Title	A Strategic Model of Technology Adoption and Labor Specialization
Sub Title	
Author	Dasgupta, Sugato
Publisher	Keio Economic Society, Keio University
Publication year	2008
Jtitle	Keio economic studies Vol.45, (2008.) ,p.17- 37
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260 492-20080000-0017

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A STRATEGIC MODEL OF TECHNOLOGY ADOPTION AND LABOR SPECIALIZATION

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First version received May 2005; final version accepted January 2009

Abstract: What determines a worker's skill level? I build a model that emphasizes strategic interactions between workers and firms. Suppose a worker invests in skills corresponding to a specific technology, denoted *t*. In a situation where wage determination occurs through bilateral bargaining, the worker is unable to extract her entire marginal product. Rather the worker's bargaining power and, hence, her compensation depend on the number of firms with technology *t*. An increase in the number of potential employers raises the fraction of the marginal product captured by the worker and encourages skill development. In my model, the technology adoption decisions of firms and the skill acquisition decisions of workers are jointly determined. The model shows that in equilibrium, all firms adopt the same technology. However, this equilibrium allocation is generally inefficient.

Key words: Hold-up, Search, Technology adoption. JEL Classification Number: L11, L13.

1. INTRODUCTION

In the *Wealth of Nations*, Adam Smith's well-known parable of the pin factory compelling demonstrates the enormous gain in productivity when labor is specialized. After decades of inexplicable neglect, a sizable literature has in recent years re-addressed the various issues related to labor specialization. I mention here a few of the important papers.

Within a partial equilibrium framework, Rosen (1978) shows how the requirements of technology and the distribution of worker skills interact to determine labor specialization. Barzel and Yu (1984) study an economy with ex ante identical workers; they establish the advantages of labor specialization and trade when there are indivisibilities in the acquisition of skills. Exploring an environment in which workers select the range

Acknowledgments. This paper, which is based on a chapter of my dissertation at the Massachusetts Institute of Technology, has benefited from the guidance of Daron Acemoglu and Peter Diamond. An anonymous referee suggested numerous changes that vastly improved the paper's structure and quality. Of course, the usual disclaimer applies.

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of skills to acquire, Grossman and Shapiro (1982) highlight the perils of specialization: specifically, when there are skill-specific shocks to labor demand, the risk-averse specialist is exposed to fluctuations in her income. Finally, a few authors (see, e.g., Kim (1989), Yang and Borland (1991), and Borland and Yang (1992)) have constructed tractable general equilibrium models formalizing the intuition that the extent of labor specialization is increasing in the size of the market.

The twin issues of skill acquisition and labor specialization are at the heart of this paper. However, in contrast to the aforementioned literature, I highlight strategic interactions between workers, the acquirers of skills, and firms, the demanders of skills. The basic idea consists of the following three parts. (1) When a worker, attached to a particular firm, engages in job-related training, the hitherto unskilled worker becomes more productive in her current firm. However, the transferability of the acquired skill is restricted to firms with similar skill demands (and, hence, similar technologies). (2) When wage determination occurs through bargaining, it is in general impossible for a worker to extract her entire marginal product. In other words, a worker's investment in job-related training is subject to hold-up-that is, a worker bears the entire cost of training but receives, in return, only a fraction of the subsequent increase in output. The hold-up problem gives rise to a familiar result: the level of job-related training is lower than the social optimum. (3) A worker's bargaining strength is determined in part by the number of firms demanding her skills. An increase in the number of potential employers raises a worker's leverage and allows her to capture, through wage negotiations, a larger fraction of her marginal product. The consequent increase in the worker's private rate of return to skill acquisition encourages, in turn, skill formation.

The above discussion suggests that the skill acquisition decisions of workers and the technology adoption decisions of firms are interconnected. This interconnection is explored formally in a model that is outlined below.

I consider a situation with agents of two sorts: workers and entrepreneurs. Ex ante identical workers constitute a continuum of mass *L* while ex ante identical entrepreneurs are countably infinite in number. For simplicity, there is only one produced good and its price is normalized to one. There are of course various ways of producing this good and, directly following Arthur (1994, pp. 15), I identify these ways with *technologies*. Let $T \equiv \{1, 2, ..., |T|\}$ index the set of exogenously available technologies.

The goal of this paper is to build a simple model that endogenizes both the adoption of technologies and workers' subsequent investment in skills.^{1,2} This is done by requiring all agents to play the following two-stage game. The first stage is called the *technology adoption stage*. In this stage, entrepreneurs first decide whether to form a firm and enter a market. If market entry is the chosen option, a technology $t \in T$ must be adopted

¹ Of course, in any real world setting, the adoption of technologies is not only based on workers' training and wage considerations. I abstract from the other determinants of technology choice not because they are unimportant; rather, incorporating many considerations at once obscures the intuitions that I seek to highlight.

² Thus, my work is related to that by Grossman and Hart (1986) and Hart and Moore (1990). These papers study the implications of the hold-up problem for the optimal allocation of ownership rights over assets. In contrast, I focus on the linkages between the hold-up problem and technology adoption.

as well. I identify the type of a firm with its technology, and let α_t measure, in units of the produced good, the cost of setting up a type-*t* firm. Notice that entrepreneurs' decisions result in an outcome vector $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$, where N_t is the number of type-*t* firms in the market.³ I shall say that *technology dispersion* is exhibited if the outcome vector shows at least two distinct firm types in the market. In contrast, *technology homogeneity* prevails when all firms in the market are of the same type.

Given $(N_1, \ldots, N_{|T|})$, each worker decides whether or not to seek employment in the market. Two cases arise. First, if the market employment option is not exercised, the worker has access to a technology called *home production* which transforms her unit endowment of unskilled labor into a unit of output; hence, every worker is guaranteed a reservation wage of one. Second, if the market employment option is taken up, the worker specifies the firm type in which she seeks employment. Notice therefore that workers' decisions result in a labor allocation vector $(L_1, \ldots, L_{|T|}) \in \mathfrak{R}^{|T|}_+$, where L_t is the mass of workers seeking employment in type-*t* firms.

The second stage of the game is called the *training and production stage*. It deals with various training and production decisions that occur following entry by entrepreneurs and workers. An overview of the events in this stage is as follows. Consider a situation wherein $N_t > 0$ and $L_t > 0$. Each of the L_t workers is randomly assigned to one of the type-*t* firms and engages in job-related training. In consequence, the worker converts her unit endowment of unskilled labor into specialized units of type-*t* labor. The worker becomes physically more productive, but finds her employment opportunities restricted to the N_t firms with technology *t*. Once job-related training is over, the worker and the firm haggle over the worker's wage. As mentioned before, the wage-bargaining game is so structured that holding fixed the worker's level of training, her equilibrium wage increases with the number of type-*t* firms, N_t . Finally, when wage-bargaining ends, the worker produces the good and obtains, in return, the negotiated wage.

My model generates three interesting results. *First*, despite the availability of |T| distinct technologies and the absence of explicit coordination among entrepreneurs, I show that technology dispersion is *not* an equilibrium phenomenon. Why? To fix ideas, suppose there exists an equilibrium wherein technology t_1 is adopted by n_1 firms and technology t_2 is adopted by n_2 firms. An entrepreneur sets up a firm anticipating that it would be able to attract a workforce. So workers must be willing to seek employment in firms of both types. In turn, this requires that a worker's wage net of the cost of job-related training be equalized across the two firm types. Now recall that a worker's wage-bargaining game with a type- t_1 (type- t_2) firm is structured so that her bargaining power and, hence, net wage in that firm is increasing in n_1 (n_2). Later in the paper, I also assume that all technologies in T have identical physical attributes—that is, technologies t_1 and t_2 are different but equally productive ways of producing the final good. Thus, equalization of a worker's net wage across both firm types can occur only if $n_1 = n_2$ ($\equiv n$, say). Consider an additional entry of a type- t_1 firm relative to the putative equilibrium

³ Z_+ is the set of non-negative integers: $Z_+ \equiv \{0, 1, 2, ...\}$. $Z_+^{|T|}$ is the |T|-fold cartesian product of Z_+ .

of *n* firms of type t_i , i = 1, 2. Post-entry there are more type- t_1 firms than type- t_2 firms and all workers maximize their net wages by seeking employment in type- t_1 firms. With a workforce of L/(n + 1) assigned to each of the (n + 1) type- t_1 firms, it turns out that the entrant more than covers the fixed cost of entry. But, since this deviation is strictly profitable, the initial supposition of technology dispersion—that is, coexistence of technologies t_1 and t_2 —cannot constitute an equilibrium after all.

Second, I show that technology homogeneity is always an equilibrium phenomenon. Why? Consider a situation wherein a number of type- \overline{t} firms have already formed and a single entrepreneur contemplates adoption of a distinct technology, $\hat{t} \neq \overline{t}$. No worker, it turns out, seeks employment in the type- \hat{t} firm since skills acquired in this firm are subjected to a severe hold-up problem (recall that specialized type- \hat{t} labor is of no value to any of the type- \overline{t} firms in the market) and the net wage, in consequence, is low. Unable to attract a workforce, the single type- \hat{t} firm obviously does not form. Hence, technology homogeneity—all type- \overline{t} firms in the market—is not disturbed by the entry of a different firm type.

Third, consider the equilibrium with only type- \bar{t} firms in the market. The formation of a type- \bar{t} firm ameliorates the hold-up problem and enhances workers' incentives for skill acquisition. Since no entrepreneur contemplating entry internalizes this externality, market equilibrium may be associated with excessive or deficient entry of type- \bar{t} firms.

In a justly-famous article, Rosenstein-Rodan (1943, pp. 204–205) considered how "depressed areas" like Eastern and South-Eastern Europe could industrialize through the adoption of a superior (that is, more efficient) technology. He argued informally that such technology adoption was not possible since workers did not possess the skills required to complement the superior technology. But, this observation begets an obvious question: If non-availability of the relevant technology that is currently in use to the new and superior technology, why do workers not invest in the required skills? I suggest that it is the hold-up problem subsequent to skill acquisition that deters workers from investing in new skills. In sum, my model, which highlights the linkages between technology adoption and skill acquisition to yield the *technology lock-in* conclusion, provides a formalization of the Rosenstein-Rodan argument (see footnote for further details).⁴

Of course, my model has relevance beyond Europe of the 1940s and is applicable to experiences of the developing countries today. Lall (2000a) argues that industrial technologies are changing at unprecedented rates, driven by a key technology (microelectronics). Indeed, old technologies have become redundant at all factor prices. Yet, the adoption of modern and obviously superior industrial technology is unevenly spread

⁴ There is a slight gap between the model in the paper and the Rosenstein-Rodan (1943) argument, which centers on the possibility of a trap wherein all firms adopt an inefficient technology. In my model, for notational simplicity, all technologies in T are assumed to possess the same physical attributes (see assumption [A.4]). But, it is easy to show (proof available upon request) that all firms adopting technology $\bar{t} \in T$ remains an equilibrium even when there is a technology $\hat{t} \in T$ which is more efficient than \bar{t} as long as assumption [A.2] is satisfied.

in the developing world. Why? Summarizing research on micro-level technical change, Lall (2000b) maintains that importing and mastering technologies in developing countries is not immediate and automatic. Technology, unlike physical products, is not sold in fully embodied forms; rather, it has important tacit elements that require skills to master.^{5,6} In other words, the effective use of a new and imported technology hinges on a workforce with sufficient skills. Therefore entrepreneurs in Sub-Saharan Africa, for example, are forced to herd on an inefficient and traditional technology simply because no entrepreneur can find workers capable of mastering the advanced and superior modern technology.

While complementarities between the adoption of new industrial technologies and worker skills are not in doubt, the arguments in Lall (2000a, 2000b) remain incomplete. When pre-employment worker skills are limited, post-employment training in an enterprise can augment worker skills. Indeed, using enterprise survey data from Indonesia, Colombia, Malaysia and Mexico, Tan and Batra (1995) show that training in enterprises is an effective and economical way to develop worker skills. In addition to cost effectiveness, enterprise training appears to be an *essential* requirement when a new technology is adopted (see footnote for details).⁷ Now consider a situation wherein all entrepreneurs have herded on a traditional technology because pre-employment worker skills are in short supply. Why doesn't a *single* entrepreneur adopt an efficient modern industrial technology and simultaneously provide training to its workers so that desired skills are thereby acquired? My model provides an answer to this puzzle. On-the-job training is costly for workers in terms of effort and time expended. Since skills acquired

⁵ Lall (2000a) provides two bits of evidence to demonstrate the dependence of technology adoption decisions in developing countries on workers' skill levels. *First*, Lall argues that foreign direct investment is becoming critical to the transfer of new industrial technologies from the developed to the developing world. Indeed, FDI to the developing countries has risen rapidly from an average of \$29 billion in 1986–91 to \$149 billion in 1997. However, these flows are very concentrated with the 43 least developing countries receiving less than one half percent of the total flows to developing countries in the 1986–1997 period. The data also reveal a clear correlation between FDI flows to countries and various country-level measures of skill formation (for example, tertiary enrolment in technical subjects). In other words, the skewness in FDI flows in the developing world appears to stem from the skewness in workers' skill levels.

Second, Lall notes that when a transnational company transfers technology to affiliates in a developing country, it must choose between a range of technologies of different vintages and complexity. Lall (2000a, pp. 33) argues that the choice of technology "reflects the ability of an affiliate to deploy technology efficiently. This is why transfers to affiliates in developing countries with low skills and capabilities tend to have lower technological content than in advanced ones, and subsequent upgrading reflects the growth of skills and capabilities in the affiliate and the host country."

⁶ Lall (2000a, pp. 9) notes that technologies differ in their learning requirements. Process technologies (for example, chemicals) are more embodied in equipment and, hence, have less tacit elements than engineering technologies (for example, machinery). Since my paper derives the technology homogeneity conclusion by emphasizing workers' technology-specific skill acquisition, it applies best when entrepreneurs in the developing world contemplate adoption of a modern industrial technology of the engineering kind.

⁷ In a detailed study of approximately 6000 Malaysian firms covering 12 industries and three years (1988, 1994 and 1997), Tan (2001) estimates probit models to show that firms which had introduced a new product or process technology in the past two or three years were significantly more likely to train their workers than firms which had not introduced new technology. Tan and Batra (1997) obtain similar results for Colombia, Mexico and Taiwan.

by workers are subject to a hold-up problem, workers become unwilling to invest significantly in the training process initiated by the firm.

There is also ample anecdotal and informal evidence that attests to the importance of worker skills in the technology adoption decisions of firms in developed countries.⁸ On the basis of surveys of firms in OECD countries carried out since the early 1980s, Vickery and Northcott (1995), for example, analyze the diffusion of two relatively modern technologies-the application of microelectronics in products and the use of microelectronics-based process equipment (advanced manufacturing technology or AMT). Vickery and Northcott (1995, pp. 264-265) observe: "When managers are asked about the most important impediments to the adoption of microelectronics, they most often [italics added] mention the lack of expertise." This shortage of expertise is interpreted by the authors as a problem of finding workers with specific skills.9 But, if skill shortage makes adoption of microelectronics-based technology difficult, it is natural to ask (as in the above discussion of the papers by Rosenstein-Rodan and Lall) why workers do not acquire the relevant skills through on-the-job training. The holdup problem, identified in this paper, provides a plausible explanation for workers' skill underinvestment. Observe also that the technology homogeneity result of my model suggests that the rapid diffusion of microelectronic applications in products and the rapid diffusion of AMT are by no means guaranteed.¹⁰

I conclude this section by relating my work to the theoretical literature dealing with investments by economic agents in markets with frictions. Two sets of papers are germane. First, Acemoglu and Pischke (1998, 1999) provide explanations for why firms

⁸ For recent econometric evidence from developed countries documenting the complementarities between the adoption of new technologies and workers' skill investments, refer to Bartel and Lichtenberg (1987) and Goldin and Katz (1996).

⁹ In the context of a developing country, Malaysia, Tan (2000) studies the adoption of new information and communications technology (IT) in 12 industries. On the basis of enterprise survey data, Tan (2000, pp. 13) concludes that "the inadequate supply of IT personnel emerges as the key constraint to the introduction of IT." Observe that the conclusions of Tan (2000) and Vickery and Northcott (1995) are strikingly similar. Furthermore, Tan (2000) estimates panel production function models to identify the productivity gains from IT adoption and use. The evidence suggests that IT use is associated with a productivity gain of 4–6 percent per annum, and that these learning gains are much larger when accompanied by worker training. Tan (2000, pp. 3) concludes that the findings "lend strong support for the skill-biased technological change hypothesis and for the intermediate role of skilled labor in IT adoption and use."

¹⁰ The advanced manufacturing technology or AMT was vastly superior to the traditional production process, which was not microelectronics based. Yet, AMT was not adopted immediately by all firms; rather the diffusion of AMT in OECD countries followed the traditional "S"-shaped pattern with an initial phase of slow adoption followed by a phase of rapid adoption and eventual saturation (Vickery and Northcott, 1995, pp. 258–260). Since AMT was vastly superior to the status-quo technology, a switch occurred from the traditional technology to AMT and entrepreneurs eventually locked-in to AMT (that is, technology homogeneity holds in the long run). The initial phase of slow adoption of AMT could stem from the hold-up problem: with very few AMT-adopting firms, workers hesitate to acquire AMT-specific skills, thereby preventing entrepreneurs from adopting AMT en masse. But, once the initial phase concludes and a critical mass of entrepreneurs switch to AMT, the hold-up problem disappears: workers readily invest in AMT-specific skills and entrepreneurs adopt AMT fully anticipating the availability of skilled workers. Of course, the arguments I have put forth here are at best suggestive since the diffusion of AMT is a dynamic process that unfolds gradually whereas my model has technology adoption occurring in a single period. often pay for the general training of their workers. They build models in which labor market frictions are such that the increase in a worker's productivity subsequent to training is not matched by an equal increase in her wage. Indeed, greater the labor market frictions (and, hence, greater the wage compression), the sharper are the incentives for a firm to invest in its workers. In contrast, I study the issue of training undertaken by the worker; training incentives in this case weaken with labor market frictions (see specifically Proposition 2). Second, Acemoglu (1996), Redding (1996), and Chander and Thangavelu (2004) examine models in which entrepreneurs choose technologies and workers undertake human capital investments in the context of a search market. A pecuniary externality is shown to exist: when workers, the first movers in these models, conjecture that many entrepreneurs will adopt a skill-using technology in the future, workers' incentives for human capital investments become sharp. This is because in a random matching environment, a worker is now more likely to be matched with a skillusing firm than with a firm posting an unskilled vacancy. In contrast, I use the hold-up problem to provide the link between workers' investment incentives and the number of firms in the market with a given technology.¹¹

The remainder of this paper is structured as follows. In section 2, I analyze the training and production stage of the model. Section 3 provides a treatment of the technology adoption stage of the model. Section 4 considers the optimality of the model's equilibrium while section 5 concludes.

2. TRAINING AND PRODUCTION STAGE

Consider a situation wherein N_t firms have adopted technology t and workers of mass L_t have chosen to be employed by these firms. I now describe the training and production stage of the model and derive as well the payoffs obtained by workers and firms.

The training and production stage begins with workers randomly distributed over the N_t firms. Therefore, each firm is assigned workers of mass L_t/N_t . Consider the history of a representative worker assigned to a particular firm, say firm F_1 .

The worker moves first and engages in job-related training. Training involves costs and produces specialization. Both of these aspects are modeled starkly. It is assumed that when effort e is expended in training, the worker suffers a reduction in utility of magnitude $C_t(e)$ and transforms her unit of unskilled labor into $X_t(e)$ efficiency units

¹¹ The principal focus of the aforementioned papers is different as well. There, the main goal is to show that the identified pecuniary externality results in multiple rational expectations equilibria: in one equilibrium, a low-skill technology is used and this is accompanied by a low level of human capital investment; in the other equilibrium, a high-skill technology is used and this is accompanied by a high level of human capital investment. Such multiple equilibria considerations are ruled out in my model since I maintain (see assumption [A.4]) that all technologies have identical physical attributes. Instead, I ask whether multiple technologies can simultaneously exist in an equilibrium. As pointed out already, the answer is "no."

of type-*t* labor. I assume, as is standard, that $X_t(e)$ ($C_t(e)$) is a strictly increasing and concave (convex) function of e.¹²

With only specialized type-*t* labor at her disposal, the worker's employment opportunities shrink drastically. Specifically, I assume that the worker now becomes unemployable in all firms with technologies other than t.¹³ This assumption is admittedly extreme. However, the results of my paper continue to hold so long as skills acquired through training with technology *t* are at least partially non-transferrable across technologies.

I also assume that skills acquired through training with technology t have the same value in all the N_t firms that have adopted this technology. I model this by positing that the output of a type-t firm equals the number of efficiency units of type-t labor supplied by its workforce. Thus, if a type-t firm employs a workforce of mass K and if the cumulative distribution of efficiency units of type-t labor supplied by its workers is F(x), then total output produced by the firm equals $K \times \int x dF(x)$.

Once training is completed, the worker and firm F_1 commence bargaining over the worker's wage. An informal description of the bargaining game is given in five parts. (1) The N_t firms in market segment t are assumed to be equally spaced on the circumference of a circle of unit length. (2) With probability $\frac{1}{2}$, the worker is given the first move and makes a take-it-or-leave-it wage demand to firm F_1 . Should firm F_1 accept the worker's wage demand, the bargaining game ends: the agreed-on wage is paid, the worker produces in firm F_1 , and the output generated is $X_t(e)$. Instead, if firm F_1 and moves on to firm F_2 . (3) With probability $\frac{1}{2}$, firm F_1 makes a wage offer to the worker. As before, if the worker accepts firm F_1 's wage offer, the bargaining game ends: the worker produces for firm F_1 and obtains, in return, the negotiated wage. On the other hand, should the worker reject firm F_1 's wage offer, she is obliged to travel to firm F_2 . (4) The same bargaining process is repeated with firm F_2 . An impasse in bargaining necessitates a movement to firm F_3 , and so on. (5) The frictions associated with matching are captured with the help of *travel costs*. In particular, I assume that if

¹² When a worker undertakes job-related training, it is empirically the case that the firm bears a large part of the monetary costs involved. Introducing monetary costs in my model leaves the analysis unaltered. To see this, consider the following situation. Let the monetary cost of job-related training be X and let this be forked up entirely by the firm. Two assumptions are implicit in my analysis. First, I have assumed that how much a worker learns from training depends, at least in part, on how much effort, e, she puts in. So, upon completion of training, the efficiency units of type-t labor at the worker's disposal is increasing in e. Second, I have assumed that effort is unverifiable by a third party (e.g., a court). So, an enforceable contract cannot be written that simply requires the worker to fix effort at a certain level, say \bar{e} . In other words, effort remains a choice variable for the worker. Notice also that when effort is chosen by the worker, X is sunk and therefore not directly relevant.

¹³ In real world labor markets, a worker invests in a combination of firm-specific skills and general skills. In this paper, I have abstracted away from issues related to the acquisition of general skills; such skills can be used in any firm and are therefore largely unaffected by hold-up problems. It turns out that the results of this paper remain unchanged when workers are permitted to acquire general skills as well; refer to footnote 17 for details. *d* is the distance between adjacent firms, then the worker loses *d* units of physical labor in transit from one match to the next.¹⁴ With N_t type-*t* firms, *d* is equal to $1/N_t$.

The characteristic of a thick market that I stress in this paper is the relative ease with which a worker can find a new match (firm). The use of travel costs provides a simple way of modeling this idea: when the number of firms with technology *t* increases, the worker's cost (measured by $d = 1/N_t$) of finding another firm of the same type diminishes. Since a setup with many type-*t* firms provides the worker with substantial mobility, intuition suggests that this enhances the worker's bargaining strength and, hence, her wage. A formal analysis of the bargaining game shows this to be the case. The solution to the bargaining game (given in Appendix A) is summarized in Proposition 1.

PROPOSITION 1. The bargaining game has a unique subgame perfect equilibrium. When the worker is given the first move, the wage demanded is $X_t(e)$ and firm F_1 acquiesces. When firm F_1 has the first-mover advantage, the wage offered is $X_t(e) \times [1-(2/N_t) \times (1-1/2^{N_t})]$ and the worker acquiesces. Consequently, the expected wage for the worker after effort e has been expended in training is $X_t(e) \times (1-K(N_t))$, where $K(N_t) \equiv (1/N_t) \times (1-1/2^{N_t})$.

Proposition 1 is unsurprising. Once effort has been expended in training, ex post efficiency requires that the worker produce in firm F_1 . In a world of perfect information, this ex post efficiency is guaranteed.¹⁵ Proposition 1 also provides a convenient and intuitive resolution to the problem of the division of the output, $X_t(e)$, produced in the firm F_1 -worker match. The fraction of the output received by firm F_1 is $K(N_t)$, which declines in N_t . As $N_t \to \infty$ and mobility costs become negligible, firm F_1 loses all of its bargaining power: hence, $K(N_t) \to 0$ as $N_t \to \infty$.

Let $e_t(N_t)$ denote the equilibrium effort level of a worker when there are N_t typet firms. Thus, $e_t(N_t) = \arg \max_e [X_t(e) \times (1 - K(N_t)) - C_t(e)]$. Let e_t^* denote the effort level that is socially optimal. Since social optimality requires that net surplus be maximized, independently of its distribution between worker and firm, $e_t^* = \arg \max_e [X_t(e) - C_t(e)]$. The strict concavity of $X_t(.)$ and the strict convexity of $C_t(.)$ jointly ensure that $e_t(N_t)$ and e_t^* are unique. Proposition 2 compares $e_t(N_t)$ to e_t^* .

PROPOSITION 2. $e_t(.)$ is a strictly increasing function. Furthermore, $e_t(N_t)$ is less than $e_t^*, \forall N_t$.

Proof. Let N_h and N_l be positive integers with N_h greater than N_l . Since the worker's unique equilibrium effort level with N_h type-*t* firms is $e_t(N_h)$, it follows that:

$$X_t(e_t(N_h)) \times (1 - K(N_h)) - C_t(e_t(N_h)) > X_t(e_t(N_l)) \times (1 - K(N_h)) - C_t(e_t(N_l)) .$$
(1)

¹⁴ As an example, if there are three firms with technology t, $d = \frac{1}{3}$. When the worker bargains with the first firm, her output in that firm is $X_t(e)$. Outputs in the second and third firms are, respectively, $\frac{2}{3} \times X_t(e)$ and $\frac{1}{3} \times X_t(e)$.

¹⁵ Note, therefore, that the equilibrium of my model involves no search. However, it is the threat and associated cost of search that determines a worker's bargaining strength.

Similarly, since the worker's unique equilibrium effort level with N_l type-*t* firms is $e_t(N_l)$, it follows that:

$$X_t(e_t(N_l)) \times (1 - K(N_l)) - C_t(e_t(N_l)) > X_t(e_t(N_h)) \times (1 - K(N_l)) - C_t(e_t(N_h)).$$
(2)

Using inequalities (1) and (2), it is immediate that:

$$X_t(e_t(N_h)) \times (K(N_l) - K(N_h)) > X_t(e_t(N_l)) \times (K(N_l) - K(N_h)).$$
(3)

But, N_h greater than N_l implies that $K(N_l)$ is greater than $K(N_h)$. So, inequality (3) implies that $X_t(e_t(N_h))$ is greater than $X_t(e_t(N_l))$. Since output is strictly increasing in effort, $e_t(N_h)$ exceeds $e_t(N_l)$. In other words, $e_t(.)$ is a strictly increasing function.

I now show that $e_t(N_t)$ is less than e_t^* , $\forall N_t$. Since e_t^* is the unique maximizer of the worker-firm net surplus, it follows that:

$$X_t(e_t^*) - C_t(e_t^*) > X_t(e_t(N_t)) - C_t(e_t(N_t)).$$
(4)

Furthermore, since the worker's unique equilibrium effort level with N_t type-t firms is $e_t(N_t)$, it follows that:

$$X_t(e_t(N_t)) \times (1 - K(N_t)) - C_t(e_t(N_t)) > X_t(e_t^*) \times (1 - K(N_t)) - C_t(e_t^*).$$
(5)

Inequalities (4) and (5) combine to yield:

$$(X_t(e_t^*) - X_t(e_t(N_t))) \times K(N_t) > 0.$$
(6)

Notice that inequality (6) implies that $X_t(e_t^*)$ is greater than $X_t(e_t(N_t))$. But, since output is strictly increasing in effort, e_t^* exceeds $e_t(N_t)$ as well.

The economics behind the formalism is transparent. Given the absence of longterm contracts, the worker's effort in acquiring job-specific skills is subject to holdup. Hence, the worker's effort is always less than what is socially optimal. As the number of type-*t* firms increases, the bargaining strength of the worker is enhanced. Predictably, this increase in bargaining strength is accompanied by an increase in effort expended. With N_t type-*t* firms and L_t workers seeking employment in these firms, I define $W_t(N_t)$ to be the net wage of a worker and $\Pi_t(N_t, L_t)$ to be the profit of a firm, gross of entry costs. The expressions for $W_t(N_t)$ and $\Pi_t(N_t, L_t)$ are given below:

$$W_t(N_t) = X_t(e_t(N_t)) \times (1 - K(N_t)) - C_t(e_t(N_t)),$$

$$\Pi_t(N_t, L_t) = (L_t/N_t) \times X_t(e_t(N_t)) \times K(N_t).$$

The expression for $W_t(N_t)$ is obtained as follows. With N_t type-t firms, a worker expends effort $e_t(N_t)$ in training, thereby generating a match-specific output of $X_t(e_t(N_t))$. The worker's share of this output is $(1 - K(N_t))$ while her private cost of training is $C_t(e_t(N_t))$. A worker's net wage, $W_t(N_t)$, is computed as the difference between output received and training costs incurred. The expression for $\Pi_t(N_t, L_t)$ is obtained as follows. With N_t type-t firms and L_t workers, a representative firm obtains L_t/N_t workers. Furthermore, each worker yields a surplus of $X_t(e_t(N_t)) \times K(N_t)$ to the firm with which it is matched. The profits of a firm, $\Pi_t(N_t, L_t)$, is equal to the product of its workforce size and the firm's surplus from each of its workers. For expositional sharpness, I shall now impose assumption [A.1].

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[A.1] $X_t(e_t(N_t)) \times K(N_t)/N_t$ is a declining function of N_t .¹⁶

For a fixed mass of workers seeking employment with type-*t* firms, an increase in N_t means that each firm obtains fewer workers and a smaller fraction of a larger output per worker. Assumption [A.1] posits that, measured in terms of a firm's profit, the first two effects jointly dominate the third one. I end this section by recording some trivial, but substantively important, properties of the W_t (.) and Π_t (., .) functions.

PROPOSITION 3. $W_t(N_t)$ is strictly increasing in N_t ; $\Pi_t(N_t, L_t)$ is strictly increasing in L_t and strictly decreasing in N_t .

Proof. Let N_h and N_l be positive integers with N_h greater than N_l . Since the worker's unique equilibrium effort level with N_h type-*t* firms is $e_t(N_h)$, it follows that:

$$\begin{aligned}
W_t(N_h) &\equiv X_t(e_t(N_h)) \times (1 - K(N_h)) - C_t(e_t(N_h)) \\
&> X_t(e_t(N_l)) \times (1 - K(N_h)) - C_t(e_t(N_l)).
\end{aligned}$$
(7)

But, N_h greater than N_l implies that $1 - K(N_h)$ is greater than $1 - K(N_l)$. Hence,

$$X_t(e_t(N_l)) \times (1 - K(N_h)) - C_t(e_t(N_l)) > X_t(e_t(N_l)) \times (1 - K(N_l)) - C_t(e_t(N_l)) \equiv W_t(N_l).$$
(8)

Using inequalities (7) and (8), it is immediate that $W_t(N_h)$ is greater than $W_t(N_l)$. In other words, $W_t(.)$ is a strictly increasing function.

Now, $\Pi_t(N_t, L_t) = L_t \times [X_t(e_t(N_t)) \times K(N_t)/N_t]$. Notice that $\Pi_t(N_t, L_t)$ is proportional to L_t and, hence, strictly increasing in L_t . $\Pi_t(N_t, L_t)$ is strictly decreasing in N_t because assumption [A.1] ensures that $X_t(e_t(N_t)) \times K(N_t)/N_t$ is strictly decreasing in N_t .

Proposition 3 is intuitive. An increase in N_t gives a worker greater bargaining strength: her net wage rises as a result. Since firms are modeled as being labor constrained, a firm's profit is proportional to the mass of workers assigned to it. Consequently, an increase in L_t results in greater profit accruing to each type-*t* firm. Finally, assumption [A.1] directly implies that the profit of a type-*t* firm declines with the number of type-*t* firms in the market.¹⁷

¹⁶ I thank an anonymous referee for suggesting that I provide an example of functions, $C_t(.)$ and $X_t(.)$, that justifies assumption [A.1]. Let $X_t(e) = Ae^{\frac{1}{2}}$, A > 0 and $C_t(e) = \frac{1}{2}e^2$. Given $C_t(.)$ and $X_t(.)$, it is immediate that $e_t(N_t) = (\frac{1}{2}A \times (1 - K(N_t)))^{\frac{2}{3}}$. Thus, $R(N_t) \equiv X_t(e_t(N_t)) \times K(N_t)/N_t$ equals $\alpha \times (1 - K(N_t))^{\frac{1}{3}} \times K(N_t)/N_t$, where $\alpha \equiv 2^{-\frac{1}{3}} \times A^{\frac{4}{3}}$. Now, by viewing N_t as a non-negative real number, the domain of R(.) can be extended from Z_t to \Re_t . This extension makes R'(.) well-defined. Assumption [A.1] follows upon noting that $R'(N_t) < 0$.

¹⁷ In the training and production stage of the model, a worker is permitted to invest in technology-specific skills only. Is this assumption critical? I now outline a scenario wherein a worker is permitted to invest in general skills as well but which leaves the main results of this paper—Proposition 4 and Proposition 5 unaltered. The scenario is as follows. The worker expends effort *e* in acquiring technology-specific skills and effort *e_g* in acquiring general skills; given (*e*, *e_g*), the effort cost incurred by the worker is $C(e) + C_g(e_g)$. Upon completion of training, the amount of type-*t* labor at the worker's disposal is $X_t(e)$ while the amount of all-purpose labor is $X_g(e_g)$. Assume, now, that hold-up problems are associated with technology-specific skills only. Then, the worker's net wage is $(1 - K(N_t)) \times X_t(e) + r \times X_g(e_g)$, where *r* is a number in the

3. TECHNOLOGY ADOPTION STAGE

The technology adoption stage begins with each entrepreneur deciding whether to form a firm and conditional on market entry, the technology $t \in T$ to adopt. Entrepreneurs' decisions therefore result in an outcome vector $(N_1, \ldots, N_{|T|})$, where N_t is the number of type-*t* firms in the market.

Given $(N_1, \ldots, N_{|T|})$, each worker decides whether to enter the labor market and conditional on market entry, the type of firm in which to seek employment. Workers' behavior is represented by |T| functions; $\bar{L}_t : Z_+^{|T|} \to \Re_+$, $t = 1, \ldots, |T|$. The interpretation is as follows: $\bar{L}_t(N_1, \ldots, N_{|T|})$ is the mass of workers seeking employment in type-*t* firms when the outcome vector is $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$. I shall require $\bar{L}_t(.), t = 1, \ldots, |T|$, to satisfy three conditions:

- [E.1] $\sum_{t=1}^{|T|} \bar{L}_t(N_1, \dots, N_{|T|}) \leq L, \ \forall (N_1, \dots, N_{|T|}) \in Z_+^{|T|}.$
- $[E.2] \quad \bar{L}_t(N_1, \dots, N_{|T|}) \times [W_t(N_t) 1] \ge 0, \ \forall t \in T, \ \forall (N_1, \dots, N_{|T|}) \in Z_+^{|T|}.$
- [E.3] There does not exist $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$ and $Q \in \mathfrak{R}_+^{|T|}$ such that (1) $\sum_{t=1}^{|T|} Q_t \leq L$ and (2) $\sum_{t=1}^{|T|} Q_t \times [W_t(N_t) - 1] > \sum_{t=1}^{|T|} \tilde{L}_t(N_1, \ldots, N_{|T|}) \times [W_t(N_t) - 1].$

Condition [E.1] is a feasibility condition. It states that the total mass of workers seeking employment in the various firms cannot exceed *L*. Condition [E.2] is an individual rationality condition for workers. Should workers choose employment in type-*t* firms (that is, $L_t(N_1, \ldots, N_{|T|}) > 0$), the net wage received cannot be less than one, a worker's guaranteed return from home production. Finally, given any $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$, condition [E.3] requires that a worker's choice of firm type maximizes her net wage.

Two bits of notation need to be introduced now. Relative to $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$, let $(N_1, \ldots, N_{|T|})|_t$ denote the outcome vector with an additional type-*t* firm in the market.¹⁸ Relative to $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$, let $(N_1, \ldots, N_{|T|})|_{ij}$ denote the outcome vector when a type-*i* firm switches to a type-*j* firm.¹⁹ With the notation in place, I call $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ an *equilibrium outcome vector* if there exists { $\bar{L}_t(.) : t = 1, \ldots, |T|$ }, obviously satisfying [E.1]–[E.3], such that the following three requirements are met:

[E.4] $\bar{N}_t \times [\Pi_t(\bar{N}_t, \bar{L}_t(\bar{N}_1, \dots, \bar{N}_{|T|})) - \alpha_t] \ge 0, \ \forall t \in T.$

[E.5] $\Pi_t(\bar{N}_t+1, \bar{L}_t((\bar{N}_1, \dots, \bar{N}_{|T|})|_t)) - \alpha_t \leq 0, \forall t \in T.$

interval [0, 1]. Finally, since the worker maximizes net wage, *e* and *e*_g are chosen to maximize $[X_t(e) \times (1 - K(N_t)) - C(e)] + [r \times X_g(e_g) - C_g(e_g)]$. Notice, therefore, that *e*_g does not affect the optimal choice of *e*; as before, the optimal *e* increases with the worker's share of the output $X_t(e)$ and, hence, N_t .

¹⁸ Notice that $(N_1, \ldots, N_{|T|})|_t$ and $(N_1, \ldots, N_{|T|})$ differ in one way: the *t*'th component of $(N_1, \ldots, N_{|T|})|_t$ exceeds the *t*'th component of $(N_1, \ldots, N_{|T|})$ by one. ¹⁹ Notice that $(N_1, \ldots, N_{|T|})|_{ij}$ and $(N_1, \ldots, N_{|T|})$ differ in two ways: the *i*'th component of

¹⁹ Notice that $(N_1, \ldots, N_{|T|})|_{ij}$ and $(N_1, \ldots, N_{|T|})$ differ in two ways: the *i*'th component of $(N_1, \ldots, N_{|T|})|_{ij}$ is less than the *i*'th component of $(N_1, \ldots, N_{|T|})$ by one and the *j*'th component of $(N_1, \ldots, N_{|T|})|_{ij}$ is more than the *j*'th component of $(N_1, \ldots, N_{|T|})$ by one.

$$[E.6] \quad \Pi_j(\bar{N}_j + 1, \ \bar{L}_j((\bar{N}_1, \dots, \bar{N}_{|T|})|_{ij})) - \alpha_j \leq \Pi_i(\bar{N}_i, \ \bar{L}_i(\bar{N}_1, \dots, \bar{N}_{|T|})) - \alpha_i, \ \forall i, \ j \in T, \ i \neq j.$$

Consider $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ and assume that $\bar{N}_t > 0$. Notice that $\bar{L}_t(\bar{N}_1, \ldots, \bar{N}_{|T|})$ is the mass of workers seeking employment in type-*t* firms and $\Pi_t(\bar{N}_t, \bar{L}_t(\bar{N}_1, \ldots, \bar{N}_{|T|}))$ is the gross profit of a type-*t* firm. Condition [E.4] says that if a type-*t* firm forms in equilibrium, the gross profit must cover the fixed cost of market entry. Now, if an additional entry of a type-*t* firm takes place, the outcome vector changes from $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ to $(\bar{N}_1, \ldots, \bar{N}_{|T|})|_t$ and the net profit of the entering firm is $\Pi_t(\bar{N}_t +$ 1, $\bar{L}_t((\bar{N}_1, \ldots, \bar{N}_{|T|})|_t)) - \alpha_t$. Relative to the equilibrium outcome vector, condition [E.5] requires that an additional entry of a type-*t* firm be weakly unprofitable. Finally, suppose an entrepreneur who forms a type-*i* firm instead. This means that the outcome vector changes from $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ to $(\bar{N}_1, \ldots, \bar{N}_{|T|})|_{ij}$ and the net profit of the deviating entrepreneur is therefore $\Pi_j(\bar{N}_j + 1, \bar{L}_j((\bar{N}_1, \ldots, \bar{N}_{|T|})|_{ij})) - \alpha_j$. Condition [E.6] says that this net profit must be weakly less than what the entrepreneur obtains in equilibrium.

To simplify matters, I now employ the following additional assumptions:

- $[A.2] \quad \max_{e} [X_t(e)/2 C_t(e)] < 1, \forall t \in T.$
- [A.3] $\max_{e} [X_t(e) C_t(e)] > 1, \forall t \in T.$
- [A.4] $X_t(e) = X(e), C_t(e) = C(e), \text{ and } \alpha_t = \alpha, \forall t \in T.$

Assumption [A.2] reduces the number of cases to be considered. When only one type-*t* firm enters the market $(N_t = 1 \text{ and } K(1) = \frac{1}{2})$, assumption [A.2] implies that no worker finds it worthwhile to seek employment in this firm. If assumption [A.3] is violated, then the net surplus associated with a type-*t* firm-worker match is lower than that obtained through home production. Consequently, assumption [A.3] is necessary if the formation of type-*t* firms is to be a possibility in equilibrium. Assumption [A.4] posits that all technologies have identical physical attributes. It is invoked *solely* for notational convenience.

Given $(L, T, X(.), C(.), \alpha)$, notice that the *trivial outcome* $(\bar{N}_t = 0 | t = 1, ..., T)$ is always an equilibrium outcome vector. Why? Relative to the trivial outcome, suppose an entrepreneur deviates and sets up a type-*t* firm. Because of assumption [A.2], no worker seeks employment in this firm (that is, $\bar{L}_t((0, ..., 0)|_t) = 0$). But this means that it is unprofitable for the entrepreneur to incur the fixed cost of market entry, α , and set up the type-*t* firm. I now search for equilibria that are distinct from the trivial outcome. To this end, three bits of notation are introduced. Let N^+ be the smallest integer such that $W(N^+) \ge 1$. Let M(N) be the smallest positive real number such that $\Pi(N, M(N)) \ge \alpha$. Finally, let $\bar{l} \equiv M(N^+) \times N^+$.²⁰ Proposition 4 (proved in Appendix B) characterizes the equilibrium outcome vector(s).

²⁰ Since the technologies are identical, $W_t(.) = W(.)$ and $\Pi_t(.,.) = \Pi(.,.), \forall t \in T$. I have therefore dropped the subscript indexing the relevant technology.

PROPOSITION 4. Consider a situation summarized by $(L, T, X(.), C(.), \alpha)$. (1) For $L < \overline{l}$, the unique equilibrium outcome vector is the trivial outcome, $(\overline{N}_t = 0 | t = 1, ..., |T|)$. (2) For $L \ge \overline{l}$, besides the trivial outcome, there are |T| essentially equivalent equilibrium outcome vectors. In the first equilibrium outcome vector, all firms that enter the market adopt technology 1; in the second equilibrium outcome vector, all firms that enter the market adopt technology 2; and so on. Focus now on an equilibrium outcome vector where technology $\overline{t} \in T$ is adopted. Then, all L workers seek employment in these type- \overline{t} firms and the number of firms entering the market, denoted $\overline{N}(L)$, is the largest integer satisfying $L \ge \overline{N}(L) \times M(\overline{N}(L))$ (see the footnote for further details).²¹

What are the implications of the above proposition? Proposition 4 identifies a *threshold effect*; that is, only when $L \ge \overline{l}$ does the adoption of any technology (say, $\overline{t} \in T$) become an equilibrium possibility. The reason for this critical mass is somewhat novel. Given the presence of ex post opportunism by firms, one requires at least N^+ firms of type- \overline{t} to draw workers away from home production. Moreover, each type- \overline{t} firm requires a workforce size of no less than $M(N^+)$ to cover its fixed cost of entry. Consequently, $\overline{t} \in T$ becomes *viable* only when the mass of workers assigned to it weakly exceeds $N^+ \times M(N^+) \equiv \overline{l}$.

Consider now a situation with $L \ge \overline{l}$. Despite the availability of multiple technologies, Proposition 4 points out that all firms end up adopting the same technology in equilibrium. Put differently, technology dispersion is not an equilibrium phenomenon. What is the intuition for this finding?

Contrary to the claim in Proposition 4, suppose that distinct technologies are adopted in equilibrium. To fix ideas, let technologies t_1 and t_2 be adopted in equilibrium. Three observations are relevant now. First, since both firm types attract workers, a worker's net wage in a type- t_1 firm must be equal to that in a type- t_2 firm. But, since W(.) is a strictly increasing function, the market must therefore have an equal number of type- t_1 and type- t_2 firms. Let *n* denote the number of type- t_i , i = 1, 2, firms in the market. Second, equilibrium market entry cannot be unprofitable. So, each of the *n* firms of type t_i , i = 1, 2, obtains enough workers to generate gross profits that weakly exceed the cost of market entry, α . Third, since the total mass of workers is *L*, the mass of workers seeking employment in at least one of the two firm types (say, t_1 -type firms) has to be weakly less than L/2.

Consider now an additional entry of a type- t_1 firm relative to the putative equilibrium. Post entry, the number of type- t_1 firms, (n + 1), strictly exceeds the number of type- t_2 firms, n. Therefore, all workers seek employment in type- t_1 firms and the entrant receives workers of mass L/(n + 1). It turns out that with a workforce of this size, the entrant more than covers its fixed cost of entry. But, if additional entry of a type- t_1

²¹ Notice, therefore, that there are |T| equilibrium outcome vectors because firms can enter with any one out of the |T| available technologies. Suppose that all firms adopt technology 1. Then, the equilibrium outcome vector is $(\bar{N}(L), 0, ..., 0)$. Furthermore, $\bar{L}_1(\bar{N}(L), 0, ..., 0) = L$, and $\bar{L}_t(\bar{N}(L), 0, ..., 0) = 0$, $t \neq 1$.

firm is strictly profitable, the technology dispersion scenario with which I began cannot constitute an equilibrium.

I close this section by examining the critical role of assumption [A.1] in my analysis. Consider the equilibrium outcome vector in which all firms that enter the market do so with technology \bar{t} . Since an individual firm's gross profit, by assumption [A.1], is decreasing in the number of firms in the market, the zero profit condition ensures that $\bar{N}(L)$ is unique. What happens when assumption [A.1] is violated? Now, there can be multiple equilibria that are Pareto-ranked. Specifically, if a large number of firms enter the market with technology \bar{t} , the resulting amelioration of the hold-up problem encourages workers to invest substantially in skills. The resultant increase in labor productivity makes, in turn, large scale firm entry privately profitable. Similarly, an equilibrium with limited firm entry and limited skill investments can be justified as well. Indeed, Burdett and Smith (2002) study a model of labor market search and use essentially this argument to generate the possibility of a low skill trap.

4. EFFICIENCY OF THE EQUILIBRIUM ALLOCATION

Consider a situation wherein the mass of workers, L, weakly exceeds \overline{l} . Recall that Proposition 4 says that there is an equilibrium outcome vector in which $\overline{N}(L)$ firms form and adopt the same technology $\overline{t} \in T$. My goal is to ascertain whether this equilibrium allocation is associated with excessive or deficient entry by firms.

An omnipotent social planner, with a desire to maximize the aggregate payoffs of workers and firms, would allow one firm to form and command all workers to undertake training at the socially optimal level. In this paper, I regard such a solution as infeasible. Instead, I only invest the social planner with the ability to regulate the amount of market entry by firms. Given this formulation, the social planner faces a tradeoff: a worker's inefficiently low effort in job-related training can be raised only through incurring the fixed cost of added entry by firms.

Some additional notation is now required. When N firms adopt the same technology $\bar{t} \in T$, let e(N) denote the equilibrium effort level of a worker and let V(N)denote the aggregate payoffs of workers and firms. Notice, therefore, that e(N) = $\arg \max_e [X(e) \times (1 - K(N)) - C(e)]$ while $V(N) = L \times (X(e(N)) - C(e(N))) - N \times \alpha$. Proposition 5 shows that V'(N) evaluated at $\bar{N}(L)$ may be positive, in which case equilibrium entry by firms is *likely* to be inefficiently low, or negative, in which case equilibrium entry by firms is *likely* to be excessive.²²

²² Two observations are relevant here. First, by viewing N as a non-negative real number, the domain of V(.) can be extended from Z_+ to \Re_+ . This extension makes V'(N) well-defined. Second, why is the interpretation of V'(N) evaluated at $\bar{N}(L)$ qualified by "likely"? Suppose V'(N) evaluated at $\bar{N}(L)$ is positive. This means that $V(\bar{N}(L) + \epsilon) > V(\bar{N}(L))$ for ϵ that is small enough. It does not necessarily follow that $V(\bar{N}(L) + 1) > V(\bar{N}(L))$.

PROPOSITION 5. Let $\eta(N)$ be the elasticity of output from a match with respect to the number of firms, N—that is, $\eta(N) \equiv [dX(e(N))/dN] \times [N/X(e(N))]$.²³ If $\eta(N)$ evaluated at $\bar{N}(L)$ is more than one, V'(N) evaluated at $\bar{N}(L)$ is positive. If $\eta(N)$ evaluated at $\bar{N}(L)$ is less than $h \equiv [X(e(\bar{N}(L) + 1)) \times K(\bar{N}(L) + 1)/(\bar{N}(L) + 1)] \div [X(e(\bar{N}(L))) \times K(\bar{N}(L))/\bar{N}(L)], V'(N)$ evaluated at $\bar{N}(L)$ is negative.²⁴

Proof. Given the expression for V(N), it is immediate that:

$$V'(N) = L \times (X'(e(N)) - C'(e(N))) \times e'(N) - \alpha.$$
(9)

The first-order condition for a worker's equilibrium effort choice, e(N), is as follows:

$$X'(e(N)) - C'(e(N)) = X'(e(N)) \times K(N).$$
(10)

Using equations (9) and (10), I obtain:

$$V'(N) = L \times X'(e(N)) \times K(N) \times e'(N) - \alpha.$$
⁽¹¹⁾

Now, the $\overline{N}(L)$ firms in the market must earn profits that are sufficient to cover the fixed cost of market entry. Hence:

$$L \times X(e(N)) \times K(N)|_{N=\bar{N}(L)} \ge \bar{N}(L) \times \alpha.$$
(12)

Furthermore, free entry ensures that if $(\overline{N}(L)+1)$ firms enter the market, strictly positive profits net of entry costs are not possible. Hence:

$$L \times X(e(N)) \times K(N)|_{N=\tilde{N}(L)+1} \le (\tilde{N}(L)+1) \times \alpha.$$
(13)

Equation (11) and inequality (12) combine to yield:

$$V'(\bar{N}(L)) \ge \left[\frac{L \times K(\bar{N}(L)) \times X(e(\bar{N}(L)))}{\bar{N}(L)}\right] \times \left[\eta(\bar{N}(L)) - 1\right].$$
(14)

Thus, V'(N) evaluated at $\overline{N}(L)$ is positive if $\eta(\overline{N}(L))$ exceeds 1. Similarly, equation (11) and inequality (13) combine to yield:

$$V'(\bar{N}(L)) \le \left[\frac{L \times K(\bar{N}(L)) \times X(e(\bar{N}(L)))}{\bar{N}(L)}\right] \times \left[\eta(\bar{N}(L)) - h\right].$$
(15)

Thus, V'(N) evaluated at $\overline{N}(L)$ is negative if $\eta(\overline{N}(L))$ is less than h.

In a model where total output is unaffected by the number of firms in a market, the social planner regards all market entry beyond the first firm as inefficient. Consequently, free entry (almost) always results in excessive entry by firms. In my model, entry serves a socially useful role: it enhances the surplus generated in every worker-firm match. Since an entrepreneur contemplating entry does not internalize this positive externality,

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²³ By viewing N as a non-negative real number, the domain of e(.) can be extended from Z_+ to \Re_+ . This extension makes $\eta(N)$ well-defined.

²⁴ Notice that *h* is [gross profit per firm when there are $(\bar{N}(L) + 1)$ firms with technology $\bar{t} \in T$]÷[gross profit per firm when there are $\bar{N}(L)$ firms with technology $\bar{t} \in T$]; assumption [A.1] ensures that *h* is less than 1.

there is the novel possibility of too little entry in equilibrium.²⁵ Specifically, Proposition 5 says that if $\eta(\bar{N}(L))$ exceeds one, additional entry at the margin likely yields a net social benefit—that is, the consequent increase in total output exceeds the fixed cost of entry. The assumptions of my model are not sufficient to determine the magnitude of $\eta(\bar{N}(L))$. Hence, the equilibrium allocation may be associated with excessive, deficient or optimal entry by firms.

5. CONCLUSION

In this paper, I study a situation wherein a good can be produced in different ways, and I let $T \equiv \{1, ..., |T|\}$ index the set of exogenously available technologies. My model highlights two-way linkages between the technology adoption decisions of entrepreneurs and the skill acquisition decisions of workers. The linkages that I emphasize are as follows. *First*, when a worker's investment in skills corresponding to technology $\tilde{t} \in T$ is subject to hold-up, the structure of the market—in particular, the number of firms with technology \tilde{t} —determines the extent to which the acquired skills are transferable across firms. The number of type- \tilde{t} firms in the market therefore directly determines a worker's bargaining power and indirectly affects the worker's investment in type- \tilde{t} skills. *Second*, the technology adoption decision of an entrepreneur anticipates and responds to workers' behavior. Thus, for example, no entrepreneur forms a type- \hat{t} firm if workers are presumed to seek employment exclusively in type- \tilde{t} firms and acquire type- \tilde{t} skills as a result.

My model generates two interesting results. *First*, despite the possibility of technology dispersion in the model, I show that all entrepreneurs end up adopting the same technology in equilibrium. This technology lock-in conclusion sheds light on an otherwise puzzling empirical question: Why do firms, principally in developing countries, herd on an inefficient technology even though the adoption of an obviously superior and available technology is advocated by policy experts? *Second*, consider the equilibrium with only type- \bar{i} firms in the market. The formation of a type- \bar{i} firm ameliorates the hold-up problem and enhances workers' incentives for skill acquisition. Since no entrepreneur contemplating entry internalizes this externality, market equilibrium may be associated with excessive or deficient entry of type- \bar{i} firms.

6. APPENDIX A

The existence and uniqueness of the subgame perfect equilibrium of the bargaining game are trivial to establish. Therefore, I only specify the equilibrium wage offers at the various nodes of the bargaining game.

Consider the following situation: the worker is matched with firm F_k but is unaware as yet of who is to make the wage offer. In this circumstance, let V(k) denote the

²⁵ Note that my model complements that by Mankiw and Whinston (1986). There, when an entrepreneur enters a market, the sales volume of all existing firms is lowered. Since the entrepreneur does not internalize this negative business stealing externality, excessive entry occurs in equilibrium.

worker's expected continuation payoff in the unique subgame perfect equilibrium of the bargaining game.

Consider the case where the worker is matched with firm F_{N_t} . With probability $\frac{1}{2}$, the worker makes the wage offer. Since firm F_{N_t} acquiesces to all wage demands less than or equal to the worker's output in that firm, the worker extracts the full surplus from the match—that is, $X_t(e)/N_t$. On the other hand, with probability $\frac{1}{2}$, firm F_{N_t} is assigned the first-mover advantage. Since the worker's outside option is worth zero, the wage offered by firm F_{N_t} is also zero. Consequently, the following expression for $V(N_t)$ obtains:

$$V(N_t) = \frac{X_t(e)}{2 \times N_t} \,. \tag{16}$$

For $k \in \{1, ..., N_t - 1\}$, I now derive an expression for V(k). When the worker makes the wage demand to firm F_k , the wage demanded equals her entire value to that firm that is, $X_t(e) \times (1 - (k - 1)/N_t)$. When firm F_k moves first, it offers the worker a wage equal to the worker's outside option—that is, V(k + 1). Consequently, the following expression for V(k) obtains:

$$V(k) = \frac{1}{2} \times X_t(e) \times \left(1 - \frac{k-1}{N_t}\right) + \frac{1}{2} \times V(k+1).$$
(17)

Equations (16) and (17) are jointly employed to solve for V(1):

$$V(1) = X_t(e) \times \left(1 - \frac{1 - 2^{-N_t}}{N_t}\right).$$
(18)

By construction, observe that V(1) is the expected payoff to the worker after effort *e* has been expended in training. Observe also that the bargaining game ends instantaneously—that is, the worker and firm F_1 reach an agreement. When the worker moves first, the wage demanded is $X_t(e)$; when firm F_1 moves first, the wage offered is $V(2) = X_t(e) \times (1 - 2 \times (1 - 2^{-N_t})/N_t)$.

7. APPENDIX B

The proof of Proposition 4 proceeds in seven steps that are detailed below.

Step 1: (i) W(N) is a strictly increasing function of N, (ii) M(N) is a strictly increasing function of N, and (iii) $1 < N^+ < \infty$.

Proof. Parts (i) and (ii) follow directly from Proposition 3. Consider part (iii). [A.2] guarantees that $N^+ > 1$. Now, notice that $K(N) \to 0$ as $N \to \infty$. Therefore, [A.3] ensures that as $N \to \infty$, W(N) approaches a number exceeding 1. Since W(.) is strictly increasing, N^+ is unique and finite.

Step 2: Let $(N_1, \ldots, N_{|T|}) \in Z_+^{|T|}$. Then, $\overline{L}_t(N_1, \ldots, N_{|T|}) > 0$ implies that (i) $N_t \ge N^+$ and (ii) $N_t = \max\{N_1, \ldots, N_{|T|}\}$.

Proof. Consider part (i). It follows from [E.2] that $\overline{L}_t(N_1, \ldots, N_{|T|}) > 0$ implies $W(N_t) \ge 1$. But, by definition, this means that $N_t \ge N^+$. To prove part (ii), assume the contrary. Thus, there exists $t' \ne t$ such that $N_{t'} > N_t$. Since W(.) is a strictly

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increasing function, $W(N_{t'}) > W(N_t)$. With $W(N_{t'}) > W(N_t)$, observe that [E.3] implies $\bar{L}_t(N_1, \dots, N_{|T|}) = 0$, thereby violating the premise with which I began.

Step 3: Let $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ be an equilibrium outcome vector. Then, (i) $\bar{N}_t > 0$ implies that $\bar{N}_t \ge N^+$. Furthermore, (ii) $\bar{N}_t > 0$ and $\bar{N}_{t'} > 0$ implies that $\bar{N}_t = \bar{N}_{t'}$.

Proof. To prove part (i), assume the contrary: $0 < \bar{N}_t < N^+$. With $\bar{N}_t < N^+$, step 2 implies that $\bar{L}_t(\bar{N}_1, \ldots, \bar{N}_{|T|}) = 0$. Since no worker seeks employment in a type-*t* firm, $\Pi(\bar{N}_t, \bar{L}_t(\bar{N}_1, \ldots, \bar{N}_{|T|})) = 0$. But, this zero gross profit conclusion violates [E.4]. To prove part (ii), assume the contrary. So, let $\bar{N}_t < \bar{N}_{t'}$. Once again, step 2 implies that $\bar{L}_t(\bar{N}_1, \ldots, \bar{N}_{|T|}) = 0$. With no worker seeking employment in a type-*t* firm, [E.4] is violated.

Step 4: Consider a situation $(L, T, X(.), C(.), \alpha)$ with $L < \overline{l}$. Then, the unique equilibrium outcome vector is the trivial outcome.

Proof. Suppose, to the contrary, that there exists a non-trivial equilibrium outcome vector, $(\bar{N}_1, \ldots, \bar{N}_{|T|})$. Then, there exists *t* such that $\bar{N}_t \ge N^+$ (see step 3). For each type-*t* firm to cover its fixed cost of entry, α , the mass of workers seeking employment with type-*t* firms must weakly exceed $M(\bar{N}_t) \times \bar{N}_t$. Furthermore, since M(.) is a strictly increasing function, $M(\bar{N}_t) \times \bar{N}_t \ge M(N^+) \times N^+ \equiv \bar{l}$. Observe, now, that the mass of workers seeking employment in type-*t* firms, $\bar{L}_t(\bar{N}_1, \ldots, \bar{N}_{|T|})$, is bounded above by *L* which, in turn, is strictly less than \bar{l} . Hence, [E.4] cannot be satisfied by $(\bar{N}_1, \ldots, \bar{N}_{|T|})$.

Before proceeding further, I provide an outline of the remainder of the proof. Hereafter, I consider a situation $(L, T, X(.), C(.), \alpha)$ with $L \ge \overline{l}$. Steps 5 and 6 show that *if* there exists an equilibrium outcome vector other than the trivial outcome, it must have the following form: (1) All firms that enter the market do so with the same technology (say, $\overline{t} \in T$), and (2) the number of firms that enter is determined by the two free entry conditions, [E.4] and [E.5]. Step 7 shows that an outcome vector satisfying (1) and (2) is indeed an *equilibrium* outcome vector.

Step 5: Consider a situation $(L, T, X(.), C(.), \alpha)$ with $L \ge \overline{l}$. Let $(\overline{N}_1, \ldots, \overline{N}_{|T|})$ be an equilibrium outcome vector that is not the trivial outcome. Then, there is only one technology (say, $\overline{t} \in T$) such that $\overline{N}_{\overline{t}} > 0$. In other words, in a non-trivial equilibrium outcome vector, all firms that enter the market choose the same technology, \overline{t} .

Proof. Given $(\bar{N}_1, \ldots, \bar{N}_{|T|})$, let $A \subseteq T$ index the technologies that are adopted by firms entering the market (that is, $t \in A$ if $\bar{N}_t > 0$). Assume, contrary to the claim in step 5, that $|A| \ge 2$.

Since the total mass of workers seeking employment is bounded above by L, there exists $\hat{t} \in A$ such that $\bar{L}_{\hat{t}}(\bar{N}_1, \dots, \bar{N}_{|T|}) \leq L/|A|$. Furthermore, since [E.4] requires each of the type- \hat{t} firms to cover its fixed cost of entry, $\Pi(\bar{N}_{\hat{t}}, \bar{L}_{\hat{t}}(\bar{N}_1, \dots, \bar{N}_{|T|})) \geq \alpha$.

Because $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ is an equilibrium outcome vector, step 3 says that $\bar{N}_t = k, \forall t \in A$, where k is an integer weakly exceeding N^+ . Now, relative to $(\bar{N}_1, \ldots, \bar{N}_{|T|})$, consider an additional entry of a type- \hat{i} firm; following entry the outcome vector

changes from $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ to $(\bar{N}_1, \ldots, \bar{N}_{|T|})|_{\hat{t}}$. Notice that in $(\bar{N}_1, \ldots, \bar{N}_{|T|})|_{\hat{t}}$, the number of type- \hat{t} firms strictly exceeds N^+ and strictly exceeds the number of firms of any other type. Hence, step 2 implies that $\bar{L}_{\hat{t}}((\bar{N}_1, \ldots, \bar{N}_{|T|})|_{\hat{t}}) = L$. In other words, post entry all workers seek employment with type- \hat{t} firms.

Observe that the gross profit of the entrant is $\Pi(\bar{N}_{\hat{i}}+1,L) = [L \times X(e(\bar{N}_{\hat{i}}+1)) \times K(\bar{N}_{\hat{i}}+1)]/(\bar{N}_{\hat{i}}+1)$. But, X(.) is strictly increasing in effort and effort is, by Proposition 2, strictly increasing in the number of type- \hat{i} firms; so, $X(e(\bar{N}_{\hat{i}}+1)) > X(e(\bar{N}_{\hat{i}}))$. It is also easy to show that $K(\bar{N}_{\hat{i}}+1)/(\bar{N}_{\hat{i}}+1) > K(\bar{N}_{\hat{i}})/(|A| \times \bar{N}_{\hat{i}})$ (see the footnote for details).²⁶ Therefore, $\Pi(\bar{N}_{\hat{i}}+1,L) > [(L/|A|) \times X(e(\bar{N}_{\hat{i}})) \times K(\bar{N}_{\hat{i}})]/\bar{N}_{\hat{i}}$. But, \hat{t} is chosen such that $\tilde{L}_{\hat{i}}(\bar{N}_{1}, \ldots, \bar{N}_{|T|}) \leq L/|A|$. So, $\Pi(\bar{N}_{\hat{i}}+1,L)$ strictly exceeds $[\tilde{L}_{\hat{i}}(\bar{N}_{1}, \ldots, \bar{N}_{|T|}) \times X(e(\bar{N}_{\hat{i}})) \times K(\bar{N}_{\hat{i}})]/\bar{N}_{\hat{i}} \equiv \Pi(\bar{N}_{\hat{i}}, \bar{L}_{\hat{i}}(\bar{N}_{1}, \ldots, \bar{N}_{|T|}))$.

Since $(\bar{N}_1, \ldots, \bar{N}_{|T|})$ is an equilibrium outcome vector, [E.4] says that $\Pi(\bar{N}_{\hat{t}}, \bar{L}_{\hat{t}}(\bar{N}_1, \ldots, \bar{N}_{|T|})) - \alpha \ge 0$. But, this means that $\Pi(\bar{N}_{\hat{t}} + 1, L) - \alpha > 0$, thereby violating [E.5]. So, the premise with which the proof begins– $|A| \ge 2$ -cannot be true after all.

Step 6: Consider a situation $(L, T, X(.), C(.), \alpha)$ with $L \ge \overline{l}$. Suppose $(\overline{N}_1, \ldots, \overline{N}_{|T|})$ is an equilibrium outcome vector with $\overline{N}_{\overline{l}} > 0$. Then, $\overline{N}_{\overline{l}}$ is fixed at $\overline{N}(L)$, where $\overline{N}(L)$ is the largest integer satisfying $L \ge \overline{N}(L) \times M(\overline{N}(L))$.

Proof. If $\bar{N}_{\bar{l}}$ exceeds $\bar{N}(L)$, the type- \bar{l} firms, by construction, cannot cover the fixed cost of market entry, thereby violating [E.4]. If $\bar{N}_{\bar{l}}$ is less than $\bar{N}(L)$, an additional entry of a type- \bar{l} firm is strictly profitable, thereby violating [E.5]. Summing up, $\bar{N}_{\bar{l}}$ must equal $\bar{N}(L)$.

Step 7: Consider a situation $(L, T, X(.), C(.), \alpha)$ with $L \ge \overline{l}$. Suppose $\overline{N}_{\overline{l}} = \overline{N}(L)$ and $\overline{N}_{t} = 0$, $\forall t \neq \overline{l}$. Then, $(\overline{N}_{1}, \ldots, \overline{N}_{|T|})$ is an equilibrium outcome vector.

Proof. Given $(\bar{N}_1, \ldots, \bar{N}_{|T|})$, it is trivial to pick $\{\bar{L}_t : t = 1, \ldots, |T|\}$ such that [E.1]–[E.6] are satisfied.

REFERENCES

- Acemoglu, D. (1996). "A Microfoundation for Social Increasing Returns in Human Capital Accumulation." *Quarterly Journal of Economics*, Vol. 111, 779–804.
- Acemoglu, D., and J.-S. Pischke. (1998). "Why do Firms Train? Theory and Evidence." Quarterly Journal of Economics, Vol. 113, 79–119.
- Acemoglu, D., and J.-S. Pischke. (1999). "The Structure of Wages and Investment in General Training." Journal of Political Economy, Vol. 107, 539–572.
- Arthur, W.B. (1994). Increasing Returns and Path Dependence in the Economy. Ann Arbor: The University of Michigan Press.
- Bartel, A., and F. Lichtenberg. (1987). "The Comparative Advantage of Educated Workers in Implementing New Technologies." *Review of Economics and Statistics*, Vol. 69, 1–11.
- Barzel, Y., and B. Yu. (1984). "The Effect of the Utilization Rate on the Division of Labor." *Economic Enquiry*, Vol. 22, 155–173.

²⁶ Since $|A| \ge 2$ and $\bar{N}_{\hat{t}} \ge N^+ > 1$, it suffices to show that $K(N+1)/(N+1) > K(N)/(2 \times N)$ for $N \ge 2$. But, this inequality follows immediately upon noting that $K(N) \equiv (1/N) \times (1 - 1/2^N)$.

- Borland, J., and X. Yang. (1992). "Specialization and a New Approach to Economic Organization and Growth." American Economic Review, Vol. 82, 386–391.
- Burdett, K., and E. Smith. (2002). "The Low Skill Trap." European Economic Review, Vol. 46, 1439– 1451.
- Chander, P., and S. Thangavelu. (2004). "Technology Adoption, Education and Immigration Policy." Journal of Development Economics, Vol. 75, 79–94.
- Goldin, C., and L. Katz. (1996). "Technology, Skill, and the Wage Structure: Insights from the Past." *American Economic Review*, Vol. 86, 252–257.
- 11. Grossman, G., and C. Shapiro. (1982). "A Theory of Factor Mobility." *Journal of Political Economy*, Vol. 90, 1054–1069.
- Grossman, S., and O. Hart. (1986). "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy*, Vol. 94, 691–719.
- 13. Hart, O., and J. Moore. (1990). "Property Rights and the Nature of the Firm." Journal of Political Economy, Vol. 98, 1119–1158.
- Kim, S. (1989). "Labor Specialization and the Extent of the Market." *Journal of Political Economy*, Vol. 97, 692–705.
- Lall, S. (2000a). "National Strategies for Technology Adoption in the Industrial Sector: Lessons of Recent Experience in the Developing Areas." University of Oxford, mimeographed.
- Lall, S. (2000b). "Technological Change and Industrialization in the Asian NIEs." In L. Kim and R. R. Nelson (eds.), *Technological Learning and Economic Development: The Experience of the Asian NIEs*, Cambridge: Cambridge University Press.
- Mankiw, G., and M. Whinston. (1986). "Free Entry and Social Inefficiency." Rand Journal of Economics, Vol. 17, 48–58.
- Redding, S. (1996). "The Low-Skill, Low-Quality Trap: Strategic Complementarities Between Human Capital and R & D." *Economic Journal*, Vol. 106, 458–470.
- 19. Rosen, S. (1978). "Substitution and the Division of Labor." Economica, Vol. 45, 235-250.
- Rosenstein-Rodan, P. (1943). "Problems of Industrialization of Eastern and South-Eastern Europe." Economic Journal, Vol. 53, 202–211.
- Tan, H. (2000). "Technological Change and Skills Demand: Panel Evidence from Malaysian Manufacturing." World Bank Institute, mimeographed.
- Tan, H. (2001). "Do Training Levies Work? Malaysia's HRDF and its Effects on Training and Firmlevel Productivity." World Bank Institute, mimeographed.
- Tan, H., and G. Batra. (1995). "Enterprise Training in Developing Countries: Overview of Incidence, Productivity Effects, and Policy Implications." World Bank, Private Sector Development Department, mimeographed.
- Tan, H., and G. Batra. (1997). "Technology and Firm Size-Wage Differentials in Colombia, Mexico, and Taiwan (China)." World Bank Economic Review, Vol. 11, 59–83.
- Vickery, G., and J. Northcott. (1995). "Diffusion of Microelectronics and Advanced Manufacturing Technology: A Review of National Surveys." *Economics of Innovation and New Technology*, Vol. 3, 253–275.
- Yang, X., and J. Borland. (1991). "A Microeconomic Mechanism for Economic Growth." Journal of Political Economy, Vol. 99, 460–482.