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REAL SECTOR, BANKS AND POLICY ISSUES: AN EXPLORATION IN A DYNAMIC MACROECONOMIC MODEL

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Abstract: This paper formulates a dynamic macroeconomic model with banks and two uses of credit, viz. for fixed capital and for working capital. The model was simulated for the dynamic path of the endogenous variables. To see the role of policy in a bad state, government expenditure was increased financed by enhanced deficit and by raising income taxes. In the money financed case long-term interest rate rises and investment falls leading to lower levels of output. On the other hand the effect of increased money supply is negative on the short-term interest rate and the price level. In the tax-financed case exactly the opposite happens. These results are contrary to the conventional wisdom.

Keywords: Macroeconomics, Dynamic Model, Banks, Fixed Capital, Working Capital.

JEL Classification Number: E44, E47, E63.

1. INTRODUCTION

In recent times a large body of literature has emerged that asserts the role of financial intermediation in the macroeconomic models.\(^1\) The emergence of banks (or other financial institutions) proceeds from the following activities of banks. (i) Banks accept deposits of household savings and lend to a large number of agents. (ii) Banks hold liquid reserves against predictable withdrawal demand. (iii) Banks issue liabilities that are more liquid than their primary assets. (iv) Banks reduce the need for self-financing of investment. The implication of the above is that holding savings in bank deposits is safe in respect of returns compared to equities or direct lending to firms that have uncertain returns. The risk averse agent would hold more of their savings in bank deposits than in

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\(^1\) See for example, Bencivenga and Smith (1991), de Mezza and Webb (1992) Gertler (1988), Greenwood and Jovanovic (1990), Bernanke and Gertler (1987) and many others.
equities or direct lending. The funds from deposit mobilization are lent to entrepreneurs to finance investment projects. Asymmetric information about the investment projects require ex-ante evaluation and ex-post monitoring which in turn require skill as well as cost. An individual investor usually does not have the necessary skill and the cost is prohibitive while banks can do the job efficiently. In the process, the banks can exploit the law of large numbers to forecast the number of unsuccessful projects and as a result the expected returns of the loans advanced. The savers can be assured of a safe return. In short, the bank is the institution through which savings are channelised into investment in the absence of a perfect insurance market for loans.

In the development economics literature credit from the banks and other financial institutions is treated, at least in the organized part of the economy, as the main source of finance for economic activities. However, it has been argued that as far as the requirement of credit is concerned, there are two uses of credit—(i) short-term requirement for financing working capital and (ii) long-term requirement for financing investment in fixed capital. These two uses of credit have different effects on the real sectors of the economy. While credit for working capital affects the supply of goods, credit for fixed capital augments the demand side. But because of their short run nature, these models largely ignore the dynamic considerations like supply side effects of credit for investment in fixed capital through its effect on productivity. On the other hand, the dynamic models ignore the institutional arrangements, like the need for distinction between fixed capital and working capital prevalent in the developing countries. Another point that deserves attention is the intertemporal spillovers in two types of loans. The credit for investment in fixed capital in the current period is determined by the expected profits in the future periods that in turn is affected by the expected credit availability of working capital credit (or the expected interest rate on such credit). This factor operates both from the standpoint of lenders as well as borrowers. In the process one can generate an appropriate investment function. The issues of budgetary operations, monetary policy etc. can then be addressed in this framework.

This paper attempts to resolve the role of different uses of credit and their interactions with the real sectors of the economy in a dynamic general equilibrium model. With this brief introduction we proceed as follows. Section 2 describes the basic model. Section 3 considers the simulation exercises and the overall conclusion is drawn in Section 4.

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2 See Williamson (1987) for a detailed discussion on this issue.
4 See particularly Rakshit (1987) on this issue. Also McKinnon (1973) and Shaw (1973) discussed these issues in detail.
5 Das (1996), (2004) formulated a two period model where investment in fixed capital takes place in the first period and investment in working capital in the second period. Thus there are two uses of credit and the firm might be constrained in either or both period. But this paper explored the effects of credit rationing on the investment and production decisions of the firm and does not address the general equilibrium issues.

Naastepad (2002) addressed some of these issues in a computable general equilibrium model. But his study also suffers from the inadequate consideration of micro foundations and role of expectations.
2. THE MODEL

We consider an overlapping generation model with the following agents: a representative household, a representative firm, a representative commercial bank (bank for short), the central bank and the government.

The households live for two periods and are born with a fixed endowment of labour in the first period. They earn income in the first period of their lives by selling the fixed endowment of labour to firms. For the sake of simplicity, we assume as in Galore and Zeira (1993)\(^\text{6}\) that the households do not consume anything in the first period; so whatever they earn in the first period net of taxes are entirely saved in the form of bank deposits. Their first period consumption is financed by their previous generation (i.e. their parents). Thus there is no optimisation problem involved for the household to allocate total wage earnings between current period consumption and saving.

Direct lending to firms or equity holding is ruled out in this model.\(^\text{7}\) The savings of the first period plus interest earned are entirely consumed in the second period. Thus denoting the nominal wage rate by \(w_t\) and normalizing the fixed endowment of labour at unity, total wage income of the households is \(w_t\). Out of this a fixed sum \(T\) is taxed by the government and the rest is saved in bank deposits, \(D_t\). Hence,

\[
D_t = w_t - T
\]

Denoting the interest rate factor (one plus interest rate) on bank deposits by \(d_t\) and the price level of \((t + 1)\) th. period by \(P_{t+1}\), the second period consumption of households is \(d_t D_t / P_{t+1}\).

A typical firm is assumed to operate in a competitive framework, where the price of its product, the wage rate and the interest rate are given. The firm is a typical producer-investor in this model. The production technology, assumed to be Cobb–Douglas with a shock parameter, is given below:

\[
Y_t = (K_t \theta_t)^\mu N_t^{1-\mu}
\]

where \(Y_t\) = the firm’s output at time \(t\), \(K_t\) = the firm’s capital stock at \(t\), \(N_t\) = labour employed by it at \(t\), \(\theta_t\) = shock to productivity of capital at \(t\). As is evident from the production function, current production depends on the current employment of labour, capital stock measured at the beginning of the period and the shock to capital realized in the \(t\) th. period, \(\theta_t\). We interpret the latter as the shock to the productivity of capital and is independently and identically distributed. The firm has no owned resources. Hence

\(^\text{6}\) This is only a simplifying assumption and the results do not change even if we relax this. Absence of the optimisation problem for the household excludes any change in the composition of current consumption and saving when there is a perturbation in the model, and therefore no change in current consumption demand. As our focus is mainly on investment in fixed and working capital and the resultant effect on output and other macroeconomic variables is affected owing to a shift in policy variables, we get sharper results as we do away with the complication arising from households’ side.

\(^\text{7}\) In India prior to liberalization in 1988, only 2% of savings were invested in equities. Even after liberalization was initiated in 1991 the corresponding figure stands at 6.3% in 1999–2000 that further drops at 3.7% in 2000–01. The scenario is not very different in majority of the developing countries.
the firm borrows from the bank to finance its fixed capital investment as well as the wage bill. We call the latter working capital. There is no other source of finance for the firm.

Investment in fixed capital made in the current period becomes productive in the next period which then combines with labour to produce output in that period. Thus the capital accumulation constraint is given by

\[ K_t = (1 - \zeta) K_{t-1} + I_{t-1} \]  

where \( \zeta \) = the rate of depreciation of capital and \( I_t \) = investment in the \( t \) th. period. For the sake of simplicity, we have set \( \zeta \) at one, implying that capital is fully exhausted after production. As a result, fixed capital in this model resembles more like working capital. However in this model investment in fixed capital has to be made one period in advance which ascribes a ‘commitment’ property of fixed capital: in period \( t \) the firm commits to the maximum amount of a particular input in period \( t+1 \). Thus the production function takes the following form.

\[ Y_t = (I_{t-1}\theta_t)^\mu N_t^{1-\mu} \]  

When investment in fixed capital is made at time \( t \), its productivity in \( t+1 \) th. period, \( \theta_{t+1} \) is uncertain—it takes a value \( \theta_1 \) with probability \( q \) and \( \theta_2 \) with probability \( (1 - q) \). This is the source of uncertainty in this model. However, this uncertainty is public information, so there is no asymmetry of information. As \( \theta_t \) is assumed to be a white noise process around a non-zero mean, we have

\[ E_t(\theta_{t+j}) = q\theta_1 + (1 - q)\theta_2 = \bar{\theta} \quad \forall j > 0 \]  

It may be noted that the real business cycle literature generally assumes that the shock to productivity follows an autoregressive (AR) scheme. In effect the time path of output follows an AR process which fits well with real life data. However, even with a white noise distribution for productivity one can generate an AR process for output in a stochastic version of Diamond (1965) overlapping generations model. However, \( \theta_t \) is the source of uncertainty for the profitability of investment and in effect the source of uncertainty for the repayment of bank loans in our model which is well captured if \( \theta_t \) is white noise. The literature on credit market imperfections makes similar assumption as in Stiglitz and Weiss (1981) or in Kiyotaki and Moore (2001) in their model on business cycle with liquidity constraint.

Given \( I_{t-1}, \theta_t \) and \( P_t \), the commodity price at time \( t \), the firm maximizes its current profit by choosing \( N_t \). The firm’s optimisation programme is given by:

\[ \max_{N_t} (P_t (I_{t-1}\theta_t)^\mu N_t^{1-\mu} - r_t w_t N_t - R_{t-1} P_{t-1} I_{t-1}) \]  

where \( r_t = (\text{one plus}) \) interest rate on working capital loans at \( t \), \( R_t = (\text{one plus}) \) interest rate on fixed capital loans of \( t \) th. period compounded for two periods.

The first order condition gives the demand for labour

\[ N_t^{d} = \frac{(1 - \mu)^{1/\mu} P_t^{1/\mu} I_{t-1} \theta_t}{(r_t w_t)^{1/\mu}} \]  

where \( r_t \) = (one plus) interest rate on working capital loans at \( t \), \( R_t = (one plus) interest rate on fixed capital loans of \( t \) th. period compounded for two periods.
The assumption of representative firm implies that this is also the aggregate demand for labour. The aggregate supply of labour is fixed, and is normalized at unity. The labour market equilibrium condition is given by:

\[ N_t^d = 1 \]  

(7)

This allows us to solve for the equilibrium money wage rate:

\[ w_t = \frac{(1 - \mu) P_t I_t^{\mu} \theta_t^{\mu}}{r_t} \]  

(8)

Assuming that the labour market is always in equilibrium, the demand for working capital loans is given by,

\[ L_t^{wd} = w_t \cdot 1 = \frac{(1 - \mu) P_t I_t^{\mu} \theta_t^{\mu}}{r_t} \]  

(9)

Forwarding (8) for one period and taking expected values we obtain

\[ E_t(w_{t+1} r_{t+1}) = (1 - \mu) E_t(P_{t+1} I_{t+1}^{\mu} \theta_{t+1}^{\mu}) = (1 - \mu) I_t^{\mu} \theta_t E_t(P_{t+1}) \]  

(10)

Now we consider the firm’s investment decision problem. This may be written as:

\[ \max_{I_t - N_{t+1}} E_t\left\{ P_{t+1} (I_t \theta_{t+1})^{\mu} N_{t+1}^{1-\mu} - r_{t+1} w_{t+1} N_{t+1} - R_t P_t I_t \right\} \]  

(P2)

The first order conditions of maximization are:

\[ \mu I_t^{\mu-1} E_t\left\{ P_{t+1} I_{t+1}^{\mu} N_{t+1}^{1-\mu} \right\} = R_t P_t \]  

(11)

\[ (1 - \mu) I_t^{\mu} E_t\left\{ P_{t+1} I_{t+1}^{\mu} N_{t+1}^{1-\mu} \right\} = E_t[r_{t+1} w_{t+1}] \]  

(12)

Combining the first order conditions we get

\[ I_t = \frac{\mu}{1 - \mu} \frac{E_t(r_{t+1} w_{t+1} N_{t+1})}{R_t P_t} \]  

(13)

By assumption aggregate supply of labour is unity in each period, so that in equilibrium \( E_t(N_{t-1}) = 1 \). Substituting this along with (10) in equation (13) we have

\[ I_t = \frac{I_t^{\mu} E_t(P_{t+1} \theta_{t+1}^{\mu})}{R_t P_t} \]  

which reduces to

\[ I_t = \mu^{\frac{1}{1-\mu}} \theta^{\frac{\mu}{1-\mu}} \frac{E_t(P_{t+1})^{\frac{1}{1-\mu}}}{(R_t P_t)^{\frac{1}{1-\mu}}} \]  

(14)

Also,

\[ P_t I_t = \mu^{\frac{1}{1-\mu}} \theta^{\frac{\mu}{1-\mu}} \frac{E_t(P_{t+1})^{\frac{1}{1-\mu}}}{(R_t P_t)^{\frac{1}{1-\mu}}} \]  

(15)
Equation (14) gives the investment demand of the firm and (15) gives the same in nominal terms. The latter is also the equation for the demand for the fixed capital loans of the firms in nominal terms.

The bank is assumed to be an infinitely lived institution specializing in intermediation activities. It accepts deposits from the households for one period and lends money to the firms for financing their fixed capital and working capital. Loans for fixed capital along with interest are repaid after two periods and those for working capital loans are repaid after one period. The bank holds both short-term and long-term assets, viz. short-term loan advanced for working capital and long-term loan for fixed investment, but holds only short-term liability, viz. bank deposits. The typical bank in this model diversifies in its asset-liability structure that differs in respect of the maturity pattern of the loans and deposits. This is one of the specialized functions of a financial intermediary. The repayment of the principal and the interest on working capital loans is certain in this model, because when loans for working capital are made, the uncertainty of the production process has already been resolved.\(^8\) The repayment of the fixed capital loans is made after paying for the working capital loans in full.

Denoting the fixed capital loans by \(L^f_t\) and working capital loans by \(L^w_t\) the bank’s expected profit in period \(t\) is given by

\[
\Pi_t^b = \delta R_t L^w_t + \delta q R_t L^f_t + (1 - q) \min(R_{t+1} L^f_{t+1}, \max(P_{t+1} Y_{t+1} - r_{t+1} L^w_{t+1}, 0)) - d_t D_t
\]

where \(\delta\) = one period subjective discount rate. There are three terms within the parenthesis on the right hand side of equation (16). The first term is the repayment for the working capital loan advanced at \(t\) that will be realized at \(t + 1\). The second term is for the repayment of the fixed capital loan which is advanced at \(t\) and will be realized at \(t + 2\). It has two components. The first component is the repayment for the fixed capital loan when the good state occurs and the second if the bad state occurs. In the event of the bad state, the bank receives the contracted amount on the fixed capital loan if the sales revenue is sufficient to cover both the working capital and the fixed capital loan. If the sales revenue is insufficient, then the bank receives either the amount left after repaying the working capital loan if it is positive or zero if nothing is left after repaying the working capital loan. They are multiplied by their respective probabilities to get the expected values. The last term within the parenthesis is the principal plus interest on bank deposits that the households is paid back at \(t + 1\).

The bank’s aggregate fund as of time \(t\) is given by

\[
F_t = (1 - \alpha) D_t + r_{t-1} L^w_{t-1} + \min(R_{t-2} L^f_{t-2}, \max(P_{t-1} Y_{t-1} - r_{t-1} L^w_{t-1}, 0)) - d_{t-1} D_{t-1} + \Lambda_{t-1}
\]

where \(\alpha\) = cash reserve ratio and \(\Lambda_t\) = accumulated profit (or loss) accrued by the bank up to time \(t\).

\(^8\) We assume away enforcement problem associated with the repayment of the two types of loans.
The bank maximizes the present value of the discounted sum of expected profits subject to fund constraint in each period. Formally, the bank’s problem is given by the following dynamic optimization problem.

$$\max_{L_t^w, L_t^f} E_t \left( \sum_{i=0}^{\infty} \delta^i \Pi_t^{bp} \right)$$

subject to

$$L_t^w + L_t^f = F_t$$

$$F_t = (1 - \alpha) D_t + r_{t-1} L_{t-1}^w$$

$$+ \min(R_{t-2} L_{t-2}^f, \max(P_{t-1} Y_{t-1} - r_{t-1} L_{t-1}^w, 0)) - d_{t-1} D_{t-1} + A_{t-1}$$

This is a stochastic dynamic programming problem that could, in principle be solved by the Bellman equation technique (Bellman (1957), Bellman and Dreyfus (1962)). The Bellman equation representation of this optimization problem is given below.

$$V(F_t, r_t, R_t, d_t) = \max_{L_t^f} \left( \delta r_t (F_t - L_t^f) + \delta (q R_t L_t^f + (1 - q) \min \left( \max \left( \frac{P_{t+1} Y_{t+1} - r_{t+1} (F_{t+1} - L_{t+1}^f), 0}\right) \right)) \right)$$

$$+ \delta^2 E_t V(F_{t+1}, r_{t+1}, R_{t+1}, d_{t+1})$$

subject to

$$F_{t+1} = (1 - \alpha) D_{t+1} + r_t (F_t - L_t^f)$$

$$+ \min(R_{t-1} L_{t-1}^f, \max(P_t Y_t - r_t (F_t - L_t^f), 0)) - d_t D_t + A_t$$

Clearly in the Bellman equation representation, \(L_t^f\) is treated as the control variable and \((F_t, r_t, R_t, d_t)\) is the vector of state variables. In principle one can solve the above optimization problem and the first order condition gives the optimal supply of two types of loans. These are equated with the respective demands for loans to get the equilibrium interest rates, \(r_t\) and \(R_t\). However, because of the complex nature of the objective function, the optimal solution obtained from the first order conditions becomes intractable. Hence we solve for the equilibrium interest rates, utilizing some other economic relations. This is done in a later part of this section when we discuss market equilibrium conditions.

In this model the role of the central bank is very limited. It holds the government’s liability which is the aggregate high-powered money in the system. It also fixes the cash reserve ratio. By altering the cash reserve ratio the central bank can change the aggregate money or the credit supply in the system.

The government, in this model, does not undertake any productive activity. It only spends on goods and services. This expenditure is met partly by taxes and partly by deficit financing. The latter is the source of injection of the high-powered money. Thus the government budget constraint is given by the relation given below.

$$G_t = T_t + H_t - H_{t-1}$$
where \( G_t \) = nominal government expenditure in period \( t \), \( T_t \) = nominal tax revenue in period \( t \), \( H_t \) = nominal high-powered money in period \( t \). We also assume that \( G_t \) and \( T_t \) are policy parameters and hence exogenously given. We have made these simplifying assumptions because we intend to focus on how increase in high-powered money, arising out of higher government expenditure, affects the generation of credit in the system and then deduce the effect of the latter on the other endogenous variables such as price, output, investment.

Putting \( G_t = G \) and \( T_t = T \) the government budget constraint could be restated as follows:

\[
H_t = H_{t-1} + G - T
\]  

(20)

\( G \) and \( T \) being determined from outside, \( H_t \) is determined endogenously by (20). Forwarding (20) for one period and taking expectation we have

\[
E_t(H_{t+1}) = H_t + G - T
\]  

(21)

Equation (21) gives the conditional expectation of the stock of high-powered money at \( t + 1 \). We will require it for later calculations.

There are three markets in this model, viz. goods, labour and credit. The labour market is assumed to be always in equilibrium attained through the instantaneous adjustment of the money wage rate. The wage rate is always at its equilibrium value given by (8) and equilibrium employment is always at unity.\(^9\) This is a somewhat unrealistic assumption, but it simplifies the analysis because in this way we can net out the labour market from the analysis and focus on the interactions of the product and credit markets.

For the aggregate banking sector, there is a relation between aggregate high-powered money in circulation and aggregate deposit. The banks are required by law to hold a fraction of their deposits either as vault cash or in the form of balances with the central bank. This is called cash reserve requirement. For the sake of simplicity, we have assumed here that the reserve requirement takes the form of vault cash. In this model no other agent holds cash. Thus denoting the aggregate high-powered money by \( H_t \) we have the following relation:

\[
D_t = H_t / \alpha
\]  

(22)

For the banking sector as a whole, the aggregate demand for two types of loans equals the respective supply so that in equilibrium

\[
L_{t}^{w} = L_{t}^{w} = \frac{H_t}{\alpha} + T \quad \text{and} \quad L_{t}^{f} = P_t I_t .
\]

Substituting (1), (8) and (9) in (22), the equilibrium interest rate on working capital loans is given by the following relation:

\[
r_t = \frac{\alpha(1 - \mu) P_t I_t^{\mu} \theta_t^{\mu}}{H_t + \alpha T}
\]  

(23)

For \( R_t \) we can recast (15) as in below:

\(^9\) This is a full employment equilibrium model.
Given the values of the other endogenous variables and the values of the parameters, (23) and (24) determine \( r_t \) and \( R_t \). To determine \( d_t \) we use the profit of the typical bank in each period. In each period, competition among the banks drives down their profit to zero. Thus setting \( \Pi_t^b = 0 \) for all \( t \) we have,

\[
d_t = r_t + \frac{\delta t}{H_t} [q R_t P_t I_t + (1 - q) \text{Min}(R_t P_t I_t, \text{Max}(E_t (P_t Y_{t+1} + r_{t+1} (H_{t+1} / \alpha + T)), 0))]
\]

Now we consider the two market equilibrium conditions mentioned earlier, viz. goods market equilibrium and credit market equilibrium given by (26) and (27) respectively.

\[
dt_{t-1} \frac{H_{t-1}}{\alpha P_t} + I_t + \frac{G}{P_t} = Y_t 
\]

\[
\left(\frac{H_t}{\alpha} + T\right) + P_t I_t = \frac{1 - \alpha}{\alpha} H_t + r_{t-1} \left(\frac{H_{t-1}}{\alpha} + T\right) + \text{Min} \left\{ R_{t-2} P_{t-2} I_{t-2}, \text{Max} \left[ P_{t-1} Y_{t-1} - r_{t-1} \left(\frac{H_{t-1}}{\alpha} + T\right), 0 \right] \right\} 
\]

\[
- d_{t-1} \left(\frac{H_{t-1}}{\alpha} + T\right) + \Lambda_{t-1}
\]

The first term on the left hand side of (26) is the consumption demand of the households of previous generation, the second term is the investment demand of the firms and the third term is the government expenditure on goods. The right hand side is the aggregate supply of output. The first term on the left hand side of (27) is the aggregate working capital loans and the second term is the demand for fixed capital loans. The right hand side is the aggregate supply of funds with the banks.

From (27) we have

\[
I_t = \frac{1}{P_t} \left( - H_t + r_{t-1} \left(\frac{H_{t-1}}{\alpha} + T\right) + \text{Min} \left\{ R_{t-2} P_{t-2} I_{t-2}, \text{Max} \left[ P_{t-1} Y_{t-1} - r_{t-1} \left(\frac{H_{t-1}}{\alpha} + T\right), 0 \right] \right\} \right) - d_{t-1} \left(\frac{H_{t-1}}{\alpha} + T\right)
\]

Substituting for \( I_t \) from (28) in (26) and simplifying we have
\[ P_t = \frac{1}{(l_{t-1} \theta_1)^\mu} \left( -H_t + G - T + r_{t-1} \left( \frac{H_{t-1}}{\alpha} + T \right) \right) \]
\[ + \min \left\{ R_{t-2} P_{t-2} I_{t-2}, \max \left\{ P_{t-1} Y_{t-1} - r_{t-1} \left( \frac{H_{t-1}}{\alpha} + T \right), 0 \right\} + A_{t-1} \right\} \]

Forwarding (29) for one period and using (4) and (21) we can derive the expression for \( E_t(P_{t+1}) \):

\[ E_t(P_{t+1}) = \frac{1}{(l_\theta)^\mu} \left( -H_t + G - T + r_t \left( \frac{H_t}{\alpha} + T \right) + \min \left\{ R_{t-1} P_{t-1} I_{t-1}, \max \left\{ P_t Y_t - r_t \left( \frac{H_t}{\alpha} + T \right), 0 \right\} + A_{t-1} \right\} \right) \]

Forwarding (4) for one period and taking conditional expectation,

\[ E_t(Y_{t+1}) = I_t^{\mu \theta} \]

Similarly from (23),

\[ E_t(r_{t+1}) = \frac{\alpha(1 - \mu) E_t(P_{t+1}) I_t^{\mu \theta}}{H_t + G - T + \alpha T} \]

Equations (31) and (32) will be used to calculate the equilibrium values of some other endogenous variables.

Given the vector of parameter values (\( \alpha, \delta, \mu, \theta_1, \theta_2, q, G, T \)), the vector of previous period values of the endogenous variables and the realization of \( \theta_t \), one can solve for the vector of current period values of the endogenous variables \( (H_t, Y_t, P_t, I_t, r_t, R_t, d_t) \) from the reduced form expressions (4), (20), (29), (28), (23), (24), (25). The equilibrium values of the other endogenous variables such as \( w_t, L_t^w, L_t^f, D_t \) whose values can be derived from the endogenous variables mentioned above.

3. MODEL SIMULATION

We are interested in the dynamic path of the endogenous variables in this study. We would like to see how the trajectories of the respective variables change as some parameter changes value. The presence of the minimum function in the expressions for \( P_t, I_t \) etc. introduces kinks in the respective functions (though they are continuous at the kink values) and renders the functions non-differentiable. Hence we opted for the numerical solution\(^\text{10} \) of the endogenous variables for a given set of parameter values for a considerable period of time, viz. 100 time points. The parameters values are chosen such that

\(^{10}\) For the simulation algorithm a computer programme was written in FORTRAN 77.
they are consistent with Indian data. The standard practice in the literature is to get the estimates of the parameter values for technological and taste parameters (the so called ‘deep’ parameters) from some econometric study. We did not attempt to estimate the model econometrically. There is no dearth of macroeconometric study on India, but structures of these models are so different that it will be inappropriate to use them for the present study. There are a number of reasons. All of these macroeconometric studies have a number of production sectors while the present study employs a model with only one good in the production sector. As a result it is difficult to reconcile the technological parameters of many production sectors into one technological parameter which is representative for the one good economy, such as $\mu$ of the Cobb–Douglas production function in equation (4).

It may also be noted that this study considers a model which incorporates the characteristic features of the post-liberalization era in India and elsewhere in the developing countries which is markedly different from the pre-liberalization period. But the econometric studies referred above generally employs same econometric model for both the pre- and post-liberalization era. Thus these econometric studies are subject to Lucas critique and makes the parameter estimates from these models inappropriate for the present study. However, consulting the existing econometric studies on India and National Income Accounts, different values for the parameter of the production function were tried and finally we settled with a value of the parameter of the production function $\mu$ for the present study as in Table 1. Needless to mention we had to depend more on judgement in the absence of appropriate econometric study.

The probability of a good state of the productivity shock, $q$ we depended on the proportion of loans from banks and financial institutions that does not become non-performing which is around 70 to 75% in India in recent past. In the absence of an econometric study on the probability of state of productivity it is quite consistent with empirical facts.

There are other parameters, which are not deep parameters of the model, such as cash reserve ratio, rate of discount for loans advanced by the banks. For these parameters this study uses data consistent with Indian economy. A value of 8% for cash reserve ratio, (so that $\alpha = 0.08$) is consistent with Indian reality at the time of writing this paper. A discount rate of 8 to 9% for long term securities of Government of India results in a value of $\delta$ equal to 0.909. The long term rate of return on Government of India securities can be treated as the appropriate rate of discount for the banks. This study assumes a constant labour supply and normalizes it at unity, so that $N = 1$. This may seem unrealistic in the first instance. However, as we are interested in the characteristics of business cycle and not in growth, we have not incorporated any growth of labour force. Thus the fluctuations are interpreted around a zero growth of the economy.

11 See Krishnamurty (2002) and Pandit (1999) for a survey of the recent literature.
12 It may be noted that shares of different factors of production (which can be calculated from National Income Accounts) are parameter of those factors for Cobb–Douglas production function.
13 Instead of unity, any other fixed number will give us qualitatively same results.
Initially we set \( G \) and \( T \) at 0.5 implying a balanced budget. The assumption of balanced budget is not true in any economy. But this was assumed so, as in a long run stationary equilibrium the budget should be balanced and starting from a stationary equilibrium we will investigate later the effect of an interventionist policy. This will be useful for comparing different regimes of interventionist policies.

The initial values of the endogenous variables, such as aggregate output, investment, price level etc. are set in such a way that they are consistent with real life data at the time of writing this paper. However, we have converted them in suitable units so that we can work with small numbers.\(^{14} \) For the interest rates we have set initial values apparently at a higher value than one would expect. Because a very small value of interest rates make the plots for consecutive values indistinguishable in the diagram. However, we were careful so that the spread of interest rates are maintained in such a way that it remains consistent with reality.

For an initial set of parameter values (given in Table 1) and the initial values for the endogenous variables (given in Table 2) we find out the dynamic paths for the endogenous variables.

These are shown in Figures 1 through 8. The dynamic path of any endogenous variable ‘\( X \)’ is denoted by ‘\( X_0 \)’ in the figures for the initial set of parameter values and initial values of the endogenous variables.

The time paths of investment and consequently output (output depends upon previous period investment and current realization of the random shock to production function \( \theta_t \)), as given in Figures 2 and 3, show that starting from their respective initial values, both of these variables fluctuate within a band, viz. (0.14 2.0) for investment and (2.0 3.0) for output. The long-term nominal interest rate, \( R \), actually converges to 1.58\(^{15} \) as

\( \begin{array}{cccccccc}
\hline
G & T & \bar{N} & \mu & \alpha & \delta & q & \theta_1 & \theta_2 \\
\hline
0.5 & 0.5 & 1 & 0.1 & 0.08 & 0.909 & 0.7 & 10000 & 190000 \\
\hline
\end{array} \)

\( \begin{array}{cccccccccc}
\hline
I_{-1} & Y_{-1} & P_{-1} & H_{-1} & r_{-1} & R_{-1} & d_{-1} & F_{-1} & A_{-1} & R_{-2} & P_{-2} & I_{-2} \\
\hline
0.093 & 1.65 & 300.5 & 15.0 & 1.61 & 1.86 & 1.78 & 49.88 & 0.0464 & 1.89 & 285.9 & 0.093 \\
\hline
\end{array} \)

Note: (1) The numerical values were rounded off to their nearest decimal for convenience of presentation.
(2) The return on account of fixed capital loan advanced two periods ago is assumed to be repaid in full in the initial period.

\(^{14} \) Output and investment are in scales of \( 10^6 \) million and \( 2 \times 10^6 \) million respectively. They are in Rupees (Indian currency) at constant prices with 1980–81 as the base. Price level corresponds to the GDP deflator with 1980–81 as the base. High powered money is in unit of \( 6 \times 10^4 \) million and accumulated profit (loss) and last period’s profit of the banking sector are in unit of \( 4 \times 10^5 \) million. All these are in current prices and in Rupees. Multiplying by the respective numbers our simulated data correspond to real life data.

\(^{15} \) With an expected rate of inflation of \(-10\% \) to \(20\% \) the real long-term rate of interest is marginally different.
in Fig. 6. Hence the fluctuation in investment is due to fluctuation in $\theta_i$. The pattern of fluctuation of investment is more dependent on current period realization of $\theta_i$ compared to that of output, because current period output is dependent on last period’s investment. However, a noticeable feature of the simulated values is that when the random shock to the production function changes from one state to the other, the corresponding change in investment or output is much more than when the new random shock is of the same kind (positive or negative). That is to say, when the random shock changes from a bad state to a good one, investment or output immediately jumps to a higher level, but another good state in the next period does not raise them with a jump but raises them only marginally.

The time path of price level displays a rising trend (exponential) throughout, with fluctuation around the trend depending upon the realization of $\theta_i$.\textsuperscript{16} Though the series of investment and output fluctuate within a band, the price series shows a rising trend. This is due to the fact that though there is no change in the parameters of the model, including that in the supply of the high-powered money, there is a continuous increase in the nominal profit of the banking sector. The latter enters the reduced form of the price equation as a component of the aggregate fund of the banking sector for lending purposes and hence the price level continuously rises. As the capital stock is exhausted in each period after production is over and the loan for the fixed capital is repaid, the bank’s credit creation capacity is renewed. The whole of the bank fund is supplied either as the fixed capital loan or as the working capital loan. But the supply of working capital loan cannot exceed $(H_i/\alpha + T)$ given the relations between wage bill, bank deposit and high-powered money. Investment in real terms is also within a band so the nominal supply of funds for fixed capital can only increase. This implies an increase in demand for investment in nominal terms, with no change in real demand for investment. Thus

\textsuperscript{16} In Fig. 4 the fluctuations in the price level for earlier iterations are not discernible. However when earlier iterations were plotted separately they show similar properties as the latter iterations.
all adjustments take place through the commodity price and the interest rate on working capital loan and they increase\(^\text{17}\) in each period. It may, however, be noted that the dynamic path of the price level so exhibited, is contingent on the particular time path of the random shock, \(\theta_t\) and the assumed probability of the occurrence of good state (and the bad state as its complement). If the time path of \(\theta_t\) is different the endogenous variables will trace out different dynamic paths. Owing to the proportionality between the short-term interest rate and the commodity price (see equation (23)) the dynamic path of the short-term interest rate is similar to that of the commodity price. The movement of the nominal interest rate on bank deposits, \(d_t\) is governed by the movement of the short-term interest rate, because \(R_t\) is convergent. Hence the time path of \(d_t\) is not shown separately in the figure.

The phenomenon that the price level or the short-term interest continuously rises without any change in the policy parameters may sound queer. It may be noted that it occurs mainly due to two assumptions of the model that the fixed capital is fully exhausted in each period and that the income of the households and their savings in the bank deposits in period \(t\) comes also from the working capital credit advanced in period \(t\). Relaxing both these assumptions makes the model more realistic, but only at a cost of more complexity; even the numerical solutions become very difficult to derive. But this

\(^{17}\) This is a single good economy. So increase in the price of the investment good means a rise in the general price level.
unrealistic feature of the model is removed to some extent if one looks at the real bank profit. The real bank profit and the real supply of aggregate bank funds fluctuate within a band where the nature of the fluctuation is governed by the realization of \( \theta_t \). Also, the interest earnings of the bank from short-term loans or long-term loans in real terms fluctuate within a band. We also tried another formulation where aggregate bank profit is distributed for consumption purposes so that it becomes part of the aggregate demand for goods. But even then the basic result remains the same because the aggregate bank
profit enters the reduced form of the price equation through the commodity market equilibrium condition.

The effect of $\theta_t$ on the price level is opposite to that of output—a bad state of $\theta_t$ leads to a higher price and vice versa around the trend. For the price series also two consecutive random shocks of the same type leads to a marginal change while when the shock changes regime, the extent of change is quite substantial. However, the time path of the expected price level is almost smooth with a rising trend (exponential). It is because of the fact that in the calculation of the expected price level the only random variable for which one has to form expectation is output. The value of the expected output is an average of the output values corresponding to the bands. Hence the expected price level moves with the other endogenous variables known in the current period. Accordingly
the actual rate of inflation and expected rate of inflation show similar movements.\footnote{The actual rate of inflation are denoted by INFLA, in Figures 7 and 14 and the expected rate of inflation by INFLE in Figures 8 and 15.} The actual rate of inflation is, however, higher than the expected one as there is less fluctuation in the expected price.

It may be noted that it is often desirable to undertake a sensitivity analysis of the parameters of the model in terms of the simulation exercise to judge the robustness of the model. This paper does not carry such exercises for two reasons. Firstly, it will make the paper too long and it is not the purpose of the paper to test the robustness of this model. Rather the purpose of this paper has been to highlight the role of fixed capital and working capital and the associated bank loans in a macroeconomic model. Secondly, the model had been found to be robust as shown by Das (2003). A change in the probability of good state of productivity or the value of the good state does not change the basic tenets of the results.

Now we consider some simulation exercises to compare solution properties of the dynamic paths of the endogenous variables when some policy changes are undertaken. From iterations 16 to 19, there are four consecutive bad states (generated stochastically). As a result, investment and output (i.e. income) are lower. It is often recommended in the literature that government expenditure be increased in order to raise total income. This additional government expenditure can be financed in several ways. Two commonly adopted methods are (i) money creation and (ii) the raising of an income tax.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Time path of Actual Rate of Inflation.}
\end{figure}
In our simulation exercise government expenditure, $G$ is raised from 0.5 to 4.0 over four consecutive iterations. Thereafter it is again lowered to its previous value. In the case of money-financed increase in $G$, high-powered money $H$ also increases by the same amount for all the four consecutive periods. Thereafter as $G$ reverts to its original level, $H$ increases no more but it remains at its higher level. In the case of tax-financed increase in $G$ there is no change in $H$. We take the same series of random shocks to production function in both these cases. Hence any change in the dynamic path of any of the endogenous variables is due not to different realization of the random shock to production function, but it is due to increase in $G$.

We consider both the cases of money-financed and the tax-financed increase in $G$. The dynamic paths of the endogenous variables for the two cases are shown in Figures 9 through 15. For comparison with the respective original series we also plot the original series of the endogenous variables in these figures. It may be noted that the dynamic path of an endogenous variable ‘$X$’ for the initial set of parameters is denoted by ‘$X_{or}$’. To distinguish ‘$X_{or}$’ from the dynamic paths in the money-financed case and the tax-financed case, the dynamic path in the money-financed case is denoted by ‘$X_{mp}$’ and in the tax-financed case ‘$X_{fp}$’.

The effects of increased government expenditure on the endogenous variables are found to be the following. In the current period (i.e. in iteration 16) investment falls by the same amount in both the cases compared to that in the original series. As a result output also falls in the next period by the same amount in both the cases. In the next period onwards investment increases; and output increases with one period lag in the tax-financed case. As a matter of fact both investment and output (with one period lag) in the tax-financed regime increase for a considerable time even after the government expenditure is brought back to its original level (at iteration 20) and then converge with the original series at iteration 57 (i.e. after 36 iterations) for investment and 58 (i.e. after 37 iterations) for output. But in the money-financed case output and investment remain all along at a lower level—even lower than the original series. Even
Figure 9. Policy Intervention and the Transition Dynamics of Investment.

Figure 10. Policy Intervention and Transition Dynamics of Output.

when the perturbation is removed investment and output do not come back to its original levels. Interest rate on long-term loans in the current period rises by the same amount in both the cases. There after it drastically falls in the case of tax-financed regime, then converges to the original series in iteration 64. This occurs much later than the iteration at which real investment converges. In the money-financed case, on the other hand, the long-term interest rate converges to a higher level. The dynamic path of price level is
all along higher for the tax-financed regime than for the money-financed regime,\textsuperscript{19} with fluctuations depending upon the state of the random shock.

It may be noted that at each $t$ there is an expectation of price estimated to prevail in $t + 1$. The actual price in $t + 1$ will be different from the expected price (where expectation is made at $t$) depending upon the state of the economy in $t + 1$. The expected price level series also exhibit similar paths as the actual price level but with a lesser

\textsuperscript{19} It is even higher than the original series.
degree of fluctuations. The actual and expected rates of inflation fluctuate within a band in both the cases of money-financed and tax-financed increase in $G$, but the bands for the tax-financed regime are on the higher side. The series for short-term interest rate and the deposit rate also exhibit properties similar to those of the expected price level. The short-term interest in all iterations after the perturbation initiated is also the lowest for the money-financed case and the highest in the tax-financed case while the original series lies in between the two.

In this model output in the current period cannot increase, it is governed by investment made in the previous period and the state of productivity, output does not rise
owing to the fact that this is a supply-constrained regime; hence the results differ from usual demand-constrained regime models. Thus an increase in $G$ whether financed by imposing higher tax or by creating additional high-powered money, crowds out private investment or private consumption expenditure. The latter is given in $t$ th. period, hence only private investment is fully crowded out. This becomes clear from (27) and (28). So there is no change in commodity price. The decrease in private investment is achieved by an increase in long-term interest rate in the current period.

The fact that working capital plays a very important role in this model is explained in the few sentences below. Let us consider the following relationships:
\[ L^{w}_t = \frac{H_t}{\alpha} + T \quad \text{and} \quad L^{f}_t = P_t I_t \quad \text{and equation (23)}. \]

It is clear that in the tax-financed case the increase in the equilibrium working capital resulting from an increase in $G$ is less than in the money-financed case. This is because in the former, the corresponding change is same as the increase in $G$ while it is $(1/\alpha)$ times the change in $G$ for the latter. Thus for the same level of increase in $G$, relatively more working capital is required in the money-financed case than what is needed in the tax-financed case. As a result, lesser amount of funds remains available for the fixed capital loan in the money-financed case.

It is observed that in the period when $G$ changes (i.e. for $t = 16$), investment in both the money-financed and the tax-financed regimes falls as compared to the original value of investment and the extent of the fall is the same but output remains at the original level when there was no perturbation. The current tax increase does not affect current consumption, as households only consume in the second period of their lives. The increased government expenditure matches the increase in tax in the tax-financed case so that there is no effect on current stock of high-powered money and the increased tax in the current period does not also enter the equation for the current price level. Thus there is no effect on the current price level. In the money-financed case also the increased government expenditure exactly balances increase in the high-powered money so that the net effect on the price level is nil in the current period. But the expected price level in the next period increases as is clear from equation (30).

An increase in the amount of tax or high-powered money implies an increase in the total wage bill and hence the requirement of total working capital loans. The working capital loans crowd out fixed capital loans. This is achieved by a lower interest rate for working capital loans in both the cases of money-financed and tax-financed regimes.\(^{20}\) But the decrease in the short-term interest rate in the case of money-financed regime is higher because in the case of tax-financed regime the effect on short-term interest rate is a fraction ($\alpha$) of the change in $G$ in the money-financed case [see equation (23)]. The crowding out of fixed investment in the two cases is identical [see equation (28)] in the current period. A higher expected price in the next period coupled with a lower fund for fixed capital reduces the long-term interest rate. In this respect there is no difference between the two regimes. Thus the long-term interest rate rises and investment falls in the current period in both the cases.

In $t + 1$ th. period output in both the money-financed and tax-financed cases is lower by the same amount. Now consumption demand is higher in the money-financed case because consumption expenditure is met out of savings held as bank deposits in the previous period and an increased government expenditure financed by high-powered money in the previous period leads to a higher bank deposits in the previous period. As a matter of fact a unit increase in $G$ leads to $(1/\alpha)$ times increase in bank deposit. This will become clear from equation (26) forwarded one period. So demand for goods is

\(^{20}\) It can is easily be verified that if there is no working capital loan then price rise is higher in the money-financed case than the tax-financed case.
more in the money-financed case than for the tax-financed case. Again in the money-financed case working capital fetches more funds in equilibrium than in the tax-financed regime. So lesser amount of funds remain available for fixed capital loan. These two factors together leads to lower investment in fixed capital in the money-financed case. This is achieved by a higher long-term interest rate in the money-financed case.

A lesser amount of working capital fund in the tax-financed case compared to that in the money-financed case raises the price level more in the former than in latter. This is because of the fact that in this model equilibrium level of employment is fixed and a lower working capital which is used to purchase inputs (in this model only labour) raises nominal wage rate.\(^\text{21}\) Hence goods price rises more in the tax-financed case than in the money-financed case. A higher investment in this period leads to a higher output in the next period in the tax-financed regime. In the next iteration, a similar development occurs in the tax-financed case and so on for the next round of iterations. Even after the perturbation is removed and \(G\) is back to its original level, the effect of the perturbation remains for a considerable period of time. Finally when the effect of higher tax dies down, long-term interest rate converges to the original series in the tax-financed case and investment and output return to their original levels.

In the money-financed case, however, high-powered money is permanently raised to a higher level and since this constitutes the working capital loan, it permanently crowds out equilibrium fixed capital loans. These are achieved by a lower short-term interest rate and a higher long-term rate all along for the series. The permanently high level of high-powered money in the money-financed case also lowers both the price and expected price compared to that in the original series or the series in the tax-financed case for all iterations after the initial perturbation. The dynamic paths of bank deposit rate in both the cases resemble the respective dynamic paths of the price level.

To sum up the results let us note that an increase in government expenditure has adverse effect on current investment irrespective of whether it is financed by increased money supply or by raising an income tax because increased government expenditure fully crowds out investment. However in the next period investment increases in the tax-financed case and falls in the money-financed case. Price level and short-term interest rate rise in the tax-financed case while they fall in the money-financed case. But the effect on the long-term interest rate is just the opposite in the two cases. In the conventional wisdom an increased money supply leads to a rise in the price level and a fall in the interest rate. The negative effect of an increase in money supply on the rate of interest operates for the short-term loans only while it is just the reverse for the long-term loans. It is the presence of working capital in this model that gives results that are contrary to conventional wisdom.

It may be noted that several simplifying assumptions were made to make the model suitable for simulation exercise. The assumption on a 100% rate of depreciation is one such simplifying assumption. However, a less than 100% rate of depreciation would not change the basic tenets of the results in connection with the two kinds of policy

\(^\text{21}\) In this model wage cost is the only variable cost while capital cost is committed one period ahead.
interventions that were considered. Whether or not the capital stock depreciates in part or in full a change in the long term interest rate due to a change in the availability of long term bank loans changes the investment demand. Thus the basic results remain same. Similarly relaxing the assumption of second period consumption for the household will not change the fundamental results of the paper.

4. CONCLUSION

We set up a dynamic general equilibrium model with banks. Banks accept deposits from the households and lend to firms. The latter require loans for two uses, viz. investment in fixed capital and in working capital. As there are two sets of equilibrium values of the endogenous variables depending upon the realization of the productivity shock, we simulated the model for an initial set of parameters. The time paths of real variables, such as investment and output remain within a range determined by the realization of stochastic shock to production function. The dynamic path of the nominal long-term interest rate converges towards a particular value. However the nominal variables, such as the current price level, the expected price level, the short-term interest rate and the bank deposit rate exhibit a rising trend. The dynamic paths of the latter two variables are similar to that of the expected price level. As both the current and the expected price levels exhibit a rising trend, the rate of inflation, both the actual and the expected rate, remain within a range as in the case of output or investment. However, the range of fluctuations of the expected rate of inflation is much less than that of the actual rate.

To see the role of interventionist policy we perturbed some of the policy parameters in a bad state with lower output. In this respect we used two types of policies: raising government expenditure by new money creation and by additional taxation. It was found from the dynamic paths of the endogenous variables that in the money-financed case, the effects on investment and output are exactly the opposite of what it is expected in the usual exposition. The dynamic paths of both investment and output follow a lower level in the money-financed case than in the tax-financed case even when the perturbation is removed. The dynamic paths are in fact give even lower levels of investment and output compared to that in the original series. However price level, rate of inflation (both actual and expected) and short-term interest rate are lower in the money-financed case while long-term interest rate is higher in the money-financed case. Thus demand management policy is more effective in this model by raising an income tax while monetary policy is more effective to control inflation.

Finally it was argued that relaxing the simplifying assumption would not make any difference to the basic tenets of the results. Regarding the extension and future research it may be noted that this model could be used as a base line model to address macroeconomic problems of developing countries where the financial institutions have differences from their counterparts in the developed world. One can introduce equity market, international finance etc. in the model to address different types of problems.
REFERENCES


