finally, we present the purpose functions of the monopolist and the social planner, respectively. In view of (1) and (2), the profit function of the monopolist is expressed by:

$$\Pi(x, q, \beta) = R(x, q, \beta) - C(x, q, \beta)\,, \tag{3}$$

where $R(x, q, \beta) = \{q - (q - \beta)x\}\,x$. Furthermore, consumer surplus is given by:

$$CS(x, q, \beta, p) = \int_{\theta_m}^{1} (\theta q + \beta x - p)d\theta = \left\{q - \left(\frac{q}{2} - \beta\right)x\right\}x - px\,. \tag{4}$$

From (3) and (4), social surplus is thus represented by:

$$W(x, q, \beta) = CS(x, q, \beta, p) + \Pi(x, q, \beta) = U(x, q, \beta) - C(x, q, \beta)\,, \tag{5}$$

where $U(x, q, \beta) = \left\{q - \left(\frac{q}{2} - \beta\right)x\right\}x$.

In what follows, to look at the endogenous decision of the degree of network function at the expense of network function-improving investments costs, we assume that a monopolist (or a social planer) sequentially makes decisions in two stages: in the first stage, the monopolist (social planner) decides on investments to improve the degree of network function; and in the second stage, given the degree of network function, it chooses the quantity and quality of the product. By backward induction, we derive a monopoly (socially optimal) equilibrium.

3. UNREGULATED MARKET AND SOCIAL PLAN

3.1. *Monopoly Equilibrium*

In the second stage, the monopolist determines both quantity and quality, given the degree of network function. In view of (3), the first-order conditions (FOCs) for profit maximization of the monopoly with respect to quantity and quality are respectively given by:

$$\frac{\partial \Pi}{\partial x} = q - 2(q - \beta)x - c(q) = 0\,, \tag{6}$$

$$\frac{\partial \Pi}{\partial q} = \{1 - x - c'(q)\}x = 0\,. \tag{7}$$

From (6), we obtain $x = \frac{q-c(q)}{2(q-\beta)} = x_M(q,\beta)$, where subscript M denotes the monopolist case. The amount of quantity is determined by the level of quality and the degree of network function. Furthermore, based on (7), we have $1 - x = c'(q)$, of which the LHS is marginal profit with respect to quality and the RHS is marginal cost. We express $q = q_M(x)$, because the level of quality depends on the amount of quantity. Thus, it holds that $\frac{\partial x_M}{\partial q} < 0$, $\frac{\partial x_M}{\partial \beta} > 0$, and $\frac{\partial q_M}{\partial x} < 0$.

In view of (6), the case of partial market coverage holds if and only if $\frac{q+c(q)}{2}$ $(= p_M) > \beta$. However, if $\beta \geq \frac{q+c(q)}{2}$, the monopolist provides the product to all consumers in the market, i.e., $x_M = 1$, and, thus, the price is given by $p_M = \beta$. Taking (7) into account, we obtain the corner solution with respect to the level of quality in the case of full market coverage such as $q_M = 0$. Accordingly, we define the lower bound of quality, i.e., $\underline{q}_M \equiv \left\{q \middle| \frac{q+c(q)}{2} = \beta\right\}$. We thus assume the following inequality holds: $q > \underline{q}_M$.

Substituting $x_M(q,\beta)$ into (7), we derive $1 - x_M = \frac{q+c(q)-2\beta}{2(q-\beta)} \equiv G_M(q,\beta) = c'(q)$. Because the Hessian determinant is assumed to be positive, i.e., $2(q-\beta)c'' - x > 0$, it holds that $c''(q) > \frac{\partial G_M(q,\beta)}{\partial q} > 0$, for $q > \underline{q}_M$. Because the level of quality is determined by the degree of network function, we represent this as: $q = \hat{q}_M(\beta)$. In this case, we can represent the amount of quantity as a function of the degree of network function, i.e., $x = \hat{x}_M(\beta) \equiv x_M(\hat{q}_M(\beta),\beta)$.

The effects of an increase in the degree of network function on quantity and quality are respectively given by:

$$\frac{d\hat{x}_M}{d\beta} = \frac{2c''x}{2(q-\beta)c''-x} > 0 \quad \text{and} \quad \frac{d\hat{q}_M}{d\beta} = -\frac{2x}{2(q-\beta)c''-x} < 0. \tag{9}$$

See Appendix 1.1. As already shown in Lambertini and Orsini (2001, 2003) and Toshimitsu (2007), an increase in the network effects increases the amount of quantity, whereas it reduces the level of quality.

Now, in the first stage, the monopolist chooses the degree of network function at the expense of some costs of network function-improving investments. Taking (6) and (7) into account, the FOC is given by:

$$\begin{aligned}\frac{d\Pi}{d\beta} &= \frac{\partial \Pi}{\partial x}\frac{\partial \hat{x}_M}{\partial \beta} + \frac{\partial \Pi}{\partial q}\frac{\partial \hat{q}_M}{\partial \beta} + \frac{\partial \Pi}{\partial \beta} \\ &= \frac{\partial \Pi}{\partial \beta} = \{\hat{x}_M(\beta)\}^2 - f'(\beta) = 0.\end{aligned} \tag{10}$$

Furthermore, with regard to the second-order condition (SOC), we assume the following condition:

$$\frac{d^2\Pi}{d\beta^2} = 2\hat{x}_M\frac{d\hat{x}_M}{d\beta} - f''(\beta) < 0. \tag{11}$$

As in (10), the degree of network function depends upon the amount of quantity only. Accordingly, this implies that the degree happens to be a maximum value in the case of full market coverage, i.e., $\hat{x}_M = 1$. Here, we define the upper bound of the degree of network function as follows: $\bar{\beta} = \{\beta \mid 1 - f'(\beta) = 0\}$. However, because we do not deal with the case of full market coverage, the available range of the degree of network function is represented as: $\beta \in [0, \bar{\beta})$. Therefore, given (11), we show that there exists $\beta_M \in (0, \bar{\beta})$ to satisfy (10).

The amount of quantity and the level of quality in the monopoly equilibrium are determined by the degree of network function given by (10). Consequently, we obtain the monopoly equilibrium, i.e., $\{x_M, q_M, \beta_M\}$, where $x_M \equiv \hat{x}_M(\beta_M)$ and $q_M \equiv \hat{q}_M(\beta_M)$. The equilibrium is the same as the equilibrium in the case simultaneously determined by equations (6), (7), and (10). We summarize the result as Lemma 1.

LEMMA 1. *$\{x_M, q_M, \beta_M\}$ is an interior maximum solution in the unregulated partial coverage market, i.e., the monopoly equilibrium.*

3.2. Social Optimum

Suppose that a social planner determines quantity and quality to maximize social surplus as in (5), given the degree of network function. Hence, the FOCs are respectively given by:

$$\frac{\partial W}{\partial x} = q - (q - 2\beta)x - c(q) = 0\,, \tag{12}$$

$$\frac{\partial W}{\partial q} = \left\{1 - \frac{x}{2} - c'(q)\right\} x = 0\,. \tag{13}$$

In view of (1) and (12), we obtain $p = c(q) - \beta x$. This means that the social planner provides the product to consumers at a price less than the marginal cost of production, because of network externalities. From (12), we have $x = \frac{q-c(q)}{q-2\beta} = x_F(q, \beta)$, where subscript F denotes the first-best policy by the social planner. From (13), we have $1 - \frac{x}{2} = c'(q)$. We express $q = q_F(x)$. Thus, we have $\frac{\partial x_F}{\partial q} < 0$, $\frac{\partial x_F}{\partial \beta} > 0$, and $\frac{\partial q_F}{\partial x} < 0$.

The social planner provides the product for some proportions of consumers, $1 > x_F(q, \beta) > 0$, if and only if $\frac{c(q)}{2} > \beta$. However, if $\frac{q}{2} > \beta \geq \frac{c(q)}{2}$, then the social planner provides the product for all consumers, i.e., $x_F = 1$. Hence, we derive the level of quality in the case of full market coverage such as $q_F = \left\{q \mid \frac{1}{2} - c'(q) = 0\right\}$. Accordingly, we define the lower bound of quality, i.e., $\underline{q}_F \equiv \left\{q \mid \frac{c(q)}{2} = \beta\right\}$. Thus, we assume $q > \underline{q}_F > \underline{q}_M$.

By a similar way of analysis in the monopolist case, substituting $x_F(q, \beta)$ into (13), we obtain $\frac{q+c(q)-4\beta}{2(q-2\beta)} \equiv G_F(q, \beta) = c'(q)$, of which the LHS implies a marginal social benefit with respect to quality. Because the Hessian determinant is assumed to be positive, i.e., $(q - 2\beta)c'' - \frac{x}{4} > 0$, it holds that $c''(q) > \frac{\partial G_F(q,\beta)}{\partial q} > 0$, for $q > \underline{q}_F$. Because the level of quality is determined by the degree of network function, we express $q = \hat{q}_F(\beta)$. Furthermore, the amount of quantity is represented as a function of the degree of network function, i.e., $x = \hat{x}_F(\beta) \equiv x_F(\hat{q}_F(\beta), \beta)$. Hence, we have:

$$\frac{d\hat{x}_F}{d\beta} = \frac{8xc''}{4(q - 2\beta)c'' - x} > 0 \quad \text{and} \quad \frac{d\hat{q}_F}{d\beta} = -\frac{4x}{4(q - 2\beta)c'' - x} < 0\,. \tag{14}$$

See Appendix 1.2.

In the first stage, the social planner chooses the degree of network function at the expense of some costs of network function-improving investments. Based on (12) and (13), the FOC is given by:

$$\begin{aligned}\frac{dW}{d\beta} &= \frac{\partial W}{\partial x}\frac{\partial \hat{x}_M}{\partial \beta} + \frac{\partial W}{\partial q}\frac{\partial \hat{q}_M}{\partial \beta} + \frac{\partial W}{\partial \beta} \\ &= \frac{\partial W}{\partial \beta} = \{\hat{x}_F(\beta)\}^2 - f'(\beta) = 0\,.\end{aligned} \tag{15}$$

Furthermore, with regard to the SOC, we assume the following condition:

$$\frac{d^2 W}{d\beta^2} = 2\hat{x}_F \frac{d\hat{x}_F}{d\beta} - f''(\beta) < 0\,. \tag{16}$$

Given (16), there exists $\beta_F \in (0, \bar{\beta})$ to satisfy (15). In addition, because the amount of quantity and the level of quality are determined by the degree of network function only, we obtain the social optimal equilibrium, $\{x_F, q_F, \beta_F\}$, where $x_F \equiv \hat{x}_F(\beta_F)$ and $q_F \equiv \hat{q}_F(\beta_F)$. Thus, we summarize the result as Lemma 2.

LEMMA 2. *$\{x_F, q_F, \beta_F\}$ is an interior social optimum, which is the first-best policy.*

3.3. Comparison: Monopoly Equilibrium and Social Optimum

We progress to a comparative analysis of the monopoly equilibrium with the social optimum presented in Lemmas 1 and 2. To do so, we first consider the relationship between the level of quality in the monopoly equilibrium and that in the social optimum, given the degree of network function. For $q > \underline{q}_F > \underline{q}_M$, it holds that $G_M(q, \beta) \geq G_F(q, \beta)$, given $\beta \in [0, \bar{\beta})$, where the equation holds if and only if $\beta = 0$. Furthermore, because $c''(q) > \frac{\partial G_i}{\partial q} > 0, i = M, F$, we derive:

$$\hat{q}_M(\beta) > \hat{q}_F(\beta)\,, \quad \text{for any} \quad \beta \in [0, \bar{\beta})\,. \tag{17}$$

Second, we show that the amount of quantity in the social optimum is larger than that in the monopoly equilibrium. Using the results shown above and taking (17) into account, we can illustrate the following equations.

$$\begin{aligned}\hat{x}_F(\beta) &= x_F(\hat{q}_F(\beta), \beta) > x_M(\hat{q}_F(\beta), \beta) \\ &> x_M(\hat{q}_M(\beta), \beta) = \hat{x}_M(\beta)\,,\end{aligned} \quad \text{for any } \beta \in [0, \bar{\beta})\,. \tag{18}$$

Third, taking (10), (15), and (18) into account, we derive:

$$\beta_F > \beta_M\,. \tag{19}$$

Therefore, taking (17), (18), and (19) into account, we present Proposition 1 as follows.

PROPOSITION 1. *We suppose that the monopolist chooses the quantity, the quality, and the degree of network function at the expense of some costs of network function-improving investments. When the marginal cost of production is increasing in quality, compared with the social optimum, the monopolist has an incentive to undersupply a* *product attached to a less efficient network function and to the overprovision of quality:* $\beta_F > \beta_M, q_F < q_M$, *and* $x_F > x_M$.

In this case, the government should let the monopolist more increase network function-improving investment to raise the degree of network function. This, in turn, reduces the overprovision of quality while it increases the undersupply. As a result, social welfare is improved by the government's regulation of the network function.

4. ON THE TYPE OF A COST FUNCTION OF QUALITY

Motta (1993) presents two types of a cost function of quality. Based on his definition, the type of a variable cost function of quality is assumed in the previous sections. That is, the cost function of quality is positively related to quantity, i.e., $\frac{\partial^2 C}{\partial q \partial x} = c'(q) > 0$ in (2). However, in this section, we assume another cost function of quality, and reconsider Proposition 1.

We assume the type of a fixed cost function of quality as follows:

$$C(q, \beta, x) = ex + c(q) + f(\beta), \tag{20}$$

where it should be that $e > \beta \geq 0$. The cost function (20) implies that quality and network function are unrelated to quantity: $\frac{\partial^2 C}{\partial q \partial x} = 0$ and $\frac{\partial^2 C}{\partial \beta \partial x} = 0$. For example, we can imagine that a firm incurs fixed costs such as R&D investments to improve the quality grade as well as the utilities of network system acquired with goods and services in advance.

First, in the case of the monopolist, the FOCs with respect to the quantity and quality are respectively given by:

$$\frac{\partial \Pi}{\partial x} = q - 2(q - \beta)x - e = 0, \tag{6$'$}$$

$$\frac{\partial \Pi}{\partial q} = (1 - x)x - c'(q) = 0. \tag{7$'$}$$

From (6′), we have $x = \frac{q-e}{2(q-\beta)} = x_M^f(q, \beta)$, where superscript f denotes the type of a fixed cost function. If and only if $q > e > \beta$, the case of partial market coverage prevails for a monopoly, i.e., $1 > x_M^f > 0$. Furthermore, based on (7′), we express $q = q_M^f(x)$. In the type of a fixed cost function of quality, it holds that $\frac{\partial x_M^f}{\partial q} > 0$, $\frac{\partial x_M^f}{\partial \beta} > 0$, and $\frac{\partial q_M^f}{\partial x} > 0$.

Substituting $x_M^f(q, \beta)$ into (7′), we can represent the quality level as a function of the degree of network function, $q = \hat{q}_M^f(\beta)$. Furthermore, with regard to the amount of quantity, we express $x = \hat{x}_M^f(\beta) \equiv x_M(\hat{q}_M^f(\beta), \beta)$. In this case, the effects of an increasing degree of network function on quantity and quality are respectively given by:

$$\frac{d\hat{x}_M^f}{d\beta} = \frac{2c''x}{2(q-\beta)c'' - (1-2x)^2} > 0 \text{ and } \frac{d\hat{q}_M^f}{d\beta} = \frac{2x(1-2x)}{2(q-\beta)c'' - (1-2x)^2} > 0. \tag{9$'$}$$

See Appendix 2. An increase in the degree of network function increases not only the quantity but also the quality (Toshimitsu, 2007). That is, because quality is a complement to quantity in the type of a fixed cost function, the larger the quantity reduces the costs of improving quality, which, in turn, allows the monopolist to increase the quality grade of a product.

The FOC with respect to the degree of network function is given by:

$$\frac{d\Pi}{d\beta} = \{\hat{x}_M^f(\beta)\}^2 - f'(\beta) = 0\,. \tag{10$'$}$$

Furthermore, the condition to satisfy the SOC is given by:

$$2\hat{x}_M^f \frac{d\hat{x}_M^f}{d\beta} - f''(\beta) < 0\,. \tag{11$'$}$$

Given (11′), there exists $\beta_M^f \in (0, \bar{\beta})$ to satisfy (10′). Therefore, we obtain the monopoly equilibrium, $\{x_M^f, q_M^f, \beta_M^f\}$, where $x_M^f \equiv \hat{x}_M^f(\beta_M^f)$ and $q_M^f \equiv \hat{q}_M^f(\beta_M^f)$.

Next, similarly, the FOCs in the case of the social planner are given by:

$$\frac{\partial W}{\partial x} = q - (q - 2\beta)x - e = 0\,, \tag{12$'$}$$

$$\frac{\partial W}{\partial q} = \left(1 - \frac{x}{2}\right)x - c'(q) = 0\,. \tag{13$'$}$$

Given (12′) and (13′), we obtain $x = \frac{q-e}{q-2\beta} = x_F^f(q, \beta)$ and $q = q_M^f(x)$. In this case, if and only if $\frac{q}{2} > \frac{e}{2} > \beta$, the case of partial market coverage holds, i.e., $1 > x_F^f > 0$. Furthermore, we have $\frac{\partial x_F^f}{\partial q} > 0$, $\frac{\partial x_F^f}{\partial \beta} > 0$, and $\frac{\partial q_F^f}{\partial x} > 0$.

By a similar procedure, we can derive $q = \hat{q}_F^f(\beta)$ and $x = \hat{x}_F^f(\beta) \equiv x_F(\hat{q}_F^f(\beta), \beta)$. The effects of an increasing degree of network function are given by:

$$\frac{d\hat{x}_F^f}{d\beta} = \frac{2xc''}{(q-2\beta)c'' - (1-x)^2} > 0 \text{ and } \frac{d\hat{q}_F^f}{d\beta} = \frac{2x(1-x)}{(q-2\beta)c'' - (1-x)^2} > 0\,. \tag{14$'$}$$

See Appendix 2.

For the FOC and the SOC with respect to the degree of network function, we have

$$\frac{dW}{d\beta} = \{\hat{x}_F^f(\beta)\}^2 - f'(\beta) = 0\,, \tag{15$'$}$$

$$\frac{d^2W}{d\beta^2} = 2\hat{x}_F^f \frac{d\hat{x}_F^f}{d\beta} - f''(\beta) < 0\,. \tag{16$'$}$$

Given (16′), there exists $\beta_F^f \in (0, \bar{\beta})$ to satisfy (15′). We define as follows: $x_F^f \equiv \hat{x}_F^f(\beta_F^f)$ and $q_F^f \equiv \hat{q}_F^f(\beta_F^f)$. Thus, we obtain the social optimum, $\{x_F^f, q_F^f, \beta_F^f\}$.

Finally, we compare the monopoly equilibrium and the social optimum in the type of a fixed cost function of quality, and then reconsider Proposition 1.

Given $\frac{e}{2} > \beta \geq 0$, and taking (6′) and (12′) into account, we have $x_F^f(q, \beta) > x_M^f(q, \beta)$, for any $q (> e > 0)$. Similarly, taking (7′) and (13′) into account, we have $q_F^f(x) > q_M^f(x)$, for any $x \in (0, 1)$. Because quantity and quality are positively related to each other in both cases, as mentioned above, we can show the following:

$$\hat{q}_F^f(\beta) > \hat{q}_M^f(\beta)\,, \tag{17′}$$

$$\hat{x}_F^f(\beta) > \hat{x}_M^f(\beta)\,. \tag{18′}$$

Based on (10′), (15′), and (18′), we derive $\beta_F^f > \beta_M^f$. Furthermore, taking (9′), (14′), (17′), and (18′), into account, the following equations hold:

$$q_F^f = \hat{q}_F^f(\beta_F) > \hat{q}_F^f(\beta_M) > \hat{q}_M^f(\beta_M) = q_M^f\,, \quad \text{and}$$
$$x_F^f = \hat{x}_F^f(\beta_F) > \hat{x}_F^f(\beta_M) > \hat{x}_M^f(\beta_M) = x_M^f\,.$$

Therefore, we summarize the results as follows.

PROPOSITION 2. *We suppose that the monopolist chooses the quantity, the quality, and the degree of network function at the expense of some costs of network function-improving investments. When the cost of upgrading quality is unrelated to quantity, the monopolist has an incentive to undersupply a product attached to a less efficient network function and to the underprovision of quality:* $\beta_F^f > \beta_M^f$, $q_F^f > q_M^f$, *and* $x_F^f > x_M^f$.

First, the lower degree of network function in the monopoly equilibrium depends upon a complementarily between the amount of quantity and the network effects. In general, it is likely for a private firm to undersupply a product, compared with the social optimum. Thus, based on Propositions 1 and 2, we suggest that, regardless of the nature of the cost function, the monopolist provides a product that has a less efficient network function.

Second, however, the opposing results with regard to the quality level obtained in Propositions 1 and 2 are due to the nature of the cost function. Quality is a complement to quantity in the case of a fixed cost function, which implies supply-side economies of scale. That is, the larger (smaller) quantity reduces (increases) the costs of improving quality, which, in turn, allows the monopolist to increase (reduce) the quality grade of a product. Thus, undersupply of production by the monopolist implies that the quality grade in the monopoly equilibrium is lower than that of the social optimum. In this case, the government lets the monopolist have an incentive to invest more to raise the degree of network function. This, in turn, remedies undersupply of a product associated with a lower quality grade.

Here, we address other cases of cost functions as follows.

$$\text{(i)} \quad C(x, q, \beta) = \{c(q) + f(\beta)\}x \quad \text{and} \quad \text{(ii)} \quad C(x, q, \beta) = c(q) + f(\beta)x\,.$$

For (i), we obtain the same results as those in the case of a variable cost function as in (2). Furthermore, for (ii), we obtain the same results as those in the case of a fixed cost function as in (20). The point of these cases is to determine whether the cost function of quality is related to quantity. That is, when the cost of quality is related (unrelated) to quantity, a substitute (complement) relationship between quality and quantity holds. Hence, the monopoly results in the overprovision (underprovision) of quality and the undersupply of production, compared with the social optimum. Furthermore, because there is a complement relationship between quantity and network function, the monopolist provides a product associated with a lower marginal efficiency of network in the monopoly equilibrium, compared with the social optimum.

5. CONCLUDING REMARKS

In this paper, we addressed the implications of the monopolist's choice regarding network function and quality for social welfare. Furthermore, to look at the effect of network function on quantity and quality, we assumed a two-stage decision, in which the monopolist chooses the degree of network function at the expense of costs of network function-improving investments before deciding quantity and quality. Hence, the degree of network function is positively related to the amount of quantity, irrespective of the nature of the cost function. Because, in general, a private firm is likely to undersupply production, compared with the social optimum, the firm necessarily chooses a less efficient network function. However, the relationship between the level of quality and the amount of quantity depends upon the nature of the cost function of quality. That is, if cost function of quality is positively related to (independent of) the amount of quantity, the firm provides an overprovision (underprovision) of quality.

Based on the results derived above, we show the policy implications for a government that regulates the degree of network function, i.e., network function-improving investments. Namely, because a private firm necessarily chooses a degree of network function that is lower than that in the social optimum, the government should raise the firm's incentive to invest in the network function. In this case, the undersupply of a product accompanied by socially distorted quality grade must be remedied to be better off, so that the social welfare improves.

With regard to the monopoly model with endogenous network effects in this paper, we need to address some issues. Two of them are concerned with specific forms of the functions assumed in the model. First, we have assumed that network effects are the same for all consumers and independent of the valuation of quality by an individual consumer. However, we must consider that the degree of network function and the valuation of quality by an individual consumer are interdependent. For example, we suppose $u = \theta\{q + N(x)\}$. In this case, network effects depend on the valuation of an individual consumer. Even if network effects are zero, the value of utility is not zero. This formulation, as assumed in the model, implies indirect system network externalities, such as hardware/software in video games and personal computers. Furthermore, we present another utility function, i.e., $u = \theta q N(x)$. In this case, if $N(0) = 0$, the value of utility becomes to be zero. This implies direct communication network effects in a communication service industry.

Second, related to the first issue, assuming that quality and network function are separable from each other in the utility function given by (1) and in the cost functions given by (2) and (20), we derive $\frac{\partial^2 \Pi}{\partial q \partial \beta} = 0$ and $\frac{\partial^2 W}{\partial q \partial \beta} = 0$. See (a.2) and (a.5) in Appendixes 1.1 and 1.2. Thus, the effect of network function on the level of quality indirectly works only through the effect on the amount of quantity. However, assuming a multiplicative form with respect to the utility and cost functions, it holds that $\frac{\partial^2 \Pi}{\partial q \partial \beta} \neq 0$ and $\frac{\partial^2 W}{\partial q \partial \beta} \neq 0$. For example, as mentioned above, if we assume the utility function such as $u = \theta q N(x)$, it follows that $\frac{\partial^2 \Pi}{\partial q \partial \beta} > 0$ and $\frac{\partial^2 W}{\partial q \partial \beta} > 0$, even with the cost functions given by (2) and (20). Accordingly, the degree of network function depends upon the level of quality as well as the amount of quantity.

Third, because there do not exist strategic effects on rivals in the case of monopoly, the role of endogenous network effects is not important in the model. Thus, we examine the cases of duopoly or oligopoly to consider strategic effects on the rivals because of endogenous network effects.

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APPENDIX

1.1. Equilibrium in the Second Stage, given the Degree of Network Function in the Monopolist Case

$$\frac{\partial^2 \Pi}{\partial x^2} = -2(q-\beta) < 0\,, \quad \frac{\partial^2 \Pi}{\partial q^2} = -c''x < 0\,, \quad \text{and } \Pi_{qx} = \Pi_{xq} = -x < 0\,. \quad \text{(a.1)}$$

From the first and second expressions of (a.1), the second-order conditions (SOCs) always hold. The third expression implies that quality is negatively related to quantity. Furthermore, we have

$$\frac{\partial^2 \Pi}{\partial x \partial \beta} = 2x \quad \text{and} \quad \frac{\partial^2 \Pi}{\partial q \partial \beta} = 0\,. \quad \text{(a.2)}$$

Taking (a.1) and (a.2) into account, we derive the following:

$$\begin{bmatrix} -2(q-\beta) & -x \\ -x & -c''x \end{bmatrix} \begin{bmatrix} dx \\ dq \end{bmatrix} = \begin{bmatrix} -2x \\ 0 \end{bmatrix} d\beta\,, \quad \text{(a.3)}$$

where we assume that the Hessian determinant is positive, i.e., $\Delta_M = x\{2(q-\beta)c'' - x\} > 0$.

1.2. The Social Planner Case

We obtain the second-order properties as follows.

$$\frac{\partial^2 W}{\partial x^2} = -(q-2\beta) < 0\,, \quad \frac{\partial^2 W}{\partial q^2} = -c''x < 0\,, \text{ and } \frac{\partial^2 W}{\partial q \partial x} = \frac{\partial^2 W}{\partial x \partial q} = -\frac{x}{2} < 0\,, \quad \text{(a.4)}$$

where we assume that $\frac{q}{2} > \beta$. Moreover, the effects of an increase in the degree of network function on the marginal social surplus are given by:

$$\frac{\partial^2 W}{\partial x \partial \beta} = 2x \quad \text{and} \quad \frac{\partial^2 W}{\partial q \partial \beta} = 0\,. \quad \text{(a.5)}$$

Taking (a.4) and (a.5) into account, we obtain:

$$\begin{bmatrix} -(q-2\beta) & -\frac{x}{2} \\ -\frac{x}{2} & -c''x \end{bmatrix} \begin{bmatrix} dx \\ dq \end{bmatrix} = \begin{bmatrix} -2x \\ 0 \end{bmatrix} d\beta\,, \quad \text{(a.6)}$$

where we assume that the Hessian determinant is positive, i.e., $\Delta_F = x\left\{(q-2\beta)c'' - \frac{x}{4}\right\} > 0$.

2. The Case of a Fixed Cost Type

The SOCs of the monopolist's profit maximizing and of the social planner's welfare maximizing are satisfied. However, with respect to the partial cross-derivatives, given (6′) (or (7′)) and (127) (or (13′)), we obtain:

$$\frac{\partial^2 \Pi}{\partial q \partial x} = \frac{\partial^2 \Pi}{\partial x \partial q} = 1 - 2x \quad \text{and} \quad \frac{\partial^2 W}{\partial q \partial x} = \frac{\partial^2 W}{\partial x \partial q} = 1 - x > 0 \,. \tag{a.7}$$

Hence, it follows that $\frac{\partial^2 W}{\partial q \partial x} > \frac{\partial^2 \Pi}{\partial q \partial x}$. In general, we cannot omit the case in which these signs are different, i.e., $\frac{\partial^2 W}{\partial q \partial x} > 0 > \frac{\partial^2 \Pi}{\partial q \partial x}$. However, as assumed in Sheshinski (1976), we deal with the same-sign case, i.e., $\frac{\partial^2 W}{\partial q \partial x} > \frac{\partial^2 \Pi}{\partial q \partial x} > 0$. This means that $\frac{\partial^2 \Pi}{\partial q \partial x} = 1 - 2x = \frac{e-\beta}{q-\beta} > 0 \Leftrightarrow e > \beta$, given (6′). In this case, quality is positively related to quantity.

Taking (a.1), (a.2), and (a.7) into account, the effects of an increase in the degree of network function in the case of the monopolist are given by:

$$\begin{bmatrix} -2(q-\beta) & 1-2x \\ 1-2x & -c'' \end{bmatrix} \begin{bmatrix} dx \\ dq \end{bmatrix} = \begin{bmatrix} -2x \\ 0 \end{bmatrix} d\beta \,, \tag{a.8}$$

where the Hessian determinant is assumed to be positive, i.e., $\Delta_M^f = 2(q-\beta)c'' - (1 - 2x)^2 > 0$.

Similarly, in the case of the social planner, the effect of an increase in the degree of network function, we obtain:

$$\begin{bmatrix} -(q-2\beta) & 1-x \\ 1-x & -c'' \end{bmatrix} \begin{bmatrix} dx \\ dq \end{bmatrix} = \begin{bmatrix} -2x \\ 0 \end{bmatrix} d\beta \,, \tag{a.9}$$

where the Hessian determinant is assumed to be positive, i.e., $\Delta_F^f = (q-2\beta)c'' - (1 - x)^2 > 0$.