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# GENERAL EQUILIBRIUM PRODUCTION WITH CONSTANT ELASTICITY OF SUBSTITUTION

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*Abstract*: Properties of general equilibrium models with constant elasticity of substitution CES production may be underappreciated. Comparative static price elasticities are generallyinsensitive to the degree of CES substitution, a surprise with substitution ranging from nearly zero to nearly perfect. The influence of the degree of CES substitution is examined and properties are illustrated in an application to the United Arab Emirates.

Keywords: substitution, production, constant elasticity, general equilibrium. JEL Classification Number: F11, C02, D20, D50

Comparative static elasticities in competitive general equilibrium models of production generally depend on the degree of substitution as well as factor intensities or factor shares. In models with the same number of factors and products, Stolper-Samuelson (1941) elasticities of prices on factor prices are known to be independent of substitution.

The present note shows that in models with more factors than products, including specific factors models, these general equilibrium elasticities are independent of the degree of constant elasticity of substitution. Output adjustments absorb the effects of substitution leaving factor price effects unchanged, and similar results hold for Rybczynski endowment elasticities of outputs. A higher degree of substitution does, however, lower endowment elasticities of factor prices and price elasticities of outputs.

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These properties are illustrated in a model calibrated for the United Arab Emirates economy with four major sectors and six types of labor along with capital input.

#### 1. CES SUBSTITUTION IN COMPETITIVE GENERAL EQUILIBRIUM PRODUCTION

Substitution elasticities describe the adjustment potential in cost minimizing inputs with respect to factor prices. The cross price elasticity between the price of factor k and the input of factor i in product j is

$$E_{ij}^{k} = \hat{a}_{ij} / \hat{w}_{k} = \theta_{kj} S_{ij}^{k} \tag{1}$$

where hats  $\hat{}$  represent percentage changes,  $a_{ij}$  is the cost minimizing input of factor *i* in product *j*,  $w_k$  is the price of factor *k*,  $\theta_{kj}$  is the share of factor *k* in industry *j* revenue, and  $S_{ij}^k$  is the Allen (1938) partial elasticity of substitution from the production function. With Cobb-Douglas production  $S_{ij}^k = 1$  and the cross price elasticity equals the *k* factor share. Linear homogeneity implies  $\sum_k E_{ij}^k = 0$  and the own price elasticity  $E_{ij}^i$  is derived in practice as the negative of the sum of the cross price elasticities.

The homogeneous CES production function introduced by Arrow, Chenery, Minhas, and Solow (1961)

$$x = (a_1 v_1^{\rho} + a_2 v_2^{\rho})^{1/\rho}$$
(2)

has output is produced by inputs  $v_i$ , i = 1, 2. The  $a_i$  are constant positive distribution parameters and the substitution parameter  $\rho$  must lie in the range  $0 < \rho < 1$  for diminishing returns. CES generalizes Cobb-Douglas to any degree of the Allen partial elasticity of substitution, converging to Cobb-Douglas in the limit as  $\rho \to 0$  with constant returns to scale (CRS). To see this property take the log of (2) ln  $x = \ln(a_1 K^{\rho} + a_2 L^{\rho})/\rho$ and use l'Hospital's rule as  $\rho \to 0$  to find ln  $x = [d(a_1 v_1^{\rho} + a_2 v_2^{\rho})/d\rho]/(a_1 v_1^{\rho} + a_2 v_2^{\rho}) = (a_1 v_1^{\rho} \ln v_1 + a_2 v_2^{\rho} \ln v_2)/(a_1 v_1^{\rho} + a_2 v_2^{\rho}) = a_1 \ln v_1 + a_2 \ln v_2$  where  $a_1 + a_2 = 1$  with CRS.

Chung (1994) shows CES Allen elasticities  $S_{ij}^k$  are constant, independent of factor prices, and identical for every pair of factors as suggested by Uzawa (1962). CES generalizes Cobb-Douglas in the strong sense that CES Allen elasticities are independent of input levels and identical for every pair of factors. The degree of CES is denoted by  $\alpha > 0$  and the Allen and cross price elasticities  $E_{ij}^i$  in (1) are scaled by  $\alpha$ .

Aggregate substitution elasticities  $\sigma_{ik}$  for the economy are a weighted average of the cross price elasticities for each sector as described by Takayama (1982) and Thompson (1994),

$$\sigma_{ik} \equiv \hat{a}_i / \hat{w}_k = \sum_j \lambda_{ij} E_{ij}^k = \sum_j \lambda_{ij} \theta_{kj} S_{ij}^k , \qquad (3)$$

with factor shares  $\theta_{kj}$  and industry shares  $\lambda_{ij}$  of factor *i* in industry *j* in the derivation of aggregate substitution elasticities.

Full employment and competitive pricing are stated  $\sum_{j} a_{ij} x_j = v_i$  and  $\sum_{i} a_{ij} w_i = p_j$  where  $x_j$  is the output of good j,  $v_i$  is the endowment of factor k,  $w_i$  is the price of

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factor *i*, and  $p_j$  is the price of good *j*. Differentiating the two conditions leads to

$$\sum_{i} \sigma_{ki} \, \hat{w}_i + \lambda_{kj} \, \hat{x}_j = \hat{v}_k \quad \text{and} \tag{4}$$

$$\sum_{i} \theta_{im} \, \hat{w}_i = \hat{p}_m \tag{5}$$

for input k and output m where (5) simplifies with the cost minimizing envelope property.

The competitive general equilibrium model from (4) and (5) with exogenous factor endowments and prices in matrix form is

$$\begin{pmatrix} \sigma & \lambda \\ \theta' & 0 \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{x} \end{pmatrix} = \begin{pmatrix} \hat{v} \\ \hat{p} \end{pmatrix}$$
(6)

where  $\sigma$  is the matrix of substitution elasticities,  $\lambda$  the matrix of industry shares,  $\theta'$  the transposed matrix of factor shares, and 0 a null matrix.

## 2. SOLUTION OF THE CES MODEL

The model relates exogenous comparative static changes in prices and endowments to endogenous adjustments in factor prices and outputs given full employment and competitive pricing. Comparative static partial elasticities are found by inverting (6). For reference,

$$F = \hat{w}/\hat{v} \quad S = \hat{w}/\hat{p} \quad R = \hat{x}/\hat{v} \quad P = \hat{x}/\hat{p}.$$
 (7)

The economy wide cross price elasticities of CES substitution in matrix  $\sigma$  of (6) are

$$\sigma_{ik} = \alpha \sum_{j} \lambda_{ij} \theta_{kj} , \qquad (8)$$

where  $\alpha$  is the Allen elasticity of substitution. Letting  $\sigma_{CD}$  represent the matrix of Cobb-Douglas substitution elasticities derived with unit Allen elasticities, the CES substitution matrix is  $\alpha \sigma_{CD}$ . Multiply the CES system matrix by its inverse to find the identity matrix

$$\begin{pmatrix} \alpha \sigma_{CD} & \lambda \\ \theta' & 0 \end{pmatrix} \begin{pmatrix} F & S \\ R & P \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{9}$$

implying

$$\alpha \sigma_{CD} F + \lambda R = 1 \tag{10}$$

$$\alpha \sigma_{CD} S + \lambda P = 0 \tag{11}$$

$$\theta' F = 0 \tag{12}$$

$$\theta' S = 1. \tag{13}$$

Although the CES scalar  $\alpha$  does not appear in (12) or (13) it may affect F and S in the solution of the full set of equations depending on model dimensionality.

## 3. CES SUBSTITUTION IN EVEN MODELS

In "even" models with the same number of factors and products, the trivial solution to the homogenous  $\theta' F = 0$  in (10) is the null F matrix. The factor price equalization FPE result is that endowment differences do not affect factor prices between economies with identical prices due to free trade. Deriving cofactors of the  $\sigma$  terms in (6) to solve for elements of F, the null matrix dominates the cofactors leading to the FPE property  $\hat{w}/\hat{v} = 0$ .

Pre-multiply  $\theta' S = 1$  in (10) by  $\theta'^{-1}$  to find another familiar property of even models, that the Stolper-Samuelson matrix *S* is the inverse of  $\theta'$ . Cofactors of  $\hat{w}/\hat{p}$  terms contain the determinant det  $\lambda$  but that is cancelled by the determinant  $\Delta = \det \lambda \det \theta$ . Stolper-Samuelson results do not depend on substitution when *S* and  $\theta'$  in (10) are square.

When F = 0 the implication of (10) is that  $R = \lambda^{-1}$  in the familiar Rybczynski reciprocity property. Cofactors of  $\hat{x}/\hat{v}$  terms in (6) contain det $\theta$  but it is cancelled by the determinant. The effects of endowments on outputs are independent of substitution in even models.

With the Stolper-Samuelson terms in S unaffected by the degree of substitution  $\alpha$  it follows from (11) that P must change by a factor of  $\alpha$ . Cofactors of the  $\hat{x}/\hat{p}$  terms in (6) contain the determinant det  $\sigma$  and they change by  $\alpha$ . The production frontier becomes less concave with increased CES substitution implying magnified output effects of price changes.

Figure 1 illustrates the adjustment process in the 2 × 2 model. Industry 1 uses factor 1 intensively and an increase in the relative price  $p_1/p_2$  raises the relative price of factor 1 from  $w_2/w_1$  to  $w'_2/w'_1$ . The factor price adjustment is identical regardless of



Figure 1. Stolper-Samuelson Price Adjustment with CES.

substitution while output adjustments depend on substitution. With homothetic production there is less of a decline in the input ratio  $v_{ij}/v_{2j}$  and less output adjustment with a higher degree of substitution. A lower degree of substitution results in more of increase in  $x_1$  and more of a decrease in  $x_2$ .

## 4. CES SUBSTITUTION IN UNEVEN MODELS

In uneven models with more factors than products, including specific factors models, there are nontrivial solutions to the homogenous system (12) with nonzero terms in the F matrix implying factor price equalization does not hold.

In the solution to (6) the null matrix 0 has lower dimensions than the substitution matrix  $\sigma$  implying nonzero cofactors for the  $\sigma$  matrix terms. For given Rybczynski terms *R*, factor price elasticities in *F* uniformly scale down by  $\alpha$  according to  $\alpha \sigma_{CD}F + \lambda R = 1$  in (10). Rybczynski terms in R are insensitive to the degree of CES substitution in  $\alpha$ .

The 3 × 2 model of Ruffin (1981), Takayama (1982), and Thompson (1985) has a  $\sigma$  matrix with one more column than  $\lambda$  and one more row than  $\theta'$ . Its system determinant  $\Delta$  contains a linear component of substitution terms  $\alpha \sigma_{ik}$  and the degree of CES substitution  $\alpha$  can be factored out. See the literature for the complete derivation.

As apparent from (9) cofactors of the *F* matrix in the  $3 \times 2$  model are corresponding terms in the  $\sigma$  matrix, eliminating a column and row of the substitution matrix  $\sigma$ . As a result, cofactors of *F* contain no substitution terms. Increasing the degree of CES substitution  $\alpha$  then reduces elements of *F* accordingly as the cofactors of *F* are divided by the determinant  $\Delta$ .

Cofactors of the  $\theta'$  matrix in the 3 × 2 Stolper-Samuelson matrix *S* in (6) contain a linear component of substitution terms  $\alpha \sigma_{ik}$  with rows or columns of the substitution matrix in each cofactor. This linear dependence on  $\alpha$  matches that in the determinant, implying the degree of CES substitution has no effect on *S*. Rybczynski terms in *R* are similar in their linear component of  $\alpha \sigma_{ik}$  terms matching that in  $\Delta$  making Rybcynski terms insensitive to the degree of CES substitution  $\alpha$  in the 3 × 2 model. The same property applies to any model with one more factor than product including specific factors models.

With S not affected by CES substitution in the  $3 \times 2$  model (11) implies concavity in the production frontier P matrix decreases with the degree of CES substitution. Cofactors in P include quadratic substitution terms since the  $\sigma$  matrix has two more rows or columns than  $\lambda$  or  $\theta'$  in the cofactors. The  $\hat{x}/\hat{p}$  terms scale by  $\alpha$  in any model with one more factor than product.

The adjustment process is illustrated in the specific factors model in Figure 2 focusing on a single sector with specific factor 2 and shared factor 1, and unit value isoquants  $x_j = 1/p_j$ . The input ratio is  $v_1/v_2$  in the original equilibrium with either High or Low CES. A higher price shifts these unit value isoquants toward the origin leading to an increase in the relative price of factor 2 from  $w_2/w_1$  to  $w'_2/w'_1$  with High or Low CES. High CES leads to more of an increase in the input ratio  $v_1/v_2$ . Given a fixed



Figure 2. Sector Adjustment in the Specific Factors Model with CES.

endowment of the specific factor, output  $x_2$  increases more with a higher degree of substitution.

Models with four or more factors raise the power of the effect of  $\alpha$  on substitution terms in both cofactors and the determinant implying the same sensitivity to  $\alpha$  as in the  $3 \times 2$  model. The present properties generalize to any number of factors beyond the number of products, including specific factors models with multiple specific inputs.

When there are more products than factors, the problem of indeterminacy arises. In the  $2 \times 3$  model, the  $3 \times 3$  null matrix in (9) implies a zero determinant. Assumptions such as maximized utility and balanced trade can close the model but are beyond the scope of the present pure production models.

## 5. PRICES AND FACTOR PRICES IN THE UAE

A calibration of the United Arab Emirates economy illustrates these properties of CES substitution. Table 1 presents the factor share matrix  $\theta$  derived directly from national income accounts for capital K and six types of labor

$L_G$	Managers	$L_P$	Professional
$L_C$	Clerks	$L_L$	Sales
$L_R$	Agricultural	$L_O$	Operators

across four sectors

	x <sub>A</sub>	<i>x</i> <sub>1</sub>	x <sub>M</sub>	xs
$L_G$	0.03	0.02	0.03	0.05
LP	0.10	0.04	0.06	0.10
L <sub>C</sub>	0.03	0.00	0.01	0.03
$L_L$	0.02	0.00	0.01	0.09
$L_R$	0.07	0.00	0.00	0.00
Lo	0.02	0.00	0.01	0.00
K	0.72	0.93	0.89	0.72

Table 1. Factor Shares  $\theta_{ij}$ .

Table	2.	Industry	Shares	λii.
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	x <sub>A</sub>	x <sub>I</sub>	х <sub>М</sub>	xs
$L_G$	0.05	0.35	0.19	0.41
LP	0.08	0.33	0.20	0.39
$L_C$	0.11	0.18	0.14	0.57
$L_L$	0.04	0.05	0.09	0.83
$L_R$	0.99	0.00	0.01	0.00
LO	0.28	0.19	0.31	0.22
K	0.04	0.54	0.22	0.21

$x_A$	Agriculture	$x_I$	Industry
х <sub>M</sub>	Manufactures	$x_S$	Services.

The source of the data is the Dubai Municipality Statistics Center (2003). Capital shares are derived as residuals of industry value added after labor payments. The share of value added paid to capital is overstated because data on energy and natural resources is not available. Capital can be considered an aggregate of nonlabor inputs. Table 2 reports industry shares derived assuming equal prices for factors across industries.

Agricultural labor  $L_R$  is nearly specific to Agriculture  $x_A$ . Gauging by factor shares alone, industry  $x_I$  uses capital intensively as well as labor inputs of Managers  $L_M$  and Professionals  $L_P$ . Services output  $x_S$  intensively uses Professionals  $L_P$  but also Clerks  $L_C$  and Sales  $L_L$ , and has the largest industry share of all types of labor except Agricultural  $L_R$  and Operators  $L_O$ .

Table 3 shows substitution elasticities for CES = 1 (Cobb-Douglas) production. Labor inputs are relatively sensitive to the price of capital while capital input is insensitive to wages. Labor inputs have nearly unit own elasticities and capital a much weaker own elasticity.

A higher degree of CES substitution scales substitution elasticities accordingly. The vast literature on applied production analysis typically reports weak or inelastic substitution as reviewed by Thompson (2006a, 2006b). Some recent examples include Vincent, Lang, Seok (1992), Chang (1994), Kemfert (1998), Mahmud (2000), Yi (2000),

	$w_G$	w <sub>P</sub>	w <sub>C</sub>	$w_L$	w <sub>R</sub>	wo	w <sub>K</sub>
aG	-0.97	0.07	0.02	0.04	0.00	0.00	0.83
ap	0.03	-0.93	0.02	0.04	0.01	0.01	0.82
a <sub>C</sub>	0.04	0.08	-0.98	0.06	0.01	0.01	0.78
$a_L$	0.05	0.09	0.03	-0.92	0.00	0.00	0.75
a <sub>R</sub>	0.03	0.10	0.03	0.02	-0.93	0.02	0.72
a <sub>O</sub>	0.03	0.08	0.02	0.03	0.02	-0.99	0.81
a <sub>K</sub>	0.03	0.06	0.01	0.02	0.00	0.00	-0.13

Table 3. Substitution Elasticities, CES= 1.

Table 4. Comparative Static Results CES= 1.

	vG	vp	ν <sub>C</sub>	$v_L$	v <sub>R</sub>	vo	υĸ	PΑ	РІ	Рм	ps
$w_G$	-0.89	0.21	0.09	0.17	-0.02	0.05	0.39	-0.26	-5.49	5.27	1.38
wp	0.13	-0.74	0.11	0.10	-0.01	0.12	0.28	-0.13	-11.1	12.2	-0.16
w <sub>C</sub>	0.15	0.32	-0.86	0.38	0.05	0.01	-0.04	0.66	-3.57	-0.11	4.04
$w_L$	0.19	0.38	0.17	-0.27	-0.00	-0.11	-0.35	-0.02	3.23	-11.3	9.36
w <sub>R</sub>	-0.16	0.25	0.09	-0.07	-0.01	-0.04	-0.06	14.2	20.3	-32.5	-0.38
w <sub>O</sub>	0.34	0.68	0.35	-0.52	-0.01	-0.21	-0.64	-0.08	-65.2	79.2	-14.6
w <sub>K</sub>	0.01	0.03	-0.01	-0.01	0.00	-0.01	-0.02	0.01	1.67	-0.64	-0.02
x <sub>A</sub>	-0.18	0.22	0.08	-0.04	1.02	-0.08	-0.01	13.6	24.1	-37.4	0.41
<i>x</i> 1	-0.65	-1.07	-0.62	0.72	0.40	-1.37	3.55	5.73	123	-149	23.2
х <sub>М</sub>	1.60	2.49	1.39	-2.88	-1.23	3.79	-4.08	-17.5	-329	413	-74.3
xs	0.11	0.24	0.08	1.15	0.06	-0.46	-0.19	0.82	31.3	-48.1	17.0

Balistreri, McDaniel, Wong (2002), and Urga and Walters (2003). Models with CES parameters  $\alpha = 1$  and 0.1 gauge sensitivity in the following calibrations.

Tables 4 and 5 present the comparative static elasticities. Stolper-Samuelson  $\hat{w}/\hat{p}$  and Rybczynski  $\hat{x}/\hat{v}$  elasticities are identical for CES = 1 and 0.1. The strongest positive  $\hat{w}/\hat{p}$  effects occur for larger factor shares: between agricultural wages  $w_R$  and prices  $p_A$ , the capital return  $w_K$  and industry prices  $p_I$ , professional  $w_P$  and operator wages  $w_O$  and manufacturing prices  $p_M$ , and sales wages  $w_L$  and service prices  $p_S$ .

The strongest negative  $\hat{w}/\hat{p}$  effects occur for professional wages  $w_P$  and operator wages  $w_O$  with respect to industry prices  $p_I$  as well as for capital  $w_K$  and sales wages  $w_L$  with respect to manufacturing prices  $p_M$ . Elasticities of factor prices with respect to endowments  $\hat{w}/\hat{v}$  scale inversely with CES substitution while the production frontier  $\hat{x}/\hat{p}$  elasticities scale directly.

The typically low levels of substitution in the applied production literature suggest CES = 0.1 presents more reasonable model elasticities. Exogenous vectors of price changes can be introduced to simulate factor price adjustments. As an example, if the

	$v_G$	$v_P$	v <sub>C</sub>	$v_L$	$v_R$	vo	$v_K$	РA	РІ	РМ	<i>ps</i>
$v_G$	-8.91	2.09	0.87	1.71	-0.17	0.47	3.88	-0.26	-5.49	5.27	1.38
w <sub>P</sub>	1.31	-7.43	1.13	1.05	-0.08	1.20	2.80	-0.13	-11.1	12.2	-0.16
w <sub>C</sub>	1.47	3.20	-8.60	3.76	0.45	0.12	-0.43	0.66	-3.57	-0.11	4.04
$w_L$	1.92	3.83	1.67	-2.71	-0.02	-1.14	-3.49	-0.02	3.23	-11.3	9.36
w <sub>R</sub>	-1.60	2.48	0.91	-0.74	-0.05	-0.41	-0.56	14.2	20.3	-32.5	-0.38
wo	3.44	6.84	3.54	-5.18	-0.06	-2.09	-6.37	-0.08	-65.2	79.2	-14.6
w <sub>K</sub>	0.14	0.27	-0.07	-0.08	0.01	-0.06	-0.20	0.01	1.67	-0.64	-0.02
x <sub>A</sub>	-0.18	0.22	0.08	-0.04	1.02	-0.08	-0.01	1.36	2.41	-3.74	0.04
x1	-0.65	-1.07	-0.62	0.72	0.40	-1.37	3.55	0.57	12.3	-14.9	2.32
x <sub>M</sub>	1.60	2.49	1.39	-2.88	-1.23	3.79	-4.08	-1.75	-32.9	41.3	-7.42
xs	0.11	0.24	0.08	1.15	0.06	-0.46	-0.19	0.08	3.13	-4.81	1.70

Table 5. Comparative Static Results CES = 0.1.

price of agricultural products  $p_A$  falls 10% and the price of services  $p_S$  rises 10%, the manager wage  $w_G$  would increase 16.4% while the professional wage  $w_P$  would fall 0.3%. Exogenous price changes may be due to taxes or tariffs on traded products as well as changes in international prices.

## 6. CONCLUSION

The present paper shows that comparative static price elasticities in general equilibrium production models are identical for any degree of CES substitution, from near zero to near perfect. These properties suggest applications of a wide range of general equilibrium models of production and trade can proceed without specification or estimation of factor substitution.

In even models with the same number of factors and products, the absence of endowment effects on factor prices is the factor price equalization property. Substitution plays no role either in the price elasticities of factor prices or the endowment elasticities of outputs. The output effects of price changes are diminished with a higher degree of CES substitution.

With more factors than products including specific factors models, a higher degree of CES substitution implies smaller differences in factor prices across countries with substitution increasing adjustment flexibility. Output effects of price changes also diminish with higher CES substitution. Price elasticities of factor prices are unaffected by the degree of CES substitution, as are elasticities of outputs with respect to factor endowments.

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