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EXISTENCE OF EQUILIBRIUM IN LABOR-MANAGED STACKELBERG OLIGOPOLY

Takafumi MATSUBA

Department of Economics and Information, Gifu Shotoku Gakuen University, Gifu, Japan

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Abstract: This paper analyzes Cournot and Stackelberg oligopolistic competition by labor-managed firms. The main aim of this paper is to demonstrate that there exists a global equilibrium under Stackelberg oligopolistic competition even when the second order condition is satisfied only locally.

Keywords: existence of equilibrium, Cournot, oligopoly, Stackelberg, market imperfection.
JEL Classification: D43, L13.

1. INTRODUCTION

It is widely known that, in typical cases of Stackelberg type competition by profit maximizing firms in the market for a homogeneous good, the profit and market share of the leader are larger than those of the follower (s). However, Krelle (1971) showed that if the products of duopolists are differentiated and if the duopolists engage in Bertrand-type competition, the profit of the follower becomes larger than that of the leader. In addition, Gal-Or (1985) derived the general condition that the second mover (*i.e.*, follower) will receive more advantages and disadvantages than the first mover (leader) in a Stackelberg game with a symmetrical pay-off function. As these preceding studies show, the leader's output is not always larger than the follower's.

Labor-managed firm is the firm which maximizes the wage or the profit per labor. Labor-managed firms used to be prevalent in former Yugoslavia, as they are now found in Basque district (in Spain). In addition, some industries in the United States and large Japanese firms are also considered to be labor managed. Even in the case of a labor managed firm that has a different objective function than the profit-maximizing firm, Okuguchi and Serizawa (1998) assert that the leader's output is less than the follower's. Okuguchi (1986, 1992) analyzed the oligopolistic situation in a labor managed economy. This paper also analyzes Stackelberg oligopolistic competition by labor managed firms, and will show that the optimum output level of the leader is less than that of the follower.

E-mail: tmatsuba@gifu.shotoku.ac.jp

Although there exists a large number of analysis of the imperfectly competitive labor managed markets, there are not so much analyses of the equilibrium. In the analysis of the imperfectly competitive labor managed market, it is uncertain whether the second order conditions for product optimization are satisfied globally. Hence, there could exist another equilibrium in which the leader's output exceeds the follower's. The main aim of this paper is to demonstrate that there exists a unique equilibrium in the optimization problem of labor managed firms under Stackelberg oligopolistic competition even when the second order condition is satisfied only locally.

2. THE BEHAVIOR OF LABOR MANAGED FIRMS IN A COURNOT OLIGOPOLY

Main framework follows Okuguchi (1993a). Suppose there is an oligopolistic market of labor managed firms. Let n be the number of labor managed firms in the market. Let x_i and Q be the i-th labor managed firm's output and the total industry output, respectively. ($i.e.$, $Q = x_1 + \cdots + x_n$.) Let the inverse demand function be $p = f(Q)$, where p is the price of the good, and $f' \equiv df(Q)/dQ < 0$. Furthermore, let $h(x_i)$ be the labor input necessary for producing output x_i, where $dh(x_i)/dx_i \equiv h'_i > 0$, $d^2h(x_i)/dx_i^2 \equiv h''_i > 0$. Let us assume symmetric firms and let i-th firm's cost function be $C(x_i) = wh(x_i) + k$, where the wage w and the fixed cost k are given. Assume, in addition, that all firms form expectations regarding all other rivals' output à la Cournot.

Since the firms act as Cournot oligopolists, the behaviour of the firms is described as the following optimization problem:

$$\max_{x_i} v_i = \frac{f(Q)x_i - wh(x_i) - k}{h(x_i)}. \tag{2-1}$$

The first order conditions for this optimizing problem are

$$\frac{\partial v_i}{\partial x_i} = \frac{V_i(x_i, Q)}{h(x_i)^2} = 0, \tag{2-2}$$

where

$$V_i(x_i, Q) \equiv h(x_i)\{f(Q) + x_i f' - wh'_i\} - h'_i\{x_i f(Q) - wh(x_i) - k\}, \quad i = 1, \cdots, n.$$

The second order conditions are

$$\begin{aligned} \frac{\partial^2 v_i}{\partial x_i^2} &= \frac{\partial V_i(x_i, Q)/\partial x_i}{h(x_i)^2} \\ &= \frac{h(x_i)(f' + x_i f'') + f'h(x_i) - h''_i(x_i f(Q) - k)}{h(x_i)^2} < 0, \end{aligned} \tag{2-3}$$

where $f'' \equiv d^2 f(Q)/dQ^2$.

We now introduce

ASSUMPTION 1.

$f' + x_i f'' < 0, \quad i = 1, \cdots, n.$

Assumption 1 is satisfied when $f'' \leq 0$. It holds also if $f''$is sufficiently small when $f'' > 0$. Under the nonnegativity of v_i and Assumption 1, the second order conditions are satisfied.

Next, let us derive the relationships between x_i and Q. The first order condition (2-2) is identical to $V_i(x_i, Q)$. Partially differentiating two variable function $V_i(x_i, Q)$ with respect to , we obtain

$$\frac{\partial V_i(x_i, Q)}{\partial x_i} = f'\{x_i h_i' + h_i(x_i)\} - h_i''\{x_i f(Q) - k\} < 0\,. \tag{2-4}$$

Therefore, by virtue of the implicit function theorem,

$$x_i \equiv \phi_i(Q)\,, \quad i = 1, \cdots, n\,, \tag{2-5}$$

Now, we introduce the following Assumption 2.

ASSUMPTION 2.

$$x_i f' \left\{ \frac{h(x_i)}{x_i} - h_i' \right\} + x_i f'' h(x_i) > 0\,.$$

Under Assumption 2, we have

$$\frac{\partial V_i(x_i, Q)}{\partial Q} = f' h_i(x_i) - x_i f' h_i'' + x_i f'' h(x_i) > 0\,.$$

Therefore, the function (2-5) has a positive slope.

$$\frac{dx_i}{dQ} = \frac{d\phi_i(Q)}{dQ} \equiv \phi_i' = -\frac{\partial V_i(x_i, Q)/\partial Q}{\partial V_i(x_i, Q)/\partial x_i}\,. \tag{2-6}$$

We now proceed to an analysis of the Cournot-Nash equilibrium. The Cournot-Nash equilibrium industry output is Q, which satisfies

$$Q = \sum_{i=1}^{n} \phi_i(Q) \equiv \Phi(Q)\,. \tag{2-7}$$

In the following, we denote Q^Cas the Cournot-Nash equilibrium industry output. We introduce the following additional assumption.

ASSUMPTION 3.

$$\frac{d\Phi}{dQ} \equiv \Phi'(Q) = \sum \frac{d\phi_i(Q)}{dQ} < 1\,.$$

Assumption 3 ensures the existence of the Cournot-Nash equilibrium (see Figure 1).

Since all firms are symmetric, their outputs are identical in the equilibrium. Hence, the Cournot-Nash equilibrium implies

$$Q^C = n\phi_i(Q^C) = nx^C\,, \tag{2-8}$$

where $x^C \equiv x_i^C$ is i-th firm's output in the equilibrium.

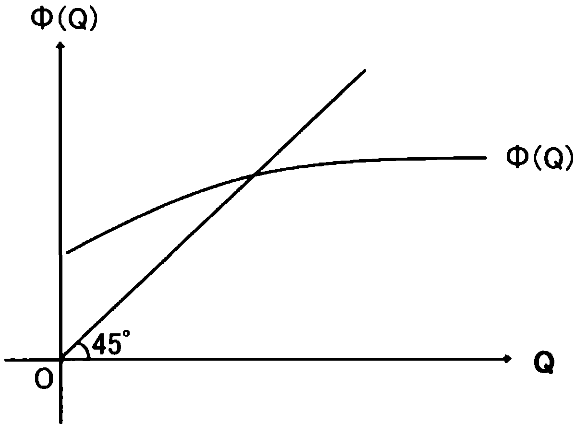

Figure 1.

3. THE BEHAVIOR OF LABOR MANAGED FIRMS IN A STACKELBERG OLIGOPOLY

We now proceed to the Stackelberg-type oligopolistic market. Let the first firm ($i = 1$) be the first mover (leader), and the remaining (n-1) firms ($i = 2, \cdots, n$) be the second movers (followers). Since the output of each follower in a Stackelberg oligopoly is shown by the functions, $x_j = \phi_j(Q)(j = 2, \cdots, n)$, the total reaction of (n-1) followers are given by $\Psi(Q) \equiv \phi_2(Q) + \cdots + \phi_n(Q)$. $\overline{Q}$ is the value of Q which satisfies $\Psi(Q) = Q$[2].

In this industry, the leader knows the reaction of all the followers. Furthermore, since the leader knows $x_i + \Psi(Q) = Q$ holds in the equilibrium, the leader solves his own optimizing problem not for his own output, x_1, but for the industry output, Q. (However, it is necessary to have a solution that $Q \geq \overline{Q}$ since $x_1 \geq 0$.)

The optimizing problem of the leader is given by

$$\max_{Q} \quad v_i = \frac{f(Q)\theta(Q) - w\mu(Q) - k}{\mu(Q)}, \tag{3-1}$$

where

$$\theta(Q) \equiv Q - \Psi(Q), \quad \mu(Q) \equiv h(\theta(Q)).$$

As shown in Figure 2, the output that the leader is able to choose is the difference between the 45 degree line and $\Psi(Q)$.

We now need further two assumptions.

ASSUMPTION 4.

$$\frac{d^2\Psi(Q)}{dQ^2} \equiv \Psi''(Q) < 0,$$

ASSUMPTION 5.

$$x_i f'' + f'(1 - \Psi') < 0, \quad i = 1, \cdots, n.$$

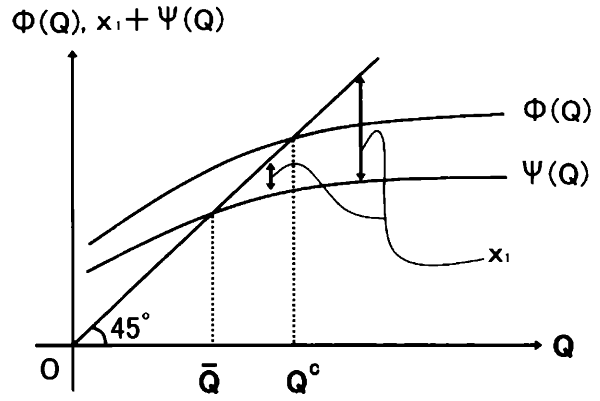

Figure 2.

Assumption 5 is a stronger condition than Assumption 1, so Assumption 1 is also valid under Assumption 5. Under Assumption 3[3] and 4, the following inequalities hold.

$$\frac{d\theta(Q)}{dQ} \equiv \theta' = 1 - \Psi' > 0\,, \quad \frac{d^2\theta(Q)}{dQ^2} \equiv \theta'' = -\Psi'' > 0\,,$$

$$\frac{d\mu(Q)}{dQ} \equiv \mu' = h'(1 - \Psi') > 0\,, \quad \frac{d^2\mu(Q)}{dQ^2} \equiv \mu'' = h''(1 - \Psi')^2 - \Psi'' h' > 0\,.$$

The first order condition of optimizing problem (3-1) is

$$\frac{dv_1}{dQ} \equiv \frac{\tilde{V}(Q)}{\mu(Q)^2} = \frac{\mu(Q)\{f'\theta(Q) + f(Q)\theta'\} - \mu'\{f(Q)\theta(Q) - k\}}{\mu(Q)^2} = 0\,, \qquad (3\text{-}2)$$

Moreover,

$$\frac{d^2 v_1}{dQ^2} = \frac{\mu(Q)^2\{d\tilde{V}(Q)/dQ\} - 2\mu(Q)\mu'\tilde{V}(Q)}{\mu(Q)^4}\,. \qquad (3\text{-}3)$$

Denoting Q^S as a Stackelberg equilibrium industry output which satisfies the first order condition (3-2), the second order condition is valid if $d\tilde{V}(Q^S)/dQ$ is negative, since $\tilde{V}(Q^S) = 0$. We assume

ASSUMPTION 6.

$$\frac{d\tilde{V}(Q^S)}{dQ} = \mu(Q)\{f''\theta(Q) + f'\theta'\} + \mu(Q)\{f'\theta' + f(Q)\theta''\}$$
$$+ \mu''\{k - f(Q)\theta(Q)\} < 0\,,$$

The second order condition is locally valid under this Assumption.

Let us now compare the Stackelberg and Cournot equilibrium industry outputs. Let $Q^S = Q^C$. First, consider the equilibrium condition for the leader. Substituting x_1 into θ in (3-2), we get the following equation for $\tilde{V}(Q)$.

$$\tilde{V}(Q^C) = h(x_1)\{x_1 f' + f(Q^C)(1 - \Psi')\} - h_1'(1 - \Psi')\{x_1 f(Q^C) - k\}\,. \qquad (3\text{-}4)$$

Keeping in mind that the followers' behaviour in production is always according to the equilibrium condition (2-2), equation (2-2) is also valid in a Stackelberg case. Consequently, the following relation holds.

$$x_j f(Q) - k = \frac{h(x_j)\{f(Q) + x_j f'\}}{h'_j}, \quad j = 2, \cdots, n. \tag{3-5}$$

Suppose $Q = Q^C$, equation (2-8) holds even in the Stackelberg equilibrium. Therefore, the leader's output is the same as the followers', *i.e.*, $x_1 = \theta(Q^C) = x_j^C$. By supposition that $Q^S = Q^C$, equation (3-4) and (3-5) are evaluated at the same value of x_i. Substituting equation (3-5) into equation (3-4),

$$\tilde{V}(Q^C) = \Psi'(Q^C)h(x_1)f' < 0. \tag{3-6}$$

This is a contradiction because the leader's first order condition is violated. Hence, the equilibrium industry outputs are different in the Stackelberg and Cournot competition.

Now, let note (3-6) as

LEMMA 1.

$$\tilde{V}(Q^C) = \Psi'(Q^C)h(x_1)f' < 0.$$

We can now show that the optimum output in Stackelberg-type competition is less than the optimum output in Cournot-type competition. If Q approaches $\overline{Q}$, *i.e.*, x_1 approaches zero on the leader's equilibrium condition (equation (3-2)), we find

$$\lim_{Q \to \overline{Q}} \theta(Q) = \lim_{Q \to \overline{Q}} Q - \Psi(Q) \to 0.$$

Hence,

$$\lim_{Q \to \overline{Q}} \frac{\tilde{V}(Q)}{\mu(Q)^2} \to \frac{\mu(\overline{Q})f(\overline{Q})\theta' + \mu' k}{\mu(\overline{Q})^2} > 0.$$

This implies that the first derivative of profit per labor is positive if the leader's output level is sufficiently close to zero. However, Lemma 1 ensures that the first derivative is negative if $x_1 = x^C$. As a result, the intermediate value theorem assures that $Q^S \in (\overline{Q}, Q^C)$ (in which Q^Ssatisfies the leader's first order condition in a Stackelberg-type competition) as shown in Figure 3.

To investigate the uniqueness of the equilibrium, let us consider the second order condition globally. In equation (3-3), the first term of the numerator is negative under our assumptions. The sign of the second term depends on the sign of $\tilde{V}$. Since the sign of the second term is always positive if $\tilde{V} > 0$, the second order condition is valid. If $\tilde{V} = 0$, the second order condition is valid as we have already demonstrated. However, it is not clear if the second order condition holds if $\tilde{V} < 0$. Hence, lemma 2 is obtained.

LEMMA 2. *$\tilde{V}(Q) < 0$ if Q is larger than Q which satisfies that $\tilde{V}(Q) = 0$.*

Proof. We denote Q^0 as a quantity, which satisfies that $\tilde{V}(Q) < 0$. As shown in Figure 4, if $Q > Q^0$, $d\tilde{V}(Q)/dQ \leq 0$ must hold when $\tilde{V}(Q) \leq 0$ at the value of Q, which satisfies $\tilde{V}(Q) = 0$.

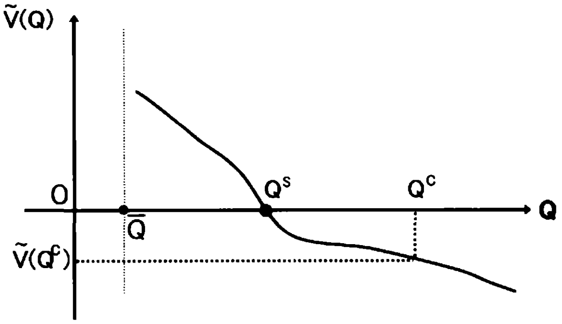

Figure 3.

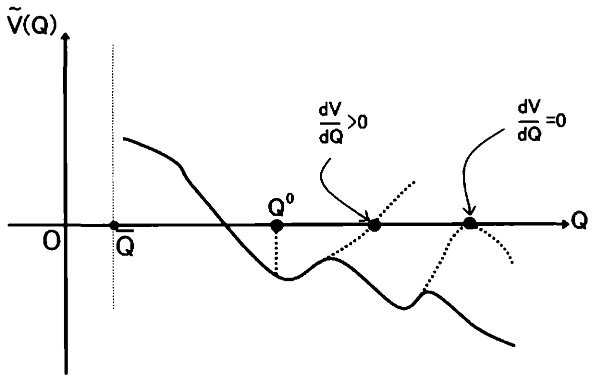

Figure 4.

However, $d\tilde{V}(Q)/dQ < 0$ holds when $\tilde{V}(Q) = 0$. Therefore, $d\tilde{V}(Q)/dQ < 0$ is valid if $Q > Q^0$. □

$\tilde{V}(Q)$ is positive when the leader's output is sufficiently close to zero. Moreover, the second order condition is satisfied if $\tilde{V}(Q) > 0$. Hence, dv_1/dQ, *i.e.*, $\tilde{V}(Q)$ approaches zero value, and finally becomes zero. At Q^S, which satisfies $\tilde{V}(Q) = 0$, the second order condition is satisfied by assumptions. Therefore, $\tilde{V}(Q) < 0$ at Q^0 that satisfies $Q > QS$ in the neighborhood of Q^S. By Lemma 2, there is no Q that satisfies the optimum condition $\tilde{V}(Q) = 0$ in the region $Q > Q^0$. This implies there exists Q uniquely, which satisfies $\tilde{V}(Q) = 0$. Now, we are ready to prove.

THEOREM.

Suppose Stackelberg-type oligopolistic competition by n symmetric labor managed firms. Suppose further that one of those labor managed firms is the leader, and the others are followers. If there exists equilibrium locally, the equilibrium is a global equilibrium. Therefore the equilibrium is unique. Furthermore, the Stackelberg industry output is less than the Cournot-Nash one.

The following corollary is derived from Figure 3 as the corollary of Theorem.

COROLLARY.

In the Stackelberg equilibrium of Theorem, the leader's output is less than the followers' output.

NOTES

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1) Since $h(x_i)$ is the inverse production function for each firm, $\frac{h(x_i)}{x_i} - h' < 0$ holds by the convexity of $h(x_i)$. Therefore, assumption 2 is valid when f'' is sufficiently close to zero.

2) Q, which satisfies $Q = \psi(Q)$, is Cournot-Nash equilibrium by $(n-1)$ firms.

3) $d\psi(Q)/dQ < 1$ holds under Assumption 3.

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