Title	Human capital accumulation, economic growth and transitional dynamics in a dual economy
Sub Title	
Author	Gupta, Manash Ranjan Chakraborty, Bidisha
Publisher	Keio Economic Society, Keio University
Publication year	2006
Jtitle	Keio economic studies Vol.43, No.1 (2006.) ,p.43- 59
JaLC DOI	
Abstract	In this paper, we analyze the transitional dynamic properties of a growth model of a dual economy in which dualism lies in the mechanism of human capital accumulation of two types of individuals. We derive the conditions under which the saddle path converging to the steady-state growth equilibrium point is unique and the conditions under which equilibrium growth path may be indeterminate.
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-20060001-0043

KEIO ECONOMIC STUDIES **43**(1), 43–59 (2006)

HUMAN CAPITAL ACCUMULATION, ECONOMIC GROWTH AND TRANSITIONAL DYNAMICS IN A DUAL ECONOMY

Manash Ranjan GUPTA

Economic Research Unit, Indian Statistical Institute, Kolkata, India

and

Bidisha CHAKRABORTY

Vijaygarh Jyotish Ray College, Economics Department, Kolkata, India

First version received June 2005; final version accepted May 2006

Abstract: In this paper, we analyze the transitional dynamic properties of a growth model of a dual economy in which dualism lies in the mechanism of human capital accumulation of two types of individuals. We derive the conditions under which the saddle path converging to the steady-state growth equilibrium point is unique and the conditions under which equilibrium growth path may be indeterminate.

Keywords: Human Capital, Economic growth, Transitional Dynamics, Uniqueness, Multiple Equilibria.
JEL Classification Number: C62, J24, O15, O41.

1. INTRODUCTION

With the emergence of the 'new' growth theory, human capital accumulation and its role on economic growth has become a major area of research in macroeconomics. The literature starts with the seminal paper of Lucas (1988) which shows that growth rate of per capita income depends on the growth rate of human capital which again depends on the time allocation of the individuals for acquiring skill. Since then many eminent economists have dealt with the issue of human capital accumulation and growth.

However, these endogenous growth models do not provide appropriate framework for analysing the problems of growth of less developed countries. Less developed economies are often characterized by the existence of opulence and poverty side by

Acknowledgements. This is a revised version of a chapter of the prospective Ph.D. work of the second author which is in progress with the first author in the Indian Statistical Institute, Kolkata. We have received helpful comments of a referee of this journal on an earlier version of this paper. All the remaining errors are of solely ours.
E-mail: bidisha_isi@yahoo.co.in

side. Rich individuals stay in contrast with the poor individuals who consume whatever they earn and thus do not have anything to save and invest to build up physical and human capital. This co-existence of rich and poor individuals leads to dualism in the less developed countries.

There exists a substantial theoretical literature dealing with the dualism and income inequalities in less developed countries.[1] However, none of the existing models focuses on the dualism in the mechanism of human capital formation of two different classes of people. In a less developed economy, the stock of human capital of the poor individuals is far lower than that of the rich individuals. Also there exists a difference in the mechanism of human capital accumulation of the rich and the poor individuals. On the one hand, there are rich families who can afford to spend a lot of time and resources for schooling of their children. On the other hand, there are poor families who have neither time nor resources to spend for education of their children. The opportunity cost of schooling of their children is very high because they can alternatively be employed as child labour. However they receive support from exogenous sources. Government sets up free public schools and introduces various schemes of paying book grants and scholarships to the meritorious students coming from the poor families. The rich individuals who are the owners of firms or industries open NGO s or give donations to them. These NGOs provide free or subsidized educational service to the poor. Government meets the cost of public education program through taxes imposed on rich individuals. So the efficiency enhancement mechanism for the wealthier individuals and the poor individuals are different. While the rich individuals can build up their human capital on their own, the poor individuals need the support of exogenous sources in accumulating their human capital.

There are substantial evidences that private individuals and firms provide voluntary services to education. Corporate giants like The Coca-Cola Company, American Express, General Electric Company, Bank of America, Nokia Corporation, Chevron Texaco Corporation are members of CECP (Committee to Encourage Corporate Philanthropy) and are providing various services including education to the underprivileged communities of both developing and developed countries. Timberland Co. reports that 95 % of its employees have in total contributed some 300, 000 service hours in 13 countries. 'Make a Connection' program undertaken by Nokia is active in 19 countries including countries of South Africa and Latin America. This program focuses on developing non academic skills like co-operation, communication skills, conflict management etc[2]. Menchik and Weisbrod (1987) report that according to a recent survey, over 80 million adults in the US volunteered 8.4 billion hours of labour to organization in 1980; and other estimates of the number of volunteer workers, relying on non-survey methods, place volunteer labour as high as 8 percent of the labour force. In India, Titan, Broadcom, Infosys Foundation, Asea Brown Boveri, Siemens Ltd, Yahoo.com

[1] This includes the work of e.g Lewis (1954), Ranis and Fei (1961), Sen (1966), Dixit (1969), Todaro (1969), Benabou (1994, 1996a, 1996b).

[2] Source: Various newsletters published by CECP

are among the many corporates who are fulfilling part of their social responsibilities by linking up with Akshaya Patra Foundation—a Bangalore based non profit organisation that provides mid day meals to unprivileged children in schools in and around Bangalore. ABB India has identified education as a key area for social and community development activities and helping the teachers of a govt. school of a village close to Peenya, Bangalore, to make their lessons more meaningful and effective.[3] Confederation of Indian Industries (CII) has initiated a program in various parts of India under which training is imparted to the unskilled labour; and a certificate recognising the skill acquired by the labourer is given. These are pure private sector initiatives.

In the present paper, we develop a growth model of an economy in which human capital accumulation is viewed as the source of economic growth and in which difference exists in the mechanism of human capital accumulation of the two types of individuals—the rich and the poor. The poor individuals lag behind the rich individuals in terms of initial endowment of human capital and in terms of the efficiency of the human capital accumulation technology. The rich individuals not only allocate their labour time between production and their own skill accumulation but also allocate a part of their labour time to the training of the poor people.[4] We have assumed the presence of external effect of their human capital on production as well as on the human capital accumulation of the poor individuals.[5]

Analysis of transitional dynamic properties of the growth models have received substantial attention in recent years. A number of studies have analyzed the transitional dynamic properties in Uzawa-Lucas model. Xie (1994) considers the Lucas (1988) model to examine global stability properties and shows that the Lucas (1988) model contains multiple equilibria in the presence of strong external effects on production. Benhabib and Perli (1994) does the local stability analysis and come to similar results. In this paper, we do the local stability analysis of our more general model and derive some interesting transitional dynamic results. We can show that the externality in production with a social IRS production technology can not explain multiple equilibria in this model. For this we need externality and increasing returns in the human capital accumulation function. The result seems to be interesting because it is contrary to that obtained from Xie (1994) and from Benhabib and Perli (1994).

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 presents the transitional dynamic analysis of the basic model. In Section

[3] Source: Various issues of Business India.

[4] This voluntary allocation of labour time to the training of the poor individual can not be supported in a world where the contribution comes mainly in the form of tax payment. However we have mentioned evidences of voluntary contributions too.

[5] There exists a large theoretical literature in both urban economics and in macroeconomics that has considered external effects emanating from human capital in explaining growth of cities, religions and countries e.g. Glaeser and Mare (1994), Glaeser (1997), Peri (2002), Ciccone and Peri (2002). In some other literature, it is found that education generates very little externalities. e.g Rudd (2000), Acemoglu and Angrist (2000). Moretti (2003) rightly points out that the empirical literature on the subject is still very young and more work is needed before we can draw convincing conclusions about the size of human capital externalities.

4, we extend the basic model introducing social IRS property in the human capital accumulation function. Concluding remarks are made in Section 5.

2. THE BASIC DUAL ECONOMY MODEL

We consider an economy with two types of individuals—the rich and the poor individuals. The poor individuals lag behind the rich individuals in terms of initial endowment of human capital and in terms of the efficiency of the human capital accumulation technology. All workers are employed in a single aggregative sector that produces a single good. By human capital we mean the set of specialized skills or efficiency level of the workers that accumulate over time. The mechanisms of human capital accumulation are different for two types of individuals. There is external effect of human capital of the rich individuals on the production and on the human capital accumulation of the poor individuals. Population size of either type of individual is normalised to unity. All individuals belonging to each group are assumed to be identical. There is full employment of both types of labour and the labour market is competitive.

The single production sector is owned by the rich individuals and they employ the poor individuals as wage labourers. Rich individuals and poor individuals form different types of human capital which are imperfectly substitute. A rich individual allocates 'a' fraction of the total non-leisure time in production. Let H_R and H_P be the skill level of the representative rich and poor individual (worker) respectively.

The production function takes the form:

$$Y = A(aH_R)^{\alpha} {H_P}^{1-\alpha} \overline{H_R}^{\epsilon_R} \overline{H_P}^{\epsilon_P} \tag{1}$$

where $0 < \alpha < 1$. Here $\overline{H_R}$ and $\overline{H_P}$ represent the average level of human capital of all the individuals belonging to the rich (R) group and to the poor (P) group respectively. $\epsilon_R > 0$ and $\epsilon_P > 0$ are the parameters representing the magnitude of the external effect of their human capital on production respectively. Production function satisfies CRS in terms of private inputs but shows social IRS if external effect is taken into consideration. Y stands for the level of output.

The representative rich individual (worker) owns the advanced type of human capital and his income is given by αY. $(1 - \alpha)Y$ is the wage income of the poor workers because the labour market is competitive. The rich household and the poor household both consume whatever they earn and hence the individuals do not save (or invest). So there is no accumulation of physical capital in this model and so capital does not enter as an input in the production function.[6] So, we have

$$C_R = \alpha Y ; \tag{2}$$

and

$$C_P = (1 - \alpha)Y . \tag{3}$$

Here C_P and C_R are the level of consumption of the representative poor worker and the representative rich worker respectively. The representative rich household (worker)

[6] Though it is assumed for simplicity, it is a serious limitation of the exercise.

maximizes discounted present value of utility over the infinite time horizon with respect to the labour time allocation variables. His instantaneous utility function is given by

$$U(C_R) = \frac{C_R^{1-\sigma}}{1-\sigma}, \quad \sigma > 0 \tag{4}$$

Here $\sigma > 0$ is the constant elasticity of marginal utility of consumption.

2.1. Difference in the mechanism of human capital accumulation

Mechanism of the human capital accumulation of the rich individual is assumed to be similar to that in Lucas (1988). The rate at which his human capital is formed is proportional to the labour time or effort devoted to acquire skill.

Hence

$$\dot{H}_R = mbH_R \tag{5}$$

where b is the fraction of the non-leisure time devoted to acquiring his own skill level. Here $0 < b < 1$; and m is a positive constant representing the productivity parameter of the human capital formation function of the rich individuals.

However the mechanism of human capital formation for the two classes of individuals are different. The skill formation of a poor person takes place through the training program conducted by the rich individuals who want to make them more efficient and productive. Every rich individual spends $(1 - a - b)$ fraction of its labour time in this training. Just for simplicity it is assumed that the poor people has surplus labour time and they improve their skill in leisure time (in the evening or in slack season). So they do not have to devote any fraction of non-leisure time for learning.[7] The additional skill acquired by the representative poor worker is assumed to be a linear homogeneous function in terms of the effort level of the rich person and of the skill level already attained by the poor person.

However, we assume that there exists a positive external effect of the average skill level of the rich individual on the human capital accumulation of the poor individuals. Hence we have

$$\dot{H}_P = \{(1 - a - b)H_R\}^{\delta} H_P{}^{1-\delta-\gamma} \overline{H_R}^{\gamma}. \tag{6}$$

Here $0 < \delta < 1$ and γ is the parameter representing the magnitude of the external effect on the skill formation of the poor individuals. The accumulation function of H_P satisfies private DRS and social CRS. However, accumulation function of H_R given by equation (5) satisfies CRS at the private level as well as at the social level. In the models of Tamura (1991), Eaton and Eckstein (1997), Lucas (2004) etc. the human capital accumulation technology is subject to external effects. In the models of Eaton and Eckstein (1997) and Tamura (1991) average human capital is affecting human capital accumulation technology where as in the model of Lucas (2004) human capital of the leader affects the human capital accumulation of all other individuals. Leader is the person with the highest skill level. In our model the rich individual has already attained

[7] This is a simplifying assumption. However, if the time devoted for production and human capital accumulation by the poor individuals are assumed to be exogenously given, that will yield the same result.

high level of human capital and the poor is lagging behind. The rich individuals and the poor individuals are assumed to be identical within their respective groups. So it is justified that the human capital of the rich individual should have external effect on the poor individual's human capital accumulation technology; and it should not be the other way round.

3. GROWTH IN THE HOUSEHOLD ECONOMY

3.1. *The optimization problem*

The objective of the representative rich individual is to maximize the discounted present value of utility over the infinite time horizon. The objective functional is given by:

$$J_H = \int_0^{\infty} U(C_R) e^{-\rho t} dt\,.$$

This is to be maximized with respect to a and b subject to the equations of motion given by

$$\dot{H}_R = mbH_R\,;$$

$$\dot{H}_P = \{(1-a-b)H_R\}^{\delta} {H_P}^{1-\delta-\gamma} {H_R}^{\gamma}\,;$$

and given the initial values of H_R and H_P. Here $U(C_R)$ is given by equation (4) and Y is given by equation (1). Here ρ is the constant positive discount rate. The control variables are a and b where $0 \le a \le 1, 0 \le b \le 1, 0 \le a+b \le 1$. The state variables are H_R and H_P. The current value Hamiltonian is given by

$$H^c = \frac{C_R^{1-\sigma}}{1-\sigma} + \lambda_R mbH_R + \lambda_P \{(1-a-b)H_R\}^{\delta} {H_P}^{1-\delta-\gamma} {H_R}^{\gamma}$$

where λ_R and λ_P are co-state variables of H_R and H_P respectively representing the shadow prices of the human capital of rich individuals and of the human capital of poor individuals. C_R is given by the equation (2). The representative household can not internalise the externalities.

3.2. *The Optimality Conditions*

(A) The first order conditions necessary for this optimization problem with respect to the control variables a and b are given by the following:

$$(\alpha Y)^{-\sigma} \alpha^2 \frac{Y}{a} - \lambda_P \delta \frac{\dot{H}_P}{(1-a-b)} = 0\,; \tag{7}$$

and

$$\lambda_R m H_R - \lambda_P \delta \frac{\dot{H}_P}{(1-a-b)} = 0\,. \tag{8}$$

(B) Time derivatives of the co-state variables satisfying the optimum growth path are given by the following:

$$\dot{\lambda_R} = \rho\lambda_R - (\alpha Y)^{-\sigma}\alpha^2 \frac{Y}{H_R} - \lambda_R m b - \lambda_P \delta \frac{\dot{H_P}}{H_R}\,; \tag{9}$$

and

$$\dot{\lambda_P} = \rho\lambda_P - (\alpha Y)^{-\sigma}\alpha(1-\alpha)\frac{Y}{H_P} - \lambda_P(1-\delta-\gamma)\frac{\dot{H_P}}{H_P}\,. \tag{10}$$

(C) Solving the system there will be family of time paths of state and costate variables satisfying the given initial condition. The member of this family that satisfies the transversality conditions given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_R(t)H_R(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_P(t)H_P(t) = 0$$

is the optimal path.

Using equations (7) and (8) we have

$$(\alpha Y)^{-\sigma}\alpha^2 \frac{Y}{a} = \lambda_R m H_R; \tag{11}$$

and using equations (7), (8) and (9) we have

$$\frac{\dot{\lambda_R}}{\lambda_R} = \rho - m\,. \tag{12}$$

Now, from equation (10) and equation (7), we have,

$$\frac{\dot{\lambda_P}}{\lambda_P} = \rho - \frac{\delta(1-\alpha)ar}{\alpha(1-a-b)} - (1-\delta-\gamma)r\,. \tag{13}$$

3.3. *The transitional dynamics*

We now turn to analyse the transitional dynamic properties around the steady state equilibrium point. We derive the equations of motion which describe the dynamics of the system. We define $z = \frac{H_R}{H_P}$ and $x = (1-a-b)$ Using equations (5) and (6) we have

$$\frac{\dot{z}}{z} = m(1-a-x) - x^{\delta}z^{\delta+\gamma}\,. \tag{14}$$

Differentiating the log of both sides of the equation (11) with respect to time and then using equations (1), (5), (12) we get

$$\begin{aligned}\frac{\dot{a}}{a} = \frac{1}{1-\alpha(1-\sigma)}[m - \rho - \{1-(\alpha+\epsilon_R)(1-\sigma)\}\\ \times (1-a-x)m + (1-\alpha+\epsilon_P)(1-\sigma)x^{\delta}z^{\delta+\gamma}]\,.\end{aligned} \tag{15}$$

Similarly differentiating the log of both sides of equation (8) with respect to time and then using equations (5), (6), (12), (13) we get

$$\frac{\dot{x}}{x} = \frac{1}{(1-\delta)}\left[m - \frac{a(1-\alpha)\delta}{\alpha}x^{\delta-1}z^{\delta+\gamma} - (1-\delta-\gamma)m(1-a-x)\right]. \tag{16}$$

The dynamics of the system is now described by the differential equations (14), (15), (16). They solve for the time path of the variables z, x and a.

3.3.1. Steady state equilibrium

Equating the growth rates of z, x, a equal to zero we get the steady state equilibrium values of the respective variables denoted by z^*, x^* and a^*. From equation (14) we have

$$z^* = (m(1 - a^* - x^*)x^{*-\delta})^{\frac{1}{(\delta+\gamma)}} . \tag{17}$$

Substituting z^* from equation (17) into equation (15) and setting $\frac{\dot{a}}{a} = 0$ we have,

$$m(1 - a^* - x^*) = \frac{m - \rho}{[1 - (1 - \sigma)(1 + \epsilon_P + \epsilon_R)]} . \tag{18}$$

Equation (18) shows that $\frac{\dot{H}_R}{H_R}$ is independent of γ but dependent on ϵ_P, ϵ_R when $\sigma \neq 1$. $\frac{\dot{H}_R}{H_R}$ varies positively (negatively) with ϵ_P, ϵ_R if $\sigma < (>)1$.

So the rate of human capital accumulation of the rich individuals is independent of the degree of externality in the human capital accumulation of the poor individuals. The value of mb should be less than the highest possible value of the growth rate of human capital, m, and for this the restriction required is

$$\sigma > 1 - \frac{\rho}{m[1 + \epsilon_P + \epsilon_R]} .$$

If the above condition is satisfied then the condition for positive mb is also satisfied since we have assumed $m > \rho$. We also find that the growth rate of H_R is positively affected by the external effects in the production sector if $\sigma < 1$ and is negatively affected by the external effects present in the production sector if $\sigma > 1$.

The steady state equilibrium rate of growth of income is denoted by χ; and it can be shown that

$$\chi = \frac{\dot{Y}}{Y} = \frac{(1 + \epsilon_R + \epsilon_P)(m - \rho)}{[1 - (1 - \sigma)(1 + \epsilon_P + \epsilon_R)]} .$$

Hence χ is always positively affected by ϵ_R and ϵ_P.

Note that, if there is no externality, i.e. if $\epsilon_R = 0$, $\epsilon_P = 0$ and $\gamma = 0$ then we have

$$mb = \frac{m - \rho}{\sigma} .$$

In this case income and human capital of both type of individuals grow at the common rate mb. This is the growth rate obtained in Lucas (1988) model in the absence of external effect on production.

Substituting z^* from equation (17) in equation (16) and setting $\frac{\dot{x}}{x} = 0$ we have,

$$a^* = \frac{\alpha}{\delta(1 - \alpha)} \left[\frac{m\{1 - (1 - \sigma)(1 + \epsilon_P + \epsilon_R)\}}{(m - \rho)} - (1 - \delta - \gamma) \right] x^* . \tag{19}$$

Using equations (18) and (19) we can solve for x^* which is given by

$$x^* = \frac{\{\rho - m(1 - \sigma)(1 + \epsilon_P + \epsilon_R)\}\delta(1 - \alpha)(m - \rho)}{[\alpha\{\rho - m(1 - \sigma)(1 + \epsilon_P + \epsilon_R)\} + (m - \rho)(\delta + \alpha\gamma)]m\{1 - (1 - \sigma)(1 + \epsilon_P + \epsilon_R)\}} . \tag{20}$$

Now equations (17), (18), (19) and (20) show that the values of z^*, x^* and a^* are uniquely determined given the pre determined values of the parameters. a^* and z^* are given by the following expressions:

$$a^* = \frac{\alpha\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R) + (\delta+\gamma)(m-\rho)\}\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)\}}{[\alpha\{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)\} + (m-\rho)(\delta+\alpha\gamma)]m\{1-(1-\sigma)(1+\epsilon_P+\epsilon_R)\}}$$

and

$$z^* = \left[\frac{m-\rho}{[1-(1-\sigma)(1+\epsilon_P+\epsilon_R)]}x^{*-\delta}\right]^{\frac{1}{(\delta+\gamma)}} \tag{21}$$

where x^* is given by the equation (20). Using equation (18) we have

$$a^* + x^* = \frac{\frac{\rho}{m} - (1-\sigma)(1+\epsilon_P+\epsilon_R)}{1-(1-\sigma)(1+\epsilon_P+\epsilon_R)}.$$

Also using equations (18) and (19) we have

$$\frac{a^*}{x^*} = \frac{\alpha}{\delta(1-\alpha)}\left[\frac{\rho - m(1-\sigma)(1+\epsilon_P+\epsilon_R)}{(m-\rho)} + (\delta+\gamma)\right].$$

If we assume $m > \rho$ and $\sigma > 1 - \frac{\rho}{m(1+\epsilon_P+\epsilon_R)}$ then using the expressions of $(a^* + x^*)$ and $\left(\frac{a^*}{x^*}\right)$ it can be easily shown that $0 < a^* + x^* < 1$ and $(a^*/x^*) > 0$. Hence $0 < x^* < 1$ and $0 < a^* < 1$. Equation (21) now clearly shows that $z^* > 0$ in this case.

So we have the following proposition.

PROPOSITION 1. *If* $m > \rho > m(1-\sigma)(1+\epsilon_P+\epsilon_R)$ *then the steady state growth equilibrium of this model is unique satisfying* $0 < a^*, x^* < 1$ *and* $z^* > 0$.

3.3.2. Uniqueness of the saddle path

Next question is regarding the uniqueness of the saddle path converging to the steady state equilibrium point. Note that it is a system of 3 differential equations. Initial values of the variable, z, is historically given; and those of other two variables x and a can be chosen by the controller. So if the roots are real then in order to get the unique saddle path converging to the steady state equilibrium point we need exactly one latent root of the Jacobian matrix to be negative and the other two to be positive.

Here the Jacobian matrix corresponding to the system of differential equations (14), (16), (15) is given by the following:

$$J = \begin{bmatrix} \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial a} \\ \frac{\partial \dot{x}}{\partial z} & \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial a} \\ \frac{\partial \dot{a}}{\partial z} & \frac{\partial \dot{a}}{\partial x} & \frac{\partial \dot{a}}{\partial a} \end{bmatrix}$$

where, the elements of the Jacobian matrix evaluated at the steady-state equilibrium values of the variables are given in Appendix (A).

The characteristic equation of the J matrix is given by

$$|J - \lambda I_3| = 0$$

where λ is an eigenvalue of the Jacobian matrix with elements being evaluated at the steady state equilibrium values. The three charateristic roots can be solved from the equation

$$a_0\lambda^3 + b_0\lambda^2 + a_1\lambda + b_1 = 0$$

where

$$a_0 = -1\,,$$

$$b_0 = \text{Trace of } J\,,$$

$$a_1 = \frac{\partial \dot{z}}{\partial x}\frac{\partial \dot{x}}{\partial z} + \frac{\partial \dot{x}}{\partial a}\frac{\partial \dot{a}}{\partial x} + \frac{\partial \dot{a}}{\partial z}\frac{\partial \dot{z}}{\partial a} - \frac{\partial \dot{x}}{\partial x}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{z}}{\partial z}\frac{\partial \dot{x}}{\partial x}\,,$$

and

$$b_1 = \text{Determinant of} J\,.$$

Clearly a_0 is negative. We can derive that

$$b_1 = \frac{ma^*\delta(1-\alpha)x^{*2\delta-1}z^{*2(\delta+\gamma)}(\delta+\gamma)}{\alpha(1-\delta)\{1-\alpha(1-\sigma)\}}(a^* + x^*)[(1-\sigma)(1+\epsilon_P+\epsilon_R) - 1]\,.$$

This is negative if

$$[(1-\sigma)(1+\epsilon_P+\epsilon_R) - 1] < 0$$

which is always true because $m(1 - a^* - x^*) > 0$ and $m > \rho$ by assumption. So the negative determinant of J implies that either all the three roots are negative or only one root of J is negative with other two roots being positive. So we have to look at the sign of b_0 which is trace of J. If $b_0 > 0$, then all the roots can not be negative. Hence, only one latent root is negative and the other two are positive.

Now it can be shown that,

$$b_0 = m(a+x) + mx\frac{(1-\delta-\gamma)}{(1-\delta)} + ma\frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}$$

If $m(1 - a^* - x^*) > 0$ and if $m > \rho$ then equation (18) shows that

$$1 - (1+\epsilon_P+\epsilon_R)(1-\sigma) > 0\,; \qquad \text{and hence} \qquad 1 - (\alpha+\epsilon_R)(1-\sigma) > 0$$

and $1 - \alpha(1-\sigma) > 0$. Also $(1-\delta-\gamma) > 0$, by assumption.

So, b_0 is always positive. Hence, in this case, there is unique saddle path converging to the steady state equilibrium point. So we have the following proposition:

PROPOSITION 2. *There exists unique saddle path converging to the steady state equilibrium point whatever be the magnitude of the external effect on production.*

So far we have considered the case of three real roots. However, $b_1 < 0$ may imply a possibility of one negative latent root and two imaginary latent roots. Since $b_0 > 0$, the sum of the two imaginary latent roots is positive.[8] In that case too, we should have

[8] A numerical example is given in the Appendix (B).

only one saddle path converging to the equilibrium point. Other trajectories may move cyclically around the equilibrium point. However, they will not converge.

The above mentioned result is important. We consider a production function satisfying private CRS and social IRS, but the presence of this aggregate external effect of human capital on production can not explain multiple equilibria in this model whatever be the magnitude of this external effect. Xie (1994), Benhabib and Perli (1994) etc. have shown that the social IRS property of the production technology may explain indeterminacy of equilibria in Lucas (1988) model. We now turn to provide the intuitive explanations of the result summarized in the above mentioned proposition. Since entire income is consumed and there is no accumulation of physical capital, economic growth is explained only by the accumulation of two human capital inputs. In a standard growth model, social IRS property of the production technology helps to raise the investment on physical capital at a very high rate because the agent makes the consumption-savings allocation rationally. With no scope of physical capital to accumulate over time the social IRS property of the production technology loses its sharpness. The human capital accumulation functions do not exhibit increasing returns to scale. Here the accumulation function of H_R shows CRS at the private as well as at the social level; and the accumulation function of H_P shows private DRS and social CRS. Without increasing returns in the production technology of the accumulable inputs, we can not explain multiple equilibria or indeterminacy. In the next section, we show the possibility of indeterminacy introducing social IRS in the human capital accumulation function of the poor individuals.

4. INCREASING RETURNS TO SCALE IN THE HUMAN CAPITAL ACCUMULATION FUNCTION

In this section we take into account the possibility of increasing returns to scale in the human capital accumulation function of the poor individuals. Hence the equation (6) is modified as follows:

$$\dot{H}_P = \{(1 - a - b)H_R\}^{\delta} H_P{}^{1-\delta-\gamma_1} H_R{}^{\gamma_2} \tag{6E}$$

Here $0 < \delta < 1$, $(1 - \delta - \gamma_1) > 0$; and $\gamma_2 \geq 0$ is the parameter representing the magnitude of the external effect on the skill formation of the poor individuals. If $\gamma_2 > \gamma_1$ then the human capital accumulation function of the poor individual satisfies social IRS and private DRS. If $\gamma_2 = \gamma_1$ then the accumulation function of H_P satisfies private DRS and social CRS. If $\gamma_2 < \gamma_1$ then the accumulation function of H_P satisfies private DRS and social DRS. We use the same definition of x; and $\hat{z}$ is defined as follows:

$$\hat{z} = H_R{}^{\delta+\gamma_2} H_P{}^{-(\delta+\gamma_1)} .$$

The equations of motions are now given by the followings.

$$\frac{\dot{\hat{z}}}{\hat{z}} = (\delta + \gamma_2)m(1 - a - x) - (\delta + \gamma_1)x^{\delta}\hat{z} , \tag{14E}$$

$$\frac{\dot{a}}{a} = \frac{1}{1-\alpha(1-\sigma)}[m - \rho - \{1 - (\alpha + \epsilon_R)(1-\sigma)\}(1-a-x)m \\ + (1 - \alpha + \epsilon_P)(1-\sigma)x^{\delta}\hat{z}], \tag{15E}$$

and

$$\frac{\dot{x}}{x} = \frac{1}{(1-\delta)}\left[m - \frac{a(1-\alpha)\delta}{\alpha}x^{\delta-1}\hat{z} - (1-\delta-\gamma_2)m(1-a-x)\right]. \tag{16E}$$

The steady state equilibrium values of the respective variables, denoted by $\hat{z}^*$, x^* and a^*, can be obtained from the following equations.

$$\hat{z}^* = \frac{(\delta+\gamma_2)}{(\delta+\gamma_1)}m(1-a^*-x^*)x^{*-\delta}, \tag{17E}$$

$$m(1-a^*-x^*) = \frac{m-\rho}{\left[1-(1-\sigma)(\alpha+\epsilon_R)-(1-\sigma)(1-\alpha+\epsilon_P)\dfrac{(\delta+\gamma_2)}{(\delta+\gamma_1)}\right]}, \tag{18E}$$

$$a^* = \frac{\alpha(\delta+\gamma_1)}{\delta(1-\alpha)(\delta+\gamma_2)}\left[\frac{m(1-A_1)}{(m-\rho)} - (1-\delta-\gamma_2)\right]x^*, \tag{19E}$$

and

$$x^* = \frac{[\rho - mA_1]/(1-A_1)}{m\left[\dfrac{\alpha(\delta+\gamma_1)}{\delta(1-\alpha)(\delta+\gamma_2)}\left[\dfrac{m(1-A_1)}{(m-\rho)} - (1-\delta-\gamma_2)\right]+1\right]}, \tag{20E}$$

where

$$A_1 = (1-\sigma)\left\{(\alpha+\epsilon_R) + (1-\alpha+\epsilon_P)\frac{(\delta+\gamma_2)}{(\delta+\gamma_1)}\right\}.$$

We assume $m > \rho$. So the condition for $0 < m(1-a^*-x^*) < m$ can be written as

$$(1-\sigma) < \frac{\rho(\delta+\gamma_1)}{m\{(\delta+\gamma_1)(\alpha+\epsilon_R)+(\delta+\gamma_2)(1-\alpha+\epsilon_P)\}} = \widehat{\Sigma_1} \tag{21E}$$

This condition ensures that $0 < x^*, a^* < 1$

Here also the determinant of the Jacobian matrix is given by

$$|J| = b_1 = \frac{ma^*\delta(1-\alpha)x^{*2\delta-1}z^{*2}}{\alpha(1-\delta)\{1-\alpha(1-\sigma)\}}(a^*+x^*)[(\delta+\gamma_1)\{(1-\sigma)(\alpha+\epsilon_R)-1\} \\ + (\delta+\gamma_2)(1-\sigma)(1-\alpha+\epsilon_P)] < 0.$$

However, the trace of J may not be positive. Here

$$b_0 = m(a+x) + mx\frac{(1-\delta-\gamma_2)}{(1-\delta)} + ma\frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}.$$

The growth rate of H_R is positive. So

$$1-(1-\sigma)(\alpha+\epsilon_R) > 0.$$

So, b_0 is positive if $(1-\delta-\gamma_2) > 0$. We have assumed that $(1-\delta-\gamma_1)$ is positive. So the sufficient condition for b_0 to be positive is $\gamma_2 \leq \gamma_1$. If this is the case then there is unique saddle path converging to the steady state equilibrium point. We have considered the case of $\gamma_1 = \gamma_2$ in the previous section. However, if there is increasing returns to scale in the human capital accumulation function of poor individuals and if γ_2 is sufficiently large then $(1 - \delta - \gamma_2)$ may be negative. Hence b_0 may be negative. If both Trace and Determinant of J are negative then all the latent roots of J may be negative. So we get a possibility of indeterminacy of the equilibrium growth path in this case. We turn to derive the condition for indeterminacy in the next subsection.

4.1. Possibility of indeterminacy

Indeterminacy of the equilibrium growth path will arise if more than one characteristic root is negative. Since the determinant is always negative all the three roots are to be negative for this. In this subsection we are considering the possibility of having three negative roots. According to Descarte's rule if all the coefficients of an equation are of same sign then no root can be positive. In this model, a_0 and b_1 of the cubic characteristic equation are always negative.

Trace of the Jacobian matrix can be written as

$$b_0 = 2m(a^* + x^*) - mx^* \frac{\gamma_2}{(1-\delta)} - ma^* \frac{(1-\sigma)\epsilon_R}{1 - \alpha(1-\sigma)} .$$

For simplicity we assume $\epsilon_R = 0$. Hence using equation (19E) we find that the negative sign of the Trace of the Jacobian matrix implies that

$$(1-\sigma) > \frac{1 - \frac{(m-\rho)}{m}\left[\left\{\frac{\gamma_2}{(1-\delta)} - 2\right\}\frac{\delta(1-\alpha)(\delta+\gamma_2)}{2\alpha(\delta+\gamma_1)} + (1-\delta-\gamma_2)\right]}{\left\{\alpha + (1 - \alpha + \epsilon_P)\frac{(\delta+\gamma_2)}{(\delta+\gamma_1)}\right\}} = \Sigma_1 .$$

Using the restriction (21E) and the above mentioned restriction we have the necessary condition[9] for indeterminacy of the equilibrium growth path which is given by

$$\Sigma_1 < (1-\sigma) < \widehat{\Sigma_1} .$$

So we have the following proposition.

PROPOSITION 3. *When $\epsilon_R = 0$ the indeterminacy of the equilibrium growth path is possible only if $\Sigma_1 < (1-\sigma) < \widehat{\Sigma_1}$.*

5. CONCLUSION

Existing endogenous growth models dealing with the role of human capital accumulation on economic growth have not considered dualism in the nature of human capital formation among different class of people. On the other hand the models which have considered dualism in less developed countries do not consider the aspect of human capital accumulation and endogenous growth. This paper attempts to bridge the gap. In this paper we have analyzed the model of an economy with two different class of individuals in which growth stems from human capital accumulation and the dualism

[9] If both Trace and Determinant of the Jacobian matrix are negative then the sufficient condition for indeterminacy is that a_1 should be negative. The expression of a_1 is derived in Appendix(C).

exists in the nature of human capital accumulation of two types of individuals. Like Lucas (1988) and Benhabib and Perli (1994) we analyze the steady state equilibrium and the transitional dynamic properties of the model and put special emphasis on the role of externalities. We consider the role of externality in the production function as well as in the human capital accumulation function of the poor individuals.

We derive some interesting transitional dynamic properties of this model. External effects on production and the social increasing returns to scale property of the production technology can not explain the indeterminacy of equilibrium growth path if there is constant returns in the human capital accumulation. We badly need strong external effects and social increasing returns on human capital accumulation to explain the indeterminacy of the equilibrium growth path. This is an interesting result because Xie (1994), Benhabib and Perli (1994) have shown that a strong external effects on production alone can explain indeterminacy in Lucas (1988) model without introducing externality and increasing returns in the human capital accumulation function.

The model, in this paper, fails to consider many important features of less developed economies. The present model does not consider the problem of rural-urban migration and the problem of marketable surplus. It does not deal with the trade problems and the problem of international factor mobility. We do not even consider the problem of physical capital accumulation; and hence assume that entire income is consumed. Ignoring the role of physical capital accumulation on economic growth is a very serious limitation of a dynamic model. However, the analysis becomes very complicated and no meaningful results can be drawn when accumulation of physical capital becomes endogenous to the present analysis. Our purpose is to focus on the dualism in the human capital accumulation in a less developed economy. In order to keep the analysis otherwise simple, we do all kinds of abstraction—a standard practice often followed in theoretical literature.

REFERENCES

Acemoglu, D, J. Angrist, 2000, How large are human capital externalities? Evidence from compulsory schooling laws, NBER Macroeconomics Annual, 15, 9–58.

Benabou, R, 1994. Human Capital, Inequality and Growth: A Local Perspective, European Economic Review, 38, 817–826.

Benabou, R, 1996a. Heterogeneity, Stratification and Growth: Macroeconomic Implications of Community Structure and School Finance, American Economic Review, 86, 584–609.

Benabou, R, 1996b. Equity and efficiency in human capital investment: the local connection, Review of Economic Studies, 63, 237–264.

Benhabib, J. and R. Perli, 1994. Uniqueness and indeterminacy: On the dynamics of endogenous growth. Journal of economic theory, 63,113–142.

Ciccone, Antonio and Giovanni Peri, April 2002. Identifying Human Capital Externalities: Theory with an Application to US Cities; IZA Discussion Paper, 488, Institute for the Study of Labor (IZA).

Dixit A. K., 1969. Marketable Surplus and Dual Development, Journal of Economic Theory, 1, 203–219

Eaton J. and Eckstein Zvi, 1997. Cities and Growth: Theory and Evidence from France and Japan, Regional Science and Urban Economics, 27, 443–474.

Gantmacher, F. R, 1959. Applications of the theory of matrices, Interscience Publication, London.

Glaeser, E. L, Learning in Cities, NBER Working Paper No.6271, 1997.

Glaeser, E. L., D. C. Mare, 1994, Cities and skills, NBER Working Paper No 4728.
Lewis, W. A., 1954. Economic Development with Unlimited Supplies of Labour. Manchester School of Economics and Social Studies, 28, 139–191.
Lucas, R. E., 1988. On the Mechanics of Economic Development, Journal of Monetary Economics, 22, 3–42.
Lucas, R. E., 2004. Life earnings and Rural-Urban Migration, Journal of Political Economy, 112, S29–S59.
Menchik, Paul L and Burton A. Weisbrod, 1987. Volunteer Labor Supply, Journal of Public Economics, 32, 159–183.
Moretti, Enrico, 2003. Human capital externalities in cities, NBER Working Paper No. 9641.
Peri, G, 2002, Young workers, learning and agglomeration, Journal of Urban Economics, 52, 582–607.
Ranis, G and C. H. Fei, 1961. A Theory of Economic Development, American Economic Review, 51, 533–556.
Rudd, Jeremy B., October 2000. Empirical Evidence on Human Capital Spillovers, FEDS Discussion Paper No. 2000–46, Board of Governors of the Federal Reserve—Macroeconomic Analysis Section.
Sen, A .K. 1966. Peasants and Dualism with or without Surplus Labour, The Journal of Political Economy, 74, 425–50.
Tamura, R, 1991. Income Convergence in an Endogenous Growth Model, Journal of Political Economy, 99, 522–540.
Todaro, M, 1969. A model of labour migration and urban unemployment in less developed countries, American Economic Review, 59, 138–148.
Uzawa, H, 1965. Optimum Technical Change in an Aggregative Model of Economic Growth, International Economic Review, 6, 18–31
Xie, D, 1994. Divergence in economic performance: Transitional dynamics with multiple equilibria, Journal of Economic Theory, 63, 97–112.

APPENDIX (A)

The elements of Jacobian matrix in the steady-state equilibrium are given as follows:

$$\frac{\partial \dot{z}}{\partial z} = -(\delta+\gamma)x^{*\delta}z^{*\delta+\gamma};$$

$$\frac{\partial \dot{z}}{\partial x} = -mz^* - \delta x^{*\delta-1}z^{*\delta+\gamma+1};$$

$$\frac{\partial \dot{z}}{\partial a} = -mz^*;$$

$$\frac{\partial \dot{x}}{\partial z} = -a^*\frac{(1-\alpha)\delta}{\alpha(1-\delta)}(\delta+\gamma)x^{*\delta}z^{*\delta+\gamma-1};$$

$$\frac{\partial \dot{x}}{\partial x} = \frac{x^*}{(1-\delta)}\left[a^*\frac{-(1-\alpha)(\delta-1)\delta}{\alpha}x^{*\delta-2}z^{*\delta+\gamma} + (1-\delta-\gamma)m\right];$$

$$\frac{\partial \dot{x}}{\partial a} = \frac{(1-\delta-\gamma)}{(1-\delta)}mx^* - \frac{(1-\alpha)\delta}{\alpha(1-\delta)}x^{*\delta}z^{*(\delta+\gamma)};$$

$$\frac{\partial \dot{a}}{\partial z} = \frac{(1-\alpha+\epsilon_P)(1-\sigma)(\delta+\gamma)x^{*\delta}a^*z^{\delta+\gamma-1}}{\{1-\alpha(1-\sigma)\}};$$

$$\frac{\partial \dot{a}}{\partial x} = ma^*\frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}} + \frac{(1-\alpha+\epsilon_P)(1-\sigma)\delta a^*}{\{1-\alpha(1-\sigma)\}}x^{*\delta-1}z^{*\delta+\gamma};$$

$$\frac{\partial \dot{a}}{\partial a} = a^* m \frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}$$

APPENDIX (B)

We consider the possibility of two positive characteristic roots being complex conjugates. This is well known that an equation of an odd degree must have at least one real root, opposite in sign to that of the last term, the leading term being positive. The charateristic equation is given by

$$a_0\lambda^3 + b_0\lambda^2 + a_1\lambda + b_1 = 0$$

where $a_0 = -1$. If we divide the both sides of the equation by a_0, b_1 being negative, the last term becomes positive. So there exists one negative real root. There exists possibility that other two roots are complex number with positive real parts. $Trace(J) > 0 > Det(J)$ ensures that the two roots are not purely imaginary and they have positive real part. Consider the following numerical specifications. For instance, let $m = 2$, $\rho = 0.3$, $\sigma = 2$, $\alpha = 0.7$, $\delta = 0.4$ $\gamma = 0.2$, $\epsilon_P = 0.01$, $\epsilon_R = 0.2$. Under this specification,$(1 - a^* - x^*) = 0.384615385$, $\frac{a^*}{x^*} = 12.83$, $x^* = 0.045$, $a^* = 0.57$, $z^* = 5.14$ and three eigen values are -0.7669, 1.92 and 1.42.

As another example, again let, $m = 2$, $\rho = 0.3$, $\sigma = 3$, $\alpha = 0.7$, $\delta = 0.4$ $\gamma = 0.2$, $\epsilon_P = 0.01$, $\epsilon_R = 0.2$. Under this specification,$(1 - a^* - x^*) = 0.25$, $\frac{a^*}{x^*} = 21.14$, $x^* = 0.03$, $a^* = 0.72$, $z^* = 2.98$ and three eigen values are -0.5411, $1.8817+0.0521i$ and $1.8817 - 0.0521i$. In both the examples, there exists a unique saddle path will converge to the steady state monotonically since the stable (negative) root is real. The diverging trajectories, on the other hand, is monotonic in the first example where all the roots are real and is cyclical in the second example where two roots are complex conjugates with positive real part.

APPENDIX (C)

Note that

$$a_1 = \frac{\partial \dot{\hat{z}}}{\partial x}\frac{\partial \dot{x}}{\partial \hat{z}} + \frac{\partial \dot{x}}{\partial a}\frac{\partial \dot{a}}{\partial x} + \frac{\partial \dot{a}}{\partial \hat{z}}\frac{\partial \dot{\hat{z}}}{\partial a} - \frac{\partial \dot{x}}{\partial x}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{\hat{z}}}{\partial \hat{z}}\frac{\partial \dot{a}}{\partial a} - \frac{\partial \dot{\hat{z}}}{\partial \hat{z}}\frac{\partial \dot{x}}{\partial x},$$

and the elements of the Jacobian matrix in the steady state equilibrium are given by the followings:

$$\frac{\partial \dot{\hat{z}}}{\partial \hat{z}} = -(\delta + \gamma_1)x^{*\delta}\hat{z}^*;$$

$$\frac{\partial \dot{\hat{z}}}{\partial x} = -m(\delta + \gamma_2)\hat{z}^* - (\delta + \gamma_1)\delta x^{*\delta - 1}\hat{z}^{*2};$$

$$\frac{\partial \dot{\hat{z}}}{\partial a} = -m(\delta + \gamma_2)\hat{z}^*;$$

$$\frac{\partial \dot{x}}{\partial \hat{z}} = -a^* \frac{(1-\alpha)\delta}{\alpha(1-\delta)} x^{*\delta};$$

$$\frac{\partial \dot{x}}{\partial x} = \frac{x^*}{(1-\delta)}\left[a^* \frac{-(1-\alpha)(\delta-1)\delta}{\alpha} x^{*\delta-2}\hat{z}^* + (1-\delta-\gamma_2)m\right];$$

$$\frac{\partial \dot{x}}{\partial a} = \frac{(1-\delta-\gamma_2)}{(1-\delta)} m x^* - \frac{(1-\alpha)\delta}{\alpha(1-\delta)} x^{*\delta}\hat{z}^*;$$

$$\frac{\partial \dot{a}}{\partial \hat{z}} = \frac{(1-\alpha+\epsilon_P)(1-\sigma)x^{*\delta}a^*}{\{1-\alpha(1-\sigma)\}};$$

$$\frac{\partial \dot{a}}{\partial x} = \frac{a^*}{\{1-\alpha(1-\sigma)\}}[m\{1-(1-\sigma)(\alpha+\epsilon_R)\} + (1-\alpha+\epsilon_P)(1-\sigma)\delta x^{*\delta-1}\hat{z}^*];$$

$$\frac{\partial \dot{a}}{\partial a} = a^* m \frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}$$

Using the elements of the Jacobian matrix we derive the expression of a_1 which is given by the following:

$$\begin{aligned} a_1 = \max^{\delta}\hat{z}\Bigg[&\frac{\delta(1-\alpha)}{\alpha(1-\delta)}(\delta+\gamma_2) + \frac{\{1-(1-\sigma)(\alpha+\epsilon_R)\}}{\{1-\alpha(1-\sigma)\}}\left\{(\delta+\gamma_1) - \frac{\delta(1-\alpha)}{\alpha(1-\delta)}\right\} \\ &+ \frac{(1-\alpha+\epsilon_P)(1-\sigma)}{\{1-\alpha(1-\sigma)\}}\left\{\frac{(1-\delta-\gamma_2)\delta}{(1-\delta)} - (\delta+\gamma_2)\right\}\Bigg] \\ &+ ax^{2\delta-1}\hat{z}^2\frac{(1-\alpha)\delta}{\alpha(1-\delta)}\left[(\delta+\gamma_1) - \delta\frac{(1-\alpha+\epsilon_P)(1-\sigma)}{\{1-\alpha(1-\sigma)\}}\right] \\ &+ mx^{\delta+1}\hat{z}\left[\frac{(\delta+\gamma_1)(1-\delta-\gamma_2)}{(1-\delta)} - \frac{a^2\{1-(1-\sigma)(\alpha+\epsilon_R)\}(1-\alpha)\delta}{x^2\{1-\alpha(1-\sigma)\}\alpha}\right] \end{aligned}$$

The sufficient condition for the indeterminacy of the equilibrium growth path is that a_1 should be negative.