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BIFURCATION AND STABILITY IN IMPERFECTLY COMPETITIVE INTERNATIONAL COMMERCIAL FISHING

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Abstract: In this paper we will conduct theoretical stability analysis and numerical analysis of international commercial fishing with two countries under imperfectly competitive condition in the markets for the fish harvested by two countries in an open-access sea. The sufficient conditions for the fish stock to become extinct, converge to a single or double equilibria, or become periodic, double periodic or chaotic are investigated. Numerical examples are presented for various values of the parameters in the fish's biological growth equation, of the harvesting costs and of the demand functions for the fish in two countries.

Keywords: international commercial fishing, imperfect competition, dynamical system, difference equation.

JEL Classification Number: D43, L13.

1. INTRODUCTION

Many authors have studied different types of commercial fishing. For example Leung and Wang (1976), and Wang and Leung (1978) have analyzed commercial fishing of a single species; Solow (1976), May et al. (1979), Okuguchi (1984) and Strobele and Wacker (1995) have studied the subject for multi species with prey-predator interaction. In most of these and some other papers the case of imperfect competition in the market for harvested fish is not considered and the price of the fish is taken to be constant. Moreover, partial or complete bifurcation analysis has never been attempted

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in these contributions. Taking into account the recent development in the theory of international trade under imperfect competition, Okuguchi (1998) has formulated international commercial fishing under imperfect competition. In this paper we will first analyze existence and stability of single and double equilibrium points for the fish stock in his model. Then we will discuss different types of periodic and chaotic solutions for various values of the parameters in his model. This paper is organized as follows. In Section 2, we briefly present international fishing model due to Okuguchi (1998) and discuss the stability property of the long-run equilibrium fish stock. We then transform his system of differential equation into discrete system and consider numerical values of the parameters which give rise to different cases considered by Okuguchi. In Section 3, we will use the maps corresponding to the discrete system and show different types of periodic and chaotic behaviors of the fish stock due to changes in the values of the parameters of the discrete system. Section 4 concludes.

2. INTERNATIONAL FISHING; STABILITY OF LONG-RUN EQUILIBRIUM

Let there be two fishing countries and le X tbe the fish stock. According to the biological growth law, its rate of change in the absence of fishing is formulated as follow.

$$\frac{dX}{dt} = X(\alpha - \beta X), \tag{1}$$

where α and β are intrinsic growth rate and carrying capacity, respectively. Rewriting (1), we have

$$\frac{dX}{dt} = \alpha X(1 - KX), \tag{2}$$

Verhulst logistic equation, where $K = \frac{\beta}{\alpha}$. If there is no fishing, the fish stock converges to carrying capacity K.

If x_{ij} be the amount of fish harvested by country i and sold in country j, $i, j = 1, 2$, the inverse demand functions for the fish in the two countries are given by

$$p_i = a_i - b_i(x_{ii} + x_{ji}), \quad i \neq j, \quad i, j = 1, 2. \tag{3}$$

where p_i is the price of the fish in country i; a_i and b_i are positive constants. The profit π_i for country i is

$$\pi_i = p_i x_{ii} + p_j x_{ij} - \frac{\gamma_i \left(x_{ii} + x_{ij}\right)^2}{X} - c_i, \quad i \neq j, \quad i, j = 1, 2, \tag{4}$$

where γ_i and c_i are positive constants and c_i is the opportunity cost of fishing for country i. According to (4), each country's harvesting cost is proportional to the square of its harvest rate and inversely proportional to the level of the fish stock.

Define $X_i = x_{i1} + x_{i2}$ and, $Y_i = x_{1i} + x_{2i}$, respectively. If each country maximizes its profits under the Cournot assumption about its rival country's harvesting and marketing of the fish, we have the following differential equation for the change of the fish stock (see Okuguchi (1998) for the detail of the derivation).

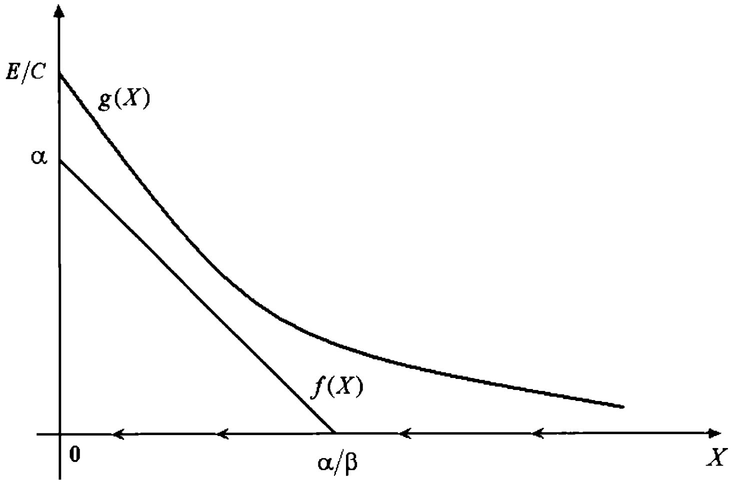

Figure 1. There is one positive equilibrium solution whenever $\alpha < E/C$ and $g(X) > f(X)$ for all X, and the fish stock converges to the zero for any initial value.

$$\frac{dX}{dt} = X(f(X) - g(X)), \tag{5}$$

where

$$f(X) = \alpha - \beta X, \tag{6}$$

$$g(X) = \frac{DX + E}{AX^2 + BX + C}, \tag{7}$$

$$A = 3b_1^2 b_1^2,\ B = 4b_1 b_2(b_1 + b_2)(\gamma_1 + \gamma_2),\ C = 4\gamma_1\gamma_2(b_1 + b_2)^2,$$
$$D = 2b_1 b_2(a_1 b_2 + a_2 b_1), \quad \text{and} \quad E = 2(b_1 + b_2)(a_1 b_2 + a_2 b_1)(\gamma_1 + \gamma_2)$$

Obviously, from biological as well as economic point of view, we are interested only in the positive equilibrium points. From equation (5) the non-trivial equilibrium points for the fish stock satisfies $f(X) = g(X)$. Note that x and y intercepts of the line $f(X)$ are $(0, \alpha)$ and $(\alpha/\beta, 0)$, respectively, and that $(0, E/C))$ is the y-intercept of $g(X)$ and the x-axis is its horizontal asymptote. Furthermore, since $g' < 0$, $g'' > 0$, the curve for $g(X)$ is always decreasing and concave. Therefore, this curve does no intersect, intersects once or intersects at most twice with the line for $f(X)$.

Case 1. As we can see in Fig. 1, if the harvest rate of the fish is always greater than its biological growth rate, i.e., no intersection between $f(X)$ and $g(X)$, then obviously the fish will become extinct in the long-run.

Case 2. If the harvest rate is always greater than the biological growth, the fish will become extinct unless the initial fish stock equals X^*.

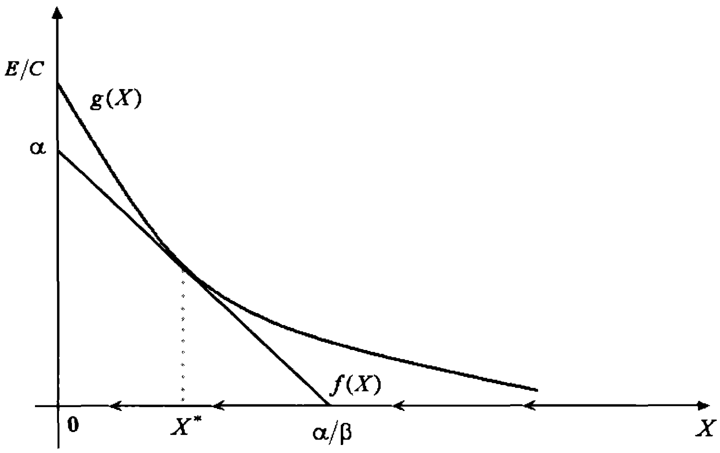

Figure 2. There is one positive unstable equilibrium point whenever $\alpha < E/C$ and $g(X) = f(X)$ at one point and $g(X) > f(X)$ for all other X.

LEMMA 1. *The equilibrium point X^* in the case 2 is unstable and the phase portrait around X^* always has decreasing direction.*

Proof. To see the stability type of this point note that the derivative of the right hand side of the equation (5) is

$$F'(X) \equiv [X(f(x) - g(X))]' = f(X) - g(X) + X(f'(X) - g'(X)). \tag{8}$$

Since $f(X^*) = g(X^*)$, $f'(X^*) = g'(X^*) = -\beta$, we have $F'(X^*) = 0$. On the other hand, $F''(X^*) < 0$, since $g''(X) > 0$. This means that the right hand side of equation (5) has maximum at X^*, hence, phase portrait around X^* has decreasing direction in both sides of X^* and the equilibrium X^* is unstable (see Hale and Kocak(1990)).

Therefore, as we can see in Fig. 2, the fish stock converges to X^* if the initial stock is greater than X^* and vanishes if the initial stock is less than X^*.

Case 3. In this case we suppose $\alpha > E/C$. Then there is again one positive equilibrium point X^*.

LEMMA 2. *The equilibrium point in case* 3 *is stable and the phase portrait around X^* has different direction.*

Proof. Since $\alpha > E/C$ and the function $f(X)$ is concave, we easily deduce that $F(X) > 0$ for all $X < X^*$, and $F(X) < 0$ for all $X > X^*$. This means that the phase portrait of equation (5) has increasing direction for $X < X^*$ and decreasing direction for $X > X^*$. Hence, the non-zero equilibrium point X^* is stable. This case is shown by Fig. 3.

Case 4. In the final case, we suppose the line for $f(X)$ intersects $g(X)$ at two points. Then as we can see in the following Lemma 3, these two equilibrium points have different stability property.

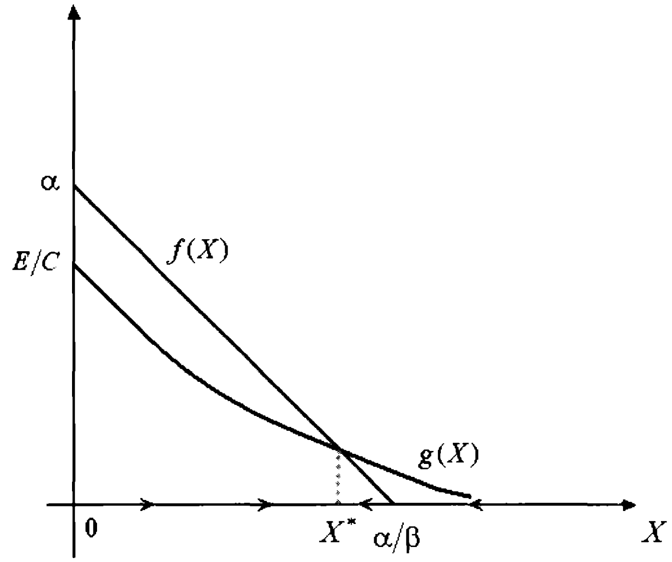

Figure 3. There is one positive stable equilibrium point for $\alpha > E/C$ and the fish stock converges to this equilibrium level regardless of the initial stock level.

LEMMA 3. *If equation* (5) *has two different equilibrium points* X^* *and* X^{**} *such that* $X^* < X^{**}$, *then* X^* *is unstable and* X^{**} *is stable.*

Proof. According to Lemma 2, since the function $F(X)$ has the same behavior around the point X^{**}, this point is stable. However, this function around X^* behaves differently, since for $X^* < X < X^{**}$, we have $F(X) > 0$ and for $0 < X < X^*$, $F(X) < 0$. That is, X^* is unstable. This case is shown by Fig. 4.

So far we have analyzed dynamics for changes in the fish stock on the basis of differential equation. We discretize it to get Euler difference equation as follows.

$$X_{n+1} = hX_n(f(X_n) - g(X_n)) + X_n\,, \tag{9}$$

where h is a discretization parameter. If we take the constants $a_1 = a_2 = 1$, $b_1 = b_2 = 0.5$, $\gamma_1 = 0.3$ and $\gamma_2 = 0.4$, then the corresponding constants in function $g(X)$ are $A = 0.1875$, $B = 0.7$, $C = 0.48$, $D = 0.5$ and $E = 1.4$. Note that the conditions $CD - BE = -0.74 < 0$ and $B(CD - DE) + ACE = -0.196 < 0$ are satisfied here, as we need them for $g'(X)$ to be negative and $g''(X)$ to be positive. We let $\beta = 1$ and take α as our variable parameter in function $f(X)$. We furthermore let $h = 0.5$.

Taking different values for α to generate the various cases corresponding cases 1–4 above, we have different types of equilibria as shown in Figs. 5–9. Now, if we take the value of α such that $\alpha < E/C$ and $g(X) > f(X)$ for all $X \geq 0$, then, as we can see in Fig. 5, the fish stock vanishes in the long-run for any initial stock.

As a numerical example of case 2 above, we take the value of α such that $\alpha < E/C$ and $g(X) > f(X)$ for all $X \geq 0$ except at one point X where $g(X) = f(X)$. This value

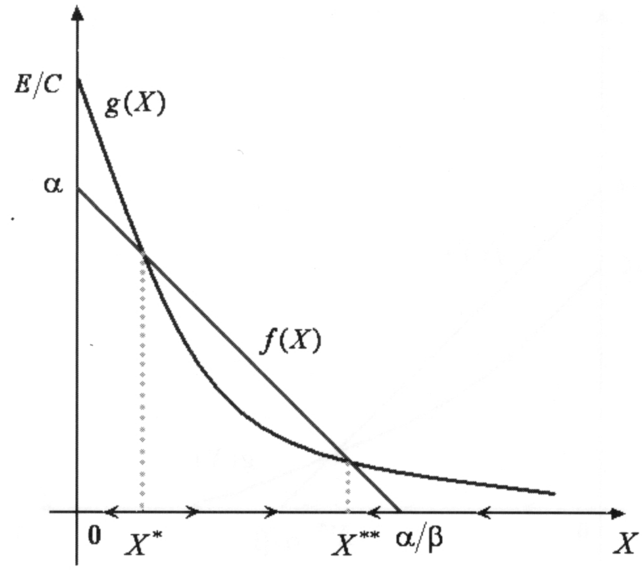

Figure 4. There are two positive equilibrium points X^* and X^{**}. The smaller one is unstable and the larger one stable.

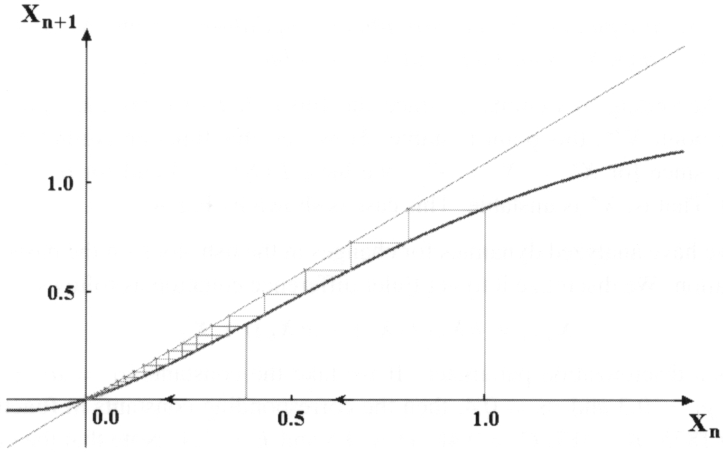

Figure 5. One dimensional-stair numerical solution of the map (9) together with phase portrait of the orbit for parameter value $\alpha = 2$, which corresponds to the situation of case 1.

of α can be calculated by solving $g'(X) = -1$ (the slope of the line $f(X)$). Note that for solving $g'(X) = -1$ from (7) we have

$$g'(X) = \frac{-ADX^2 - 2AEX + CD - BE}{(AX^2 + BX + C)^2} = -1\,,$$

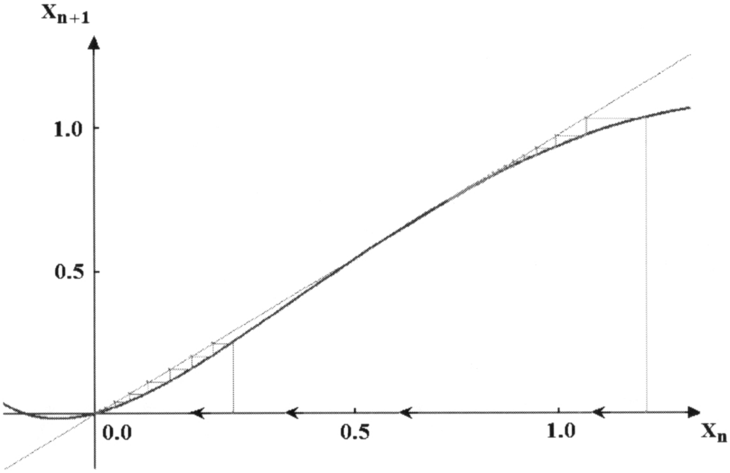

Figure 6. One unstable equilibrium point $X^* = 0.71833$ for $\alpha^* = 2.3478192$, which corresponds to the case 2 . The phase portrait is shown on the horizontal axis.

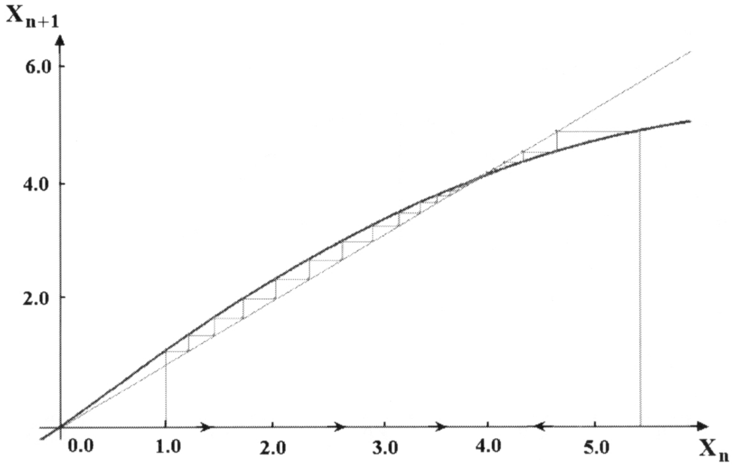

Figure 7. One dimensional-stair numerical solution for the map (9) for the case 3, where $4.5 = \alpha > E/C \approx 2.917$. The result is one stable equilibrium point $X^* = 3.9534$.

which is an equation of degree 4 with the same constant values for A, B, C, D, and E as above. Solving this equation numerically gives us $X^* = 0.71833$ as the only positive solution. The other solutions are negative and complex. Now finding $g(X^*)$ and solving equation $f(X) = \alpha - \beta X$ for this X^* and $\beta = 1$ yields $\alpha^* = 2.3478192$. As we can see in Fig. 6, in this case the equilibrium point X^* is unstable.

Now, taking $\alpha = 4.5$ we will have case 3, where $\alpha > E/C$ and $g(X)$ intersects $f(X)$ at $X^* = 3.9534$. In this case the steady state stock X^* is stable (see Fig. 7).

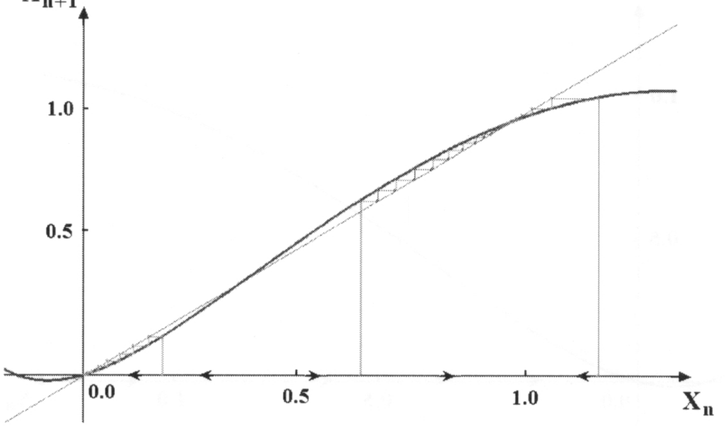

Figure 8. For $2.3478192 < \alpha < 2.917$ there are two equilibria. Here, $\alpha = 2.41$ which gives two positive equilibria $X^* = 0.42994$ and $X^{**} = 1.069$. The first one is unstable and the second one stable.

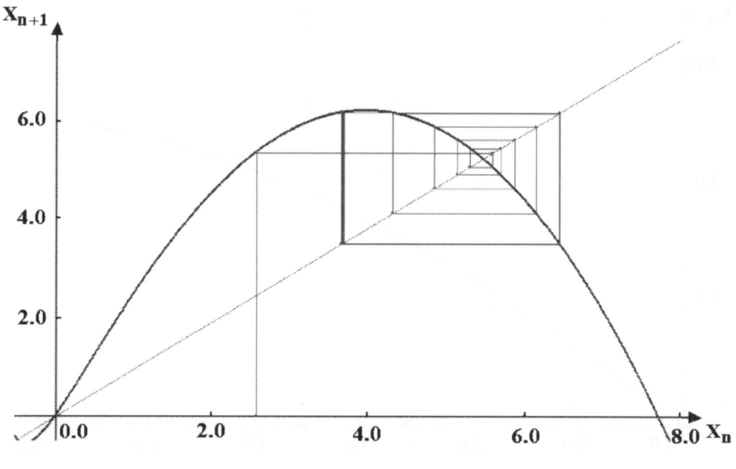

Figure 9. Periodic-2 solution at $X^* \approx 3.44$ for $\alpha = 5.57$ of the map (9).

The last case to be considered is case 4. In this case if we take α such that $E/C > \alpha > \alpha^*$ ($\alpha^* = 2.3478192$ in case 2), then there should be two equilibrium points. Indeed, if we take $\alpha = 2.41$, then we have two positive equilibria $X^* = 0.42994$ and $X^{**} = 1.069$. As we can see in Fig. 8, the first one is unstable, but the second one is stable.

3. PERIODIC DOUBLING AND CHAOTIC SOLUTIONS

Since our mathematical model (5) is non-linear and depends on different parameters, we can conceive of more complicated solutions other than the ones which we have seen in Section 2. This is in fact the case. The existence of such a solution can be established by taking different values for the parameter α. Indeed, using the same parameter values as in Section 2 and different values for $\alpha > E/C$ we observe different periodic solutions. Note that, by taking the value $\alpha > E/C$, as we have seen in case 4, the model has one non-zero solution. So, if $\alpha = 5.57$, difference equation (9) for any starting point $X_0 \in (0, 7.5)$ will have periodic solution with period-two (see Fig. 9). Indeed, the appearance of the periodic-two for all $\alpha \in (5, 5.7)$ is the beginning of other sequence of periodic solutions that leads to a complicated dynamics, namely chaos. For example, for $\alpha = 5.7$ we will have periodic solution with period-four, and for $\alpha = 5.881$ the periodic-four orbit loses its stability and gives rise to an asymptotically stable period-eight. Finally, for the parameter value $\alpha = 6.1$ the dynamics of the model (9) becomes quite complicated. Indeed, for this value of the parameter the system is chaotic and this behavior continues until α becomes close to 7 (see Fig. 10). The bifurcation diagram for the value $\alpha \in (2, 7.5)$ is illustrated in Fig. 11. Of course, there are some periodic solutions other than the ones that we have illustrated here. For example, in the area of the parameter α for which the system is chaotic, the system becomes again periodic all of a sudden in a small window of the parameter. Fig. 12 shows one of these situations where in fact a periodic-3 solution for the parameter value $\alpha = 6.4$ emerges.

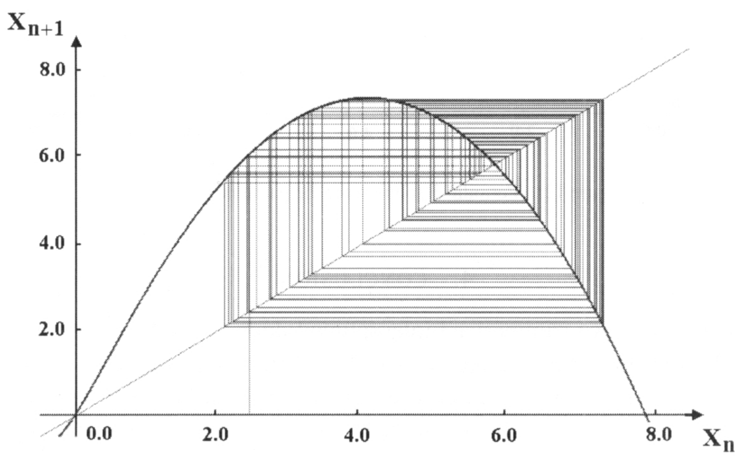

Figure 10. Chaotic solution for $\alpha = 6.1$ with starting point $X = 2.4$.

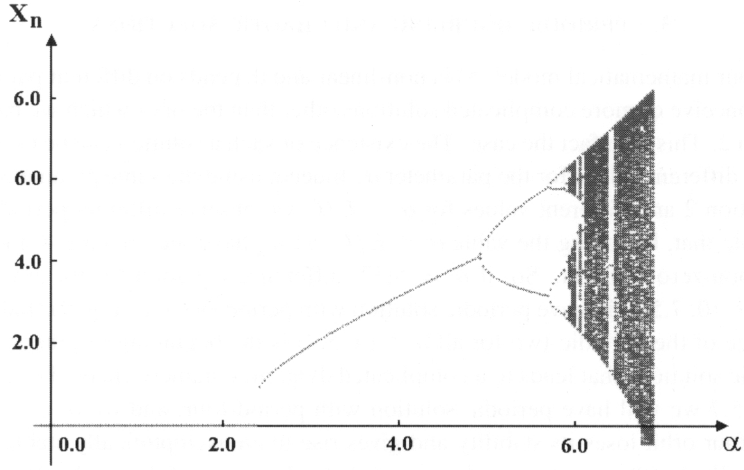

Figure 11. Bifurcation diagram for $2 < \alpha < 7.5$. As we can see, there is one solution for $0 < \alpha < 2.3478192$, different periodic solutions for $2.3478192 < \alpha < 6$ and chaotic solution for $6 < \alpha < 7$.

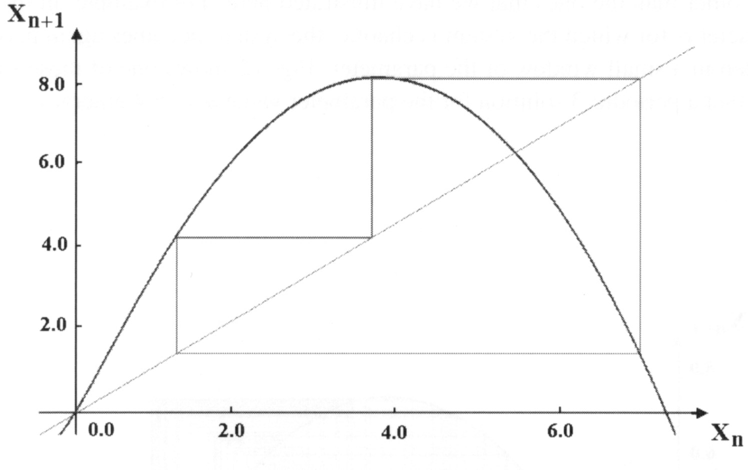

Figure 12. Periodic-3 solution for the parameter $\alpha = 6.4$.

4. CONCLUSION

In this paper we have summarized international duopoly mode of commercial fishing originally formulated by Okuguchi (1998) as differential equation. We have transformed his equation into Euler difference equation, which has been revealed to have more complicated solution paths for the fish stock. We have shown stability types of equilibrium points together with the phase portraits of the different orbits. Interestingly, periodic solutions with different even periods have been observed. In particular, the propagation of these periodic solutions has led us to find parameter values for which chaotic solutions exist.

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