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POLLUTION AND WELFARE IN THE PRESENCE OF INFORMAL SECTOR: IS THERE ANY TRADE-OFF?

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Abstract: We present a three-sector general equilibrium model with an informal sector, which produces an intermediate input for the formal sector, and analyze the effects of different policies on the environmental standard and welfare of the economy. Since the informal manufacturing sector creates pollution, higher the use of the informal sector's product, higher is the pollution created and higher the discrepancy between the actual and the permissible levels of pollution, so that the emission tax payable by the formal sector is also higher. The efficiency of a representative worker is inversely related to the level of pollution. In this setup, we show that even if the permissible level of pollution is reduced, the polluting sector may expand and worsen the environmental standard. However, this policy may be welfare improving. On the other hand, an inflow of foreign capital may reduce the pollution level but affect welfare adversely. The paper finds that there might exist a trade-off between the economy's twin objectives of lowering the level of pollution and improving national welfare. These results are new in the trade and environment literature.

Keywords: Pollution, informal sector, pollution emission tax, permissible level of pollution, foreign capital.
JEL Classification Number: O17, O22, O33, O54, Q25, L61.

1. INTRODUCTION

In recent years, pollution control is considered as one of the most important among all the policy measures facing the policy makers of the developing countries. Pollution is perhaps an unavoidable accomplice of economic growth and development. Development fosters higher consumption demand, larger population size (due to a lower death rate) and a high standard of living, generating more discharges to the environment in the form of smoke, scraps, wastes and garbage causing greater pollution. At the same time,

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people of a developed nation usually demand higher standards of living atmosphere. As the absorbing capacity of nature is already saturated, pollution cannot be allowed to increase infinitely since it poses a serious threat on the entire living world. Thus a possible solution to this problem is a trade-off between pollution level and economic growth.

Most countries have taken significant movements to protect environment, and the developed ones have successfully been able to combat pollution to a large extent. But for the developing countries, a major problem in regulating environmental standards is the persistence of an informal sector.[1] The informal sector constitutes a large part of the manufacturing and service sectors. On the basis of the works of Agenor (1996), Cole and Sanders (1985), Majumdar (1993), we find that informal sectors provide most of the employment in most of the developing countries. Empirical evidences (see for example, Papola (1981), Romatet (1983), Joshi and Joshi (1976)) also suggest that the urban informal sector units mostly produce intermediate inputs for the formal manufacturing sector on a subcontracting basis. It also suggests that this sector is a major source of environmental pollution. For example, in the city of Kolkata, India leather tanning process is handled by the informal sector. Similarly, for the garments industry the dyeing of garments are done by the informal sector participants on a subcontracting basis. Both tanning and dyeing pollute the environment considerably. Thus, it can be argued that one major reason behind the environmental degradation in the developing countries with expansion of economic activity is the wide prevalence of the urban informal sector. Usually, legislative authorities adopt two major types of environmental regulation, namely, command and control and economic incentives. In case of command and control, the regulator specifies the steps to control pollution after collecting the necessary information regarding the polluter. Economic incentives can take the form of pollution fees, marketable permits and liability.[2] Although these methods can be implemented for the formal sectors in developed countries, the unregistered informal manufacturing units cannot be forced or induced to internalize the environmental costs inflicted on the society due to two reasons. First, these units are unregistered, geographically dispersed and it is quite difficult to identify them. Hence they cannot be kept under the surveillance of the regulating authority. Secondly, the informal sector units with a nominal capital base cannot afford to pay pollution fees or install pollution abating equipments. However, the significant amount of pollution created by them cannot be left unattended.

Biller and Quintero (1995) have examined leather tanneries in Bogota, Colombia. In addition to tanneries they identify the metalworking, electroplating, and textile industries, automobile repair shops, and brick manufacturing as typical informal sector activities causing severe contamination. Blackman and Bannister (1996) have presented

[1] Enterprises, which are unregistered and engaged in manufacturing, construction, transport, trade, service sectors etc. constitute the informal sector. Unlike the formal sector firms the informal sector units do not receive any benefits from the state in the form of tax concessions and they are also outside government regulation.

[2] See Kolstad (2000) for more details.

the results of an econometric analysis of the diffusion of propane among informal 'traditional' brick-makers in Cd. Juárez, Mexico and suggested that community pressure applied by private-sector trade and neighborhood organizations can generate strong incentives for the informal firms to adopt clean technologies. Blackman (1999) has developed a list of feasible environmental management policies.

Among the many alternatives, one of the possible solutions may be to target the formal sector with the capability of bearing the external costs. Most of the informal sector products are used as intermediate goods by the formal sector (for example, in shoe industry, garment industry, etc.). This is particularly beneficial for the formal sector since labor is cheap in the informal sector and due to absence of labor legislation laws, labor can be fully exploited. Now, if the formal sector is made to pay for its use of the output of the polluting informal sector, it may work as an indirect incentive to reduce informal sector production, generating less pollution.

The central objectives of the present paper are as follows. We consider a three-sector general equilibrium model with an informal sector, the output of which is used as an input in the formal sector. We have assumed that environmental pollution occurs through the production of goods by the informal sector. The formal sector firms are made to pay a pollution emission tax if the actual level of pollution exceeds a certain permissible limit, decided by the pollution regulatory authority. The tax revenue collected from this source is transferred to the workers as they are the victims of environmental pollution. Labour endowment is measured in efficiency units where the efficiency of a representative worker is inversely related to the level of pollution. So, any changes in the actual level of pollution affect the efficiency of the workers and hence the effective labour endowment. This again causes a change in the inter-sectoral output and the level of pollution. Higher the use of the informal sector's product, higher is the pollution created and higher the discrepancy between actual and permissible levels of pollution, so that the emission taxes payable by the formal sector is also higher. In this situation, we show that even if the permissible pollution level is reduced, the polluting sector may expand and produce a perverse effect on the actual environmental pollution level of the economy. However, this policy is likely to influence national welfare favourably. Next, we have analyzed the effects of indirect policies like an inflow of foreign capital on the level of pollution and welfare of the economy. We have found that an inflow of foreign capital,[3] although may lower the level of pollution, affects welfare adversely. According to the conventional wisdom an inflow of foreign capital is likely to increase the level of pollution by increasing the size of the formal and the informal sectors but leaves welfare

[3] An inflow of foreign capital is sometimes accompanied by technology transfer including Environmentally Sound Technology (EST). As a result of foreign direct investment, residents of the host country come into contact with foreign entrepreneurs who possess superior technical skills and know how. These new ideas lead to transfer of technology from the foreigners to the residents of the host country and it takes place through observation, discussion and training. This transmission can be considered as a spillover or external effect on the host country. So technology transfer in developing countries takes place mainly through foreign direct investment. However, we here do not consider the case of technology transfer. There is a separate and well-enriched literature in this area. See for example, Mansfield (1961, 1968), Koizumi and Kopecky (1977), Findlay (1978) and Gupta (1998).

unaffected in the absence of any tariff-protection of the import-competing sector. Quite interestingly, we have found that an inflow of foreign capital may reduce the pollution level but lead to welfare deterioration even in the absence of any protectionist policy. On the other hand, a direct policy e.g. a reduction in the permissible level of pollution designed to mitigate pollution, contrary to the common belief, may become counterproductive in the given setup. Thus, in both cases the paper finds that there might exist a trade-off between the economy's objectives of improving the environmental standard and improving national welfare. These results are new in the literature on trade and environment.

2. THE MODEL

We consider a small open economy with three sectors operating at close vicinity. Sector 1 (rural sector) produces an agricultural (export) commodity using capital and labour. There are two manufacturing sectors: formal (sector 3) and informal (sector 2). The informal manufacturing sector produces a non-traded input for the formal sector using capital and labor. The formal sector is the tariff-protected import-competing sector producing a manufacturing commodity using capital, labor and the non-traded input produced by the informal sector. Capital is mobile among the three sectors. On the other hand, labour is perfectly mobile between the agricultural and informal manufacturing sectors. But the formal sector faces an imperfect labour market. It is assumed that labor in the formal sector earns a contractual wage, W^*, while the wage rate in the informal sector, W, is market determined with $W^* > W$. Owing to our small open economy assumption, we consider that the final commodity prices are given internationally. The price of the non-traded input produced by sector 2 is determined endogenously. Production functions exhibit constant returns to scale[4] with diminishing marginal productivity to each factor. The three inputs, capital, labor and the non-traded input, are fully employed. The aggregate capital stock of the economy consists of both domestic and foreign capital and these are perfect substitutes. Income from foreign capital is completely repatriated. Finally, commodity 1 is chosen as the numeraire.

The following symbols will be used in the formal presentation of the model.

a_{Ki} = capital-output ratio in the ith sector, $i = 1, 2, 3$;
a_{Li} = labour-output ratio in the ith sector, $i = 1, 2, 3$;
a_{23} = amount of intermediate input required to produce 1 unit of commodity 3 (technologically fixed);
$P_1 = 1$ (commodity 1 is the numeraire);
P_3 = world price of good 3;
P_2 = endogenously determined price of the non-traded input;
m = ad-valorem rate of tariff on the import of commodity 3;
$P_3^* = P_3(1 + m)$ = tariff-inclusive or domestic price of commodity 3;

[4] Production in the import-competing sector, apart from capital and labour inputs, requires a non-traded input, per-unit requirement of which is assumed to be technologically fixed. However, labour and capital are substitutes and the production function displays the property of constant returns to scale in these two inputs.

h = efficiency of each worker;
W = competitive wage rate (in efficiency units);
W^* = unionized wage in the formal sector (in efficiency units);
r = return to capital;
X_i = output level of the ith sector, $i = 1, 2, 3$;
L = labour endowment of the economy in physical units (normalized to unity);
K_D = domestic capital stock of the economy;
K = aggregate capital stock of the economy including foreign capital;
Z = actual level of pollution in the economy;
$\bar{Z}$ = permissible level of pollution;
$T(\cdot)$ = aggregate pollution emission tax;
θ_{ji} = distributive share of the jth input in the ith sector, $i = 1, 2, 3$;
λ_{ji} = proportion of the jth input employed in the ith sector, $i = 1, 2, 3$;
σ_i = elasticity of substitution between capital and labour in the ith sector, $i = 1, 2, 3$;
Y = national income at world prices;
$\wedge$ = percentage change.

The general equilibrium is represented by the set of following equations:

Given the assumption of perfectly competitive markets the usual price-unit cost equality conditions relating to the three sectors of the economy are given by the following three equations, respectively.

$$Wa_{L1} + ra_{K1} = 1 \tag{1}$$

$$Wa_{L2} + ra_{K2} = P_2 \tag{2}$$

$$W^*a_{L3} + ra_{K3} + P_2a_{23} = P_3(1 + m) - \frac{T(Z(X_2) - \bar{Z})}{X_3} \tag{3}$$

The rural sector does not generate any pollution[5] and without any loss of generality it is assumed that the informal sector is the only polluting sector[6] so that pollution level (industrial emission), Z, is a positive function of the production level of the informal sector, X_2, i.e.

$$Z = Z(X_2);\ Z' > 0\,.$$

For the sake of analytical simplicity, we assume that $Z(\cdot)$ is strictly proportional to X_2. So, we write

[5] This is only a simplifying assumption. A typical rural sector is assumed to produce a primary exportable commodity. Production of primary exportable commodities also vitiates the environment through use of chemical fertilizers and pesticides. However, the amount of pollution generated by the rural sector is insignificant relative to that produced by the manufacturing sectors.

[6] Qualitative results of the model remain unchanged even if the formal sector is also assumed to produce pollution. As formal manufacturing sector uses an input produced by the informal sector at a fixed proportion, an expansion of the formal sector implies an expansion of the informal sector. Thus, the qualitative effect of any policy on the informal sector's output (and hence on pollution) is equivalent to the case where both the sectors are assumed to generate pollution.

$$Z = Z'X_2;\ Z' > 0,\ Z'' = 0 \tag{4}$$

In other words, Z' is constant.

Even though the informal sector is the only polluting sector, it cannot be brought directly under government regulation simply because these are unregistered units. Hence it is only the formal sector, which can be compelled to maintain the environmental standards by making them pay emission tax for the pollution created by them indirectly by the usage of the input produced by the polluting informal sector. Now, let $\bar{Z}$ be the permissible level of pollution, which is a policy parameter of the government. Greater the discrepancy between the permissible level, $\bar{Z}$, and the actual level of pollution, Z, more is the deterioration in environmental standards and hence higher the aggregate pollution emission tax, T, borne by the formal sector.

We define the emission tax function as follows.

$$T = \begin{cases} 0 \quad \text{for} \quad X_2 \leq \bar{X}_2\,; \quad \text{and}\,, \\ T(Z(X_2) - \bar{Z})\,; \quad T' > 0 \quad \text{for} \quad X_2 > \bar{X}_2\,. \end{cases} \tag{5}$$

We explain this emission tax function as follows. We have already stated that the informal sector is the only polluting sector, the level of pollution increases with an increase in the level of production of this sector and that there is a permissible level of pollution, denoted $\bar{Z}$. Let $\bar{X}_2$ be the level of production at which $Z(\cdot) = \bar{Z}$. So, for any, $X_2 \leq \bar{X}_2$ the level of pollution in the economy does not exceed the permissible limit and the emission tax borne by the formal sector is zero. But, once X_2 surpasses $\bar{X}_2$, the pollution level goes above the permissible limit and the emission tax on the formal sector becomes positive. The amount of tax increases as the difference between X_2 and $\bar{X}_2$ (and hence between $Z(\cdot)$ and $\bar{Z}$) increases. As our policy analysis is meaningful only when $Z(\cdot) > \bar{Z}$, we concentrate solely on the case where $X_2 > \bar{X}_2$.

The entire emission tax revenue is transferred to the workers in a lump-sum fashion. The right-hand side of equation (3) denotes the unit domestic price of X_3 net of emission tax where $\frac{T(Z(X_2)-\bar{Z})}{X_3}$ is the effective emission tax per unit of output that the formal sector has to bear.

Complete utilization of capital in the economy implies that

$$a_{K1}X_1 + a_{K2}X_2 + a_{K3}X_3 = K \tag{6}$$

where $a_{Ki}X_i$ is the amount of capital employed in the ith sector with $i = 1, 2$ and 3.

The output of the informal sector, X_2, is used entirely for producing X_3, so that the supply of X_2 is circumscribed by its total demand by sector 3. The demand—supply equality condition is given by

$$X_2 = X_2^D = a_{23}X_3 \tag{7}$$

Here, a_{23} is assumed to be a constant. This means that to produce one unit of the formal

sector's product a_{23} units of the non-traded input are required.[7]

It is assumed that the efficiency of a representative worker, h, is inversely related to the level of pollution, Z, in the economy. Environmental pollution leads to health hazards[8], thus adversely affecting the worker's efficiency. Although in this model the informal sector only creates pollution, it is assumed that pollution affects the efficiency of the entire workforce, not only those who are engaged in the informal sector activities. This is because the three sectors operate at close vicinity so that environmental degradation affects all the members of the working class equally. Thus,

$$h = h(Z(X_2)); \quad h' < 0. \tag{8}$$

After normalizing the labor endowment in physical units to unity, the full-employment of labor in efficiency units implies the following:

$$a_{L1}X_1 + a_{L2}X_2 + a_{L3}X_3 = h(Z(X_2)) \tag{9}$$

where $a_{Li}X_i$ is the employment of labor in the ith sector in efficiency units for $i = 1, 2$ and 3. As the labor endowment of the economy in physical units has been normalized to unity, the labor endowment in efficiency units is $h(Z(X_2))$.

Throughout the paper, we shall make the following assumptions regarding the relative factor intensities of the different sectors of the economy. The agricultural sector is always more labour intensive than the formal manufacturing sector and that the industrial sector as a whole (formal plus informal) is more capital intensive than the agricultural sector in value terms.[9] The latter implies that the industrial sector is also more capital intensive vis-à-vis the agricultural sector in physical terms as well. In mathematical terms, we write the capital intensity conditions as follows.

$$\begin{gathered}(\lambda_{L1}\lambda_{K3} - \lambda_{L3}\lambda_{K1}) > 0; \\ \{\theta_{L1}(\theta_{K3} + \theta_{23}\theta_{K2}) - \theta_{K1}(\theta_{L3} + \theta_{23}\theta_{L2})\} > 0; \quad \text{and}, \\ \{\lambda_{L1}(\lambda_{K3} + \lambda_{23}\lambda_{K2}) - \lambda_{K1}(\lambda_{L3} + \lambda_{23}\lambda_{L2})\} > 0,\end{gathered} \tag{10}$$

From (10) the following relationships trivially follow.

[7] It rules out the possibility of substitution between the non-traded input and the other factors of production in sector 3. Although this is a simplifying assumption, it is not totally unrealistic. In industries like shoe making and garments, large formal sector firms farm out their production to the small informal sector firms under the system of subcontracting. So the production is done in the informal sector firms while labeling, packaging and marketing are done by the formal sector firms. One pair of shoes produced in the informal sector does not change in quantity when it is marketed by the formal sector as a final commodity. Thus there might exist a fixed proportion between the use of the non-traded input and the quantity of the final commodity produced and marketed by the formal sector. It may be noted that Gupta (1994), Chaudhuri (2003) and Marjit (2003) have also made this assumption in different contexts.

[8] Air pollution can lead to irritation, breathing problems and lung diseases; water pollution causes contaminated drinking water; improper waste disposal management involves significant human pathogens, all these contribute directly to reduce human performance.

[9] This assumption is quite realistic and has been extensively used in the theoretical literature on trade and development. See Chandra and Khan (1993), Gupta (1997) and Chaudhuri (2003) in this context.

$$\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\} > 0\,; \quad \text{and}\,,$$
$$[\lambda_{K1}\{\lambda_{L2} + \lambda_{L3} - (h'(\cdot)Z'X_2/h(\cdot))\} - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}] < 0 \tag{10.1}$$

However, the informal manufacturing sector can independently be either capital intensive or labour intensive relative to the agricultural sector. In the main body of the paper, we concentrate on the case where the agricultural sector is more labor intensive than the informal manufacturing sector.[10] In terms of algebra, this can be expressed as follows.

$$(\theta_{L1}\theta_{K2} > \theta_{L2}\theta_{K1}) \text{ i.e. } (\theta_{L1} > \theta_{L2})\,; \quad \text{and,} \quad (\lambda_{L1}\lambda_{K2} > \lambda_{L2}\lambda_{K1}) \tag{11}$$

There are nine endogenous variables in the system: $W, r, P_2, X_1, X_2, X_3, h, Z$ and T. This is an indecomposable production system where any changes in factor endowment affect factor prices and factor coefficients. By solving equations (1) and (2) W and r can be obtained in terms of P_2. Substituting the values of W and r into (3), solving simultaneously with (6) and (9) and using (7), the values of X_1, X_2 and P_2 can be obtained.[11] Having obtained X_2, one can find X_3 from (7). Again, Z can be obtained from (4) and h from (8) once X_2 is obtained. Having obtained Z, from equation (5) T can be found.

Before turning to analyze the policy effects, we should mention that our measure of welfare in this small open economy is national income at world prices, Y, which is expressed as follows[12]:

$$Y = W(a_{L1}X_1 + a_{L2}X_2) + W^*a_{L3}X_3 + rK_D + T(Z(X_2) - \bar{Z}) - mP_3X_3$$

Using equation (9) the above expression becomes

$$Y = Wh(\cdot) + (W^* - W)a_{L3}X_3 + rK_D + T(Z(X_2) - \bar{Z}) - mP_3X_3 \tag{12}$$

In equation (12), $W(a_{L1}X_1 + a_{L2}X_2)$ is the total wage income of the workers engaged in the first two sectors of the economy. $W^*a_{L3}X_3$ is the amount of the wage income of the laborers employed in the formal sector. rK_D is the rental income from domestic

[10] The case where the informal manufacturing sector is more labor intensive vis-à-vis the agricultural sector has been taken up in Appendix V. In that case, some of the results of the model hold under different sufficient conditions. Instead of dealing with both the cases, we consider only one case in details, since our main intention is to question the desirability of policies rendering a lower permissible level of pollution. If we can show this by considering just one case, our purpose is served.

[11] We should note that X_2 is nothing but the supply of commodity 2 i.e. X_2^S. Conversely, $a_{23}X_3$ in equation (7) gives the demand for the non-traded input i.e. X_2^D. Usually, X_2^D and X_2^S would not match if one starts from a random P_2. Therefore, we can define an excess demand function for commodity 2 as: $E(P_2) = X_2^D(P_2) - X_2^S(P_2)$. Equation (7) is valid if and only if $E(P_2) = 0$ say at $P_2 = P_2^e$ For making the entire system consistent, we assume that such a $P_2^e > 0$ exists and it is unique. See, Marjit (2003) in this context.

[12] One may argue that the national welfare function does not explicitly include any social cost due to pollution. However, the welfare function indirectly takes care of the cost due to pollution. This is because it contains the labour endowment in efficiency units, which is negatively related to the level of pollution. So as pollution level rises the endowment of labour in efficiency units falls leading to a decrease in the aggregate wage income. However, the qualitative results of the paper hold under different sufficient conditions even if we consider welfare as: $Y^* = Wh(\cdot) + (W^* - W)a_{L3}X_3 + rK_D + T(Z(X_2) - \bar{Z}) - \beta Z(X_2)$, where $\beta$is the marginal social cost due to pollution.

capital.[13] $T(Z(X_2) - \bar{Z})$ is the revenue from the emission tax, which the workers receive as transfer payments. Finally, mP_3X_3 measures the cost of tariff protection of the import-competing sector.

3. COMPARATIVE STATIC EXERCISES

According to the conventional wisdom, any policy that entails an improvement in environmental standards is welfare enhancing. Thus, a lowering of the permissible level of pollution by the pollution controlling authority appears to be a highly desirable policy. But, in this paper, we reanalyze the efficacy of such a direct environmental policy in lowering the pollution level and improving welfare in a developing country in the presence of an informal manufacturing sector generating considerable amount of pollution. We then examine the effects of an inflow of foreign capital[14] on the level of environmental pollution as well as on the welfare of the economy in the given setup.

Total differentials of (1) and (2) and use of envelope conditions[15] yield:

$$\theta_{L1}\hat{W} + \theta_{K1}\hat{r} = 0 \tag{13}$$

$$\theta_{L2}\hat{W} + \theta_{K2}\hat{r} = \hat{P}_2 \tag{14}$$

Solving (13) and (14) by Cramer's rule, one gets the following expressions:

$$\hat{W} = -(\theta_{K1}\hat{P}_2)/|\theta| \tag{15}$$

$$\hat{r} = \theta_{L1}\hat{P}_2/|\theta| \; ; \quad \text{and}, \tag{16}$$

$$(\hat{W} - \hat{r}) = -(\hat{P}_2/|\theta|) \tag{17}$$

where,

$$|\theta| = \theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2} = \theta_{L1} - \theta_{L2}\,.$$

Now, differentiation of (7), gives

$$\hat{X}_3 = \hat{X}_2 \tag{18}$$

Total differentiation of equations (3), (9) and (6) and use of (15)–(17) and (18) yield[16] the following expressions, respectively.

$$(\hat{P}_2|\theta|)[\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})] + (\theta_{23}/P_2X_2)\{T'(\cdot)Z'X_2 - T(\cdot)\}\hat{X}_2$$
$$= (\theta_{23}/P_2X_2)(T'(\cdot)\bar{\bar{Z}}\hat{\bar{Z}}) \tag{19}$$

[13] Income from foreign capital is fully repatriated. Hence, it is not included in equation (12).

[14] See footnote 3 in this context.

[15] Producers in each industry choose techniques of production so as to minimize unit costs. This leads to the condition that the distributive-share weighted averages of changes in input-output coefficients along the unit isoquant in each industry must vanish near the cost-minimization point. This states that an isocost line is tangent to the unit isoquant. In mathematical terms, cost-minimization conditions for the three industries may be written as: $\theta_{L1}\hat{a}_{L1} + \theta_{K1}\hat{a}_{K1} = 0$; $\theta_{L2}\hat{a}_{L2} + \theta_{K2}\hat{a}_{K2} = 0$; and, $\theta_{L3}\hat{a}_{L3} + \theta_{K3}\hat{a}_{K3} = 0$. These are called the envelope conditions. See Caves, Frankel and Jones (1990), pp. 732–38.

[16] See Appendix I for detailed derivation.

$$(\hat{P}_2/|\theta|)[\lambda_{L1}\theta_{K1}\sigma_1 + \lambda_{L2}\theta_{K2}\sigma_2 + (\lambda_{L3}\theta_{K3}\theta_{L1}\sigma_3/(1-\theta_{23}))] + \{\lambda_{L2} + \lambda_{L3} - (h'(\cdot)Z'X_2/h(\cdot))\}\hat{X}_2 + \lambda_{L1}\hat{X}_1 = 0 \tag{20}$$

and,

$$(\hat{P}_2/|\theta|)[-\lambda_{K1}\theta_{L1}\sigma_1 - \lambda_{K2}\theta_{L2}\sigma_2 - (\lambda_{K3}\theta_{L3}\theta_{L1}\sigma_3/(1-\theta_{23}))] + (\lambda_{K2} + \lambda_{K3})\hat{X}_2 + \lambda_{K1}\hat{X}_1 = \hat{K} \tag{21}$$

Solving equations (19)–(21) by Cramer's rule the following expressions can be obtained.

$$\hat{X}_2 = -(1/\Delta|\theta|)[\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})]\lambda_{L1}\hat{K} \quad (+)$$
$$- \hat{\bar{Z}}\{(\theta_{23}T'(\cdot)\bar{Z})/(\Delta|\theta|P_2X_2)\}[\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2}) + (\theta_{L1}\sigma_3/(1-\theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})] \tag{22}$$
$$(+)$$

and,

$$\hat{P}_2 = \hat{\bar{Z}}[\{(\theta_{23}T'(\cdot)\bar{Z})/(\Delta P_2X_2)\} \times \{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - (h'(\cdot)Z'X_2/h(\cdot))) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}] \quad (-)$$
$$+ \hat{K}[(\lambda_{L1}\theta_{23})\{T'(\cdot)Z'X_2 - T(\cdot)\}/(\Delta P_2X_2)] \tag{23}$$

where,

$$\Delta = (1/|\theta|)[\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\} \quad (+)$$
$$\times \{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}] \quad (-)$$
$$- [\theta_{23}\{T'(\cdot)Z'X_2 - T(\cdot)\}/(P_2X_2|\theta|)] \times [\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2}) + (\theta_{L1}\sigma_3/(1-\theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})] \quad (+) \tag{24}$$

As commodity 2 is non-traded, its market must clear domestically and the comparative static exercises are meaningful only if the equilibrium in the market for commodity 2 is stable. It can be checked that the stability condition in the market for the non-traded input is as follows.[17]

$$(\Delta/|\lambda|) = (\Delta'/|\theta||\lambda|) > 0\,. \tag{25}$$

[17] This has been derived in Appendix II.

where: $|\theta| = (\theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2})$;

$$\begin{aligned}\Delta' = [&\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\} \\ &\qquad (+) \\ &\times \{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}] \\ &\qquad (-) \\ &- [\theta_{23}\{T'(\cdot)Z'X_2 - T(\cdot)\}/(P_2X_2)] \\ &\times [\lambda_{K1}\lambda_{L2}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{K2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2}) \\ &+ (\theta_{L1}\sigma_3/(1 - \theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})] \qquad (26.1) \\ &\qquad (+)\end{aligned}$$

$$|\lambda| = [G(\lambda_{L1}\lambda_{K3} - \lambda_{L3}\lambda_{K1}) + H\{\lambda_{L1}\lambda_{K2} - \lambda_{K1}(\lambda_{L2} - h'(\cdot)Z'X_2/h(\cdot))\}]\,; \quad (26.2)$$

$$G = (T'(\cdot)Z'X_2/P_3^*X_3) > 0\,; \quad \text{and}, \qquad (26.3)$$

$$H = (T(\cdot)/P_3^*X_3) > 0\,. \qquad (26.4)$$

In the present case where the agricultural sector is more labour intensive vis-à-vis the informal manufacturing sector,[18] we have $|\theta| > 0$. Using (26.3) and (26.4) from (26.2) we obtain that $|\lambda| > 0$. Then using (10.1), from (25) and (26.1) we find that for the fulfillment of the stability condition in the market for the non-traded good one requires: $\Delta > 0$ and $\Delta' > 0$. The necessary condition for $\Delta' > 0$ is: $\{T'(\cdot)Z'X_2 < T(\cdot)\}$. This, in turn, implies that $(E_T < 1 - \bar{Z}/Z)$ where $E_T (= [\{dT(\cdot)/d(Z - \bar{Z})\}\{(Z - \bar{Z})/T(\cdot)\}])$ is the elasticity of the emission tax function.

3.1. Policy effects on environmental pollution

In the stable equilibrium in the market for the non-traded input we have found that $\Delta > 0$. From (22), it now follows that $\hat{X}_2 > 0$ when $\hat{\bar{Z}} < 0$; and, $\hat{X}_2 < 0$ when $\hat{K} > 0$. Differentiating equation (4) one gets

$$Z\hat{Z} = Z'X_2\hat{X}_2 \qquad (27)$$

From (27) it is easy to derive the following results.

$\hat{Z} > 0$ when $\hat{\bar{Z}} < 0$ and $\hat{Z} < 0$ when $\hat{K} > 0$. This leads to the following proposition.

PROPOSITION 1. *A reduction in the permissible level of pollution in the presence of an informal sector leads to an increase in pollution while an inflow of foreign capital lowers the level of pollution in the economy.*

To explain these results in economic terms, let us first examine the effects of these policies on the effective pollution emission tax rate, say, F, where

$$F = \frac{T(Z(X_2) - \bar{Z})}{X_3}$$

Differentiating F, using (22) and simplifying we get[19]

[18] See footnote 11 in this context.

[19] For detailed derivation see Appendix III.

$$
\begin{aligned}
F\hat{F}X_3 = {} & \hat{K}\{T'(\cdot)Z'X_2 - T(\cdot)\} \\
& \times [(1/\Delta|\theta|)\lambda_{L1}\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}] \\
& \qquad\qquad (+) \\
& - \hat{\bar{Z}}\{T'(\cdot)\bar{Z}/(\Delta|\theta|)\}[\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\} \\
& \qquad\qquad\qquad\qquad (+) \\
& \times \{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2) \\
& \qquad\quad (+) \\
& - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}] \\
& (-)
\end{aligned}
\tag{28}
$$

We have already mentioned that the comparative static exercises are meaningful only if equilibrium in the market for the non-traded input is stable. In the present case we have found that in the stable equilibriumand $\Delta > 0$ and $\{T'(\cdot)Z'X_2 < T(\cdot)\}$. From (28) it then follows that $\hat{F} > 0$ when $\hat{K} > 0$; and, $\hat{F} < 0$ when $\hat{\bar{Z}} < 0$.

If in an attempt to check further deterioration in environmental quality, the pollution control authority fixes the permissible level of pollution at a lower level, $\bar{Z}$ takes a lower value. From equation (28) it follows that the average pollution emission tax that the formal sector has to bear decreases. As a consequence, the effective price of the formal sector's product (net of average emission tax) rises leading to an expansion of the formal sector. As the formal sector uses the output of the informal sector at a fixed rate, the latter sector also expands, thereby raising the pollution level of the society. On the other hand, owing to an inflow of foreign capital the aggregate capital stock of the economy swells up. It produces a *Rybczynski effect*, leading to an expansion of the formal sector (also the informal manufacturing sector) and a contraction of the agricultural sector as the manufacturing sector as a whole (formal plus informal) is more capital-intensive than the agricultural sector. The average pollution emission tax, F, rises and the effective price of the formal sector's product decreases as $\Delta > 0$ (see equation (28)). This produces a *Stolper-Samuelson effect* and exerts downward pressures on the output levels of the two manufacturing sectors. So two opposite effects on X_3 (and hence on X_2) are generated. As the negative effect of an increase in F outweighs the positive *Rybczynski effect*, X_3 (and hence X_2) falls in the new equilibrium.

3.2. Policy effects on welfare

To analyze the welfare implications of the two policies totally differentiating equation (12) and using (15), (16), (22) and (23) the following expression can be derived.[20]

[20] See Appendix IV for detailed derivation.

$$
\begin{aligned}
Y\hat{Y} = \hat{\bar{Z}} \cdot \{&(T'(\cdot)\bar{Z}/(\Delta|\theta|P_2X_2)\} \\
&[A\theta_{23}\{\lambda_{K1}(\lambda_{L2}+\lambda_{L3}-h'(\cdot)Z'X_2/h(\cdot))-\lambda_{L1}(\lambda_{K2}+\lambda_{K3})\} \\
&\qquad\qquad (-) \\
&-B\theta_{23}C-(|\theta|\Delta P_2X_2)] \\
&\quad (+) \\
&+\hat{K}\cdot(\lambda_{L1}/|\theta|\Delta)[(A\theta_{23}/P_2X_2)\{T'(\cdot)Z'X_2-T(\cdot)\} \\
&-B\{\theta_{K3}\theta_{L1}+\theta_{23}(\theta_{L1}-\theta_{L2})\}] \\
&\qquad (+)
\end{aligned} \tag{29}
$$

where $A = [\{(W^*-W)h(\cdot)\lambda_{L3}\theta_{K3}\theta_{L1}\sigma_3/(1-\theta_{23})\}-Wh(\cdot)\theta_{K1}(1-\lambda_{L3})+rK_D\theta_{L1}]$,

$B = [(W^*-W)h(\cdot)\lambda_{L3}+\{T'(\cdot)+h'(\cdot)W\}Z'X_2-mP_3X_3]$; and,

$C = [\lambda_{K1}\lambda_{L1}\sigma_1+\sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2}+\lambda_{L1}\lambda_{K2}\theta_{L2})$
$+(\theta_{L1}\sigma_3/(1-\theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3}+\lambda_{L1}\lambda_{K3}\theta_{L3})] > 0$.

Using the stability condition from (29) we find that $\hat{Y} < 0$ when $\hat{K} > 0$ and $\hat{Y} > 0$ when $\hat{\bar{Z}} < 0$ if (i) $A > 0$; and, (ii) $B \geq 0$. This establishes the following proposition.

PROPOSITION 2. *A reduction in the permissible level of pollution in the presence of an informal sector leads to an improvement in welfare if* (i) $A > 0$; *and,* (ii) $B \geq 0$. *On the contrary, an inflow of foreign capital with full repatriation of income on foreign capital is welfare deteriorating under the same set of sufficient conditions.*

Proposition 2 can be intuitively explained as follows. With the lowering of the permissible pollution level, $\bar{Z}$, the discrepancy between the actual and permissible pollution level increases. However, from equation (28) it follows that the average pollution emission tax that the formal sector has to bear decreases. The effective price of the formal sector's product (net of average emission tax) rises and leads to an expansion of the higher wage-paying formal sector. This we call the *labour reallocation effect*, which works favourably on welfare. The polluting sector (sector 2) also expands as its output is solely used in the formal sector in a fixed proportion. The labour endowment in efficiency units decreases as the level of pollution rises. Besides, a reduction in $\bar{Z}$ lowers P_2, which in turn raises r and reduces W following a *Stolper-Samuelson effect*. Thus, the aggregate wage income is affected due to three different effects: (i) direct negative effect on W following a reduction in $\bar{Z}$, (ii) the *labour reallocation effect* as the higher (lower) wage-paying formal (agricultural) sector expands (contracts); and, (iii) changes in labour endowment of the economy in efficiency units. The net outcome on the aggregate wage income is ambiguous. There are other effects on welfare as well. Both the aggregate capital income and transfer payments to the workers (the pollution emission tax revenue collected from the formal sector) increase unambiguously. On the contrary, the cost of tariff protection rises as the formal sector expands. This lowers welfare. The net impact of all these effects would be an increase in welfare under the sufficient conditions: (i) $A > 0$; and, (ii) $B \geq 0$.

On the other hand, if foreign capital enters into the economy with full repatriation of foreign capital income the aggregate capital stock of the economy swells up. We have seen (see proposition 1) that it leads to a contraction of the formal (and informal) sector. As the higher (lower) wage-paying formal (agricultural) sector contracts (expands), the aggregate wage income falls due to the *labour reallocation effect*. However, there are two other effects on the aggregate wage income of the workers. As the polluting (informal manufacturing) sector contracts the labour endowment in efficiency units rises. Besides, an inflow of foreign capital lowers the price of the informal sector's price, P_2, which lowers W and raises r. There are again three different effects on the aggregate wage income. However, the aggregate capital income on domestic capital falls. Also, the transfer payments that the workers receive from the government also falls as the level of pollution (and hence the emission tax revenue) falls. On the contrary, the cost of tariff protection falls as the tariff-protected formal sector contracts, which works favourably on welfare. The net effect of all these effects on welfare is negative if: (i) $A > 0$; and, (ii) $B \geq 0$. However, it is easy to check that welfare may fall due to an inflow of foreign capital even in the absence of any tariff protection.[21] Actually, in the absence of any tariff the possibility of welfare deteriorating effect of foreign capital increases.

4. CONCLUDING REMARKS

In the developing countries reduction of the permissible level of pollution by pollution regulating authorities is a conventional policy to arrest further environmental degradation. Although both the formal and the informal manufacturing units cause industrial pollution, the extent of pollution generated by the informal sector firms is significantly greater than that generated by their formal sector counterparts. As the informal sector firms have limited access to Environmentally Sound Technology (EST) and as they use backward technology these firm are responsible for a major share of pollutants. The formal sector firms in the developing countries actually subcontract the informal sector firms to undertake a number of tasks and processes that are "dirty" from the environment point of view. Perrings, Bhargava and Gupta (1995) have argued that such subcontracting is an economical way for the formal sector firms to avoid investment in ESTs made

[21] This is a highly interesting result because it is contrary to the conventional Brecher-Alejandro proposition. Brecher and Alejandro (1977) have analyzed the welfare effects of foreign capital inflow in a two-commodity, two-factor full employment model. The important result is as follows: an inflow of foreign capital with full repatriation of its earnings is necessarily immiserizing if the import-competing sector is capital-intensive and is protected by a tariff. However, welfare remains unaffected in the absence of any tariff. Here welfare is defined as a positive function of national income. In the literature, the Brecher-Alejandro proposition has also been re-examined in terms of three-sector full-employment models like Beladi and Marjit (1992a, 1992b) with the third sector being a duty-free zone. As the developing countries are plagued by labour market distortion, some attempts have been made to analyze the welfare impact of foreign capital inflow using a Harris-Todaro (1970) framework. For example, Khan (1982) has considered a mobile capital Harris-Todaro model with urban unemployment. A third sector, called an urban informal sector, has been introduced in the works of Grinols (1991) and Chandra and Khan (1993). The immiserizing result of foreign capital has been found to be valid in general (Grinols 1991 is, of course, a notable exception) in the presence of a tariff protected import-competing sector. Interestingly, in the present paper the immiserizing result of foreign capital may be obtained even in the absence of any tariff.

obligatory by the regulatory authority. The informal sector firms being unregistered units are difficult to control and so they face fewer incentives to prevent pollution. In the circumstances, an indirect way to control environmental pollution is to impose pollution emission tax on the formal sector firms if the level of industrial pollution created exceeds a certain permissible level. This is expected to induce them to minimize harmful discharges by cutting down the use of inputs produced by the informal sector and thus improve the environmental quality. A pertinent question is whether this indirect way of controlling pollution can actually deliver the goods. In this paper, we have analyzed the efficacy of such a policy in a three-sector general equilibrium model with a polluting informal sector, which produces a non-traded input for the formal sector. Higher the use of informal sector's product, higher is the pollution created and higher the discrepancy between permissible and actual level of pollution, so that the pollution emission tax payable by the formal sector is also higher. Again, labour endowment is measured in efficiency units where the efficiency of a representative worker is inversely related to the level of pollution. The emission tax revenue is transferred to the workers in a lump-sum fashion. In this setup, we have shown that even if the permissible pollution level is reduced, the polluting sector may expand and lead to a deterioration in the environmental standard. Quite unexpectedly, this policy may improve welfare of the economy. This has a very important policy implication due to the counterintuitive results of direct environmental policies. On the contrary, an inflow of foreign capital may be effective in lowering the level of pollution although it may affect welfare of the economy adversely. Therefore, the paper finds that there might exist a trade-off between the economy's objectives of lowering the level of pollution and improving national welfare. These results are new in the literature on trade and environment, which would help the policy makers in the developing countries in designing appropriate policies because reduction of pollution level and improvement of national welfare are both desirable for a developing economy.

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MATHEMATICAL APPENDICES

APPENDIX I: DERIVATIONS OF CERTAIN EXPRESSIONS

Total differentiation of (3) gives

$$\begin{aligned}
a_{K3}dr + a_{23}dP_2 &= -(a_{23}/(X_2)^2)[X_2\{T'(\cdot)(Z'dX_2 - d\bar{Z})\} - T(\cdot)dX_2] \\
\text{or,}\quad \theta_{K3}\hat{r} + \theta_{23}\hat{P}_2 &= (a_{23}/X_2P_3(1+t))T(\cdot)\hat{X}_2 \\
&\quad - (a_{23}/P_3(1+t))T'(\cdot)Z'\hat{X}_2 \\
&\quad + (a_{23}/X_2P_3(1+t))T'(\cdot)\bar{Z}\hat{\bar{Z}} \\
\text{or}\quad \theta_{K3}\theta_{L1}(\hat{P}_2/|\theta|) + \theta_{23}\hat{P}_2 &= (\theta_{23}/P_2X_2)T(\cdot)\hat{X}_2 - (\theta_{23}/P_2)T'(\cdot)Z'\hat{X}_2 \\
&\quad + (\theta_{23}/P_2X_2)T'(\cdot)\bar{Z}\hat{\bar{Z}} \\
\text{or,}\quad (\hat{P}_2/|\theta|)[\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})] &\\
&\quad + (\theta_{23}/P_2X_2)(T'(\cdot)Z'X_2 - T(\cdot))\hat{X}_2 \\
&= (\theta_{23}/P_2X_2)(T'(\cdot)\bar{Z}\hat{\bar{Z}})
\end{aligned} \tag{19}$$

Again differentiation of (9) yields,

$$\begin{aligned}
&\lambda_{L1}\hat{X}_1 + (\lambda_{L2} + \lambda_{L3})\hat{X}_2 - (h'(\cdot)Z'X_2/h(\cdot))\hat{X}_2 \\
&\quad = -\lambda_{L1}\theta_{K1}\sigma_1(\hat{P}_2/|\theta|) - \lambda_{L2}\theta_{K2}\sigma_2(\hat{P}_2/|\theta|) \\
&\quad - \{\lambda_{L3}\theta_{K3}\theta_{L1}\sigma_3/(1-\theta_{23})\}(\hat{P}_2/|\theta|)
\end{aligned}$$

Rearranging terms we get

$$\begin{aligned}
&(\hat{P}_2/|\theta|)([\lambda_{L1}\theta_{K1}\sigma_1 + \lambda_{L2}\theta_{K2}\sigma_2 + (\lambda_{L3}\theta_{K3}\theta_{L1}\sigma_3/(1-\theta_{23})] \\
&\quad + [\lambda_{L2} + \lambda_{L3} - (h'(\cdot)Z'X_2/h(\cdot))]\hat{X}_2 + \lambda_{L1}\hat{X}_1 = 0
\end{aligned} \tag{20}$$

Finally, differentiation of (6) gives,

$$\begin{aligned}
&\lambda_{K1}\hat{X}_1 + (\lambda_{K2} + \lambda_{K3})\hat{X}_2 = \hat{K} + \lambda_{K1}\theta_{L1}\sigma_1(\hat{P}_2/|\theta|) + \lambda_{K2}\theta_{L2}\sigma_2(\hat{P}_2/|\theta|) \\
&+ (\lambda_{K3}\theta_{L3}\theta_{L1}\sigma_3/(1-\theta_{23}))(\hat{P}_2/|\theta|) \\
&\text{or,}\quad (\hat{P}_2/|\theta|)[-\lambda_{K1}\theta_{L1}\sigma_1 - \lambda_{K2}\theta_{L2}\sigma_2 - \{\lambda_{K3}\theta_{L3}\theta_{L1}\sigma_3/(1-\theta_{23})\}] \\
&+ (\lambda_{K2} + \lambda_{K3})\hat{X}_2 + \lambda_{K1}\hat{X}_1 = \hat{K}
\end{aligned} \tag{21}$$

APPENDIX II: DERIVATION OF STABILITY CONDITION IN THE MARKET FOR THE NON-TRADED INPUT

As commodity 2 is internationally non-traded its market must clear domestically through adjustments in its price, P_2.

The stability condition in the market for commodity 2 requires that $(d(X_2^D - X_2)/dP_2) < 0$. This implies around equilibrium, initially, $X_2^D = X_2$. Thus,

$$((\hat{X}_2^D/\hat{P}_2) - (\hat{X}_2/\hat{P}_2)) < 0\,. \tag{A.1}$$

We note that X_1, X_2 and X_3 can be simultaneously solved from equations (3), (6) and (9) as functions of P_2. Differentiating equations (3), (9) and (6) and keeping the parameters unchanged, we get the following three expressions, respectively.

$$-G\hat{X}_2 + H\hat{X}_3 = M_1\hat{P}_2 \tag{A.2}$$

$$\lambda_{L1}\hat{X}_1 + \{\lambda_{L2} - (h'(\cdot)Z'X_2/h(\cdot))\}\hat{X}_2 + \lambda_{L3}\hat{X}_3 = -M_2\hat{P}_2\,; \quad \text{and}, \tag{A.3}$$

$$\lambda_{K1}\hat{X}_1 + \lambda_{K2}\hat{X}_2 + \lambda_{K3}\hat{X}_3 = M_3\hat{P}_2 \tag{A.4}$$

where: $G = (T'(\cdot)Z'X_2/P_3^*X_3) > 0$; $H = (T(\cdot)/P_3^*X_3) > 0$;

$$\begin{aligned} M_1 &= \{(\theta_{K3}\theta_{L1} + \theta_{23}|\theta|)/|\theta|\} = \{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}(1/|\theta|) \\ M_2 &= \{\lambda_{L1}\theta_{K1}\sigma_1 + \lambda_{L2}\theta_{K2}\sigma_2 + (\lambda_{L3}\theta_{K3}\theta_{L1}\sigma_3/(1-\theta_{23}))\}(1/|\theta|)\,; \quad \text{and}, \\ M_3 &= \{\lambda_{K1}\theta_{L1}\sigma_1 + \lambda_{K2}\theta_{L2}\sigma_2 + (\lambda_{K3}\theta_{L3}\theta_{L1}\sigma_3/(1-\theta_{23}))\}(1/|\theta|) \end{aligned} \tag{A.5}$$

Solving (A.2)–(A.4) by Cramer's rule we get:

$$\hat{X}_2 = (\hat{P}_2/|\lambda|)[-M_1(\lambda_{L1}\lambda_{K3} - \lambda_{L3}\lambda_{K1}) + H(\lambda_{L1}M_3 + \lambda_{K1}M_2)]\,; \quad \text{and}, \tag{A.6}$$

$$\begin{aligned} \hat{X}_3 = (\hat{P}_2/|\lambda|)[&G(\lambda_{L1}M_3 + \lambda_{K1}M_2) + M_1\lambda_{L1}\lambda_{K2} \\ &- M_1\lambda_{K1}\{\lambda_{L2} - (h'(\cdot)Z'X_2/h(\cdot))\}] \end{aligned} \tag{A.7}$$

where

$$|\lambda| = [G(\lambda_{L1}\lambda_{K3} - \lambda_{L3}\lambda_{K1}) + H\{\lambda_{L1}\lambda_{K2} - \lambda_{K1}(\lambda_{L2} - h'(\cdot)Z'X_2/h(\cdot))\}] \tag{A.8}$$

From (A.6) we find that

$$(\hat{X}_2/\hat{P}_2) = (1/|\lambda|)[-M_1(\lambda_{L1}\lambda_{K3} - \lambda_{L3}\lambda_{K1}) + H(\lambda_{L1}M_3 + \lambda_{K1}M_2)] \tag{A.9}$$

Now the demand for the non-traded input is given by
$X_2^D = a_{23}X_3$. Differentiating this equation one gets
$\hat{X}_2^D = \hat{X}_3$. Using (A.7) one can find

$$\begin{aligned} (\hat{X}_2^D/\hat{P}_2) = (1/|\lambda|)[&G(\lambda_{L1}M_3 + \lambda_{K1}M_2) \\ &+ M_1\lambda_{L1}\lambda_{K2} - M_1\lambda_{K1}\{\lambda_{L2} - (h'(\cdot)Z'X_2/h(\cdot))\}] \end{aligned} \tag{A.10}$$

Using (A.1), (A.9) and (A.10) we find the following stability condition for equilibrium in the market for commodity 2.

$$\begin{aligned} ((\hat{X}_2^D/\hat{P}_2) = (\hat{X}_2/\hat{P}_2)) = (1/|\lambda|)[&G(\lambda_{L1}M_3 + \lambda_{K1}M_2) - H(\lambda_{L1}M_3 + \lambda_{K1}M_2) \\ &+ M_1\{\lambda_{L1}(\lambda_{K2} + \lambda_{K3}) \\ &- \lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot))\}] < 0 \end{aligned}$$

Inserting the values of G, H, M_1, M_2 and M_3 from (A.5) into the above expression and noting that $(1/P_3^*X_3) = (\theta_{23}/P_2X_2)$ we get

$$((\hat{X}_2^D/\hat{P}_2) - (\hat{X}_2/\hat{P}_2)) = (1/|\theta||\lambda|)[\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}\{\lambda_{L1}(\lambda_{K2} + \lambda_{K3})$$
$$(+)$$
$$- \lambda_{K1}(\lambda_{L2} + \lambda_{L3}) - (h'(\cdot)Z'X_2/h(\cdot))\}$$
$$(+)$$
$$+ \{\theta_{23}\{T'(\cdot)Z'X_2 - T(\cdot)\}/P_2X_2\}$$
$$\times \{\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2})$$
$$+ (\theta_{L1}\sigma_3/(1 - \theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})\}] < 0$$
$$(+)$$
$$= (\Delta/|\lambda|) = (\Delta'/|\theta||\lambda|) > 0 \tag{25}$$

where:

$$\Delta = (1/|\theta|)[\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}\{\lambda_{K1}(\lambda_{L2} + \lambda_{L3}) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})$$
$$(+) \qquad (-)$$
$$- (\lambda_{K1}h'(\cdot)Z'X_2/h(\cdot))\} - (\theta_{23}/P_2X_2)(T'(\cdot)Z'X_2 - T(\cdot))$$
$$\times \{\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2})$$
$$+ (\theta_{L1}\sigma_3/(1 - \theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})\}]; \quad \text{and,} \tag{24}$$
$$(+)$$

$$\Delta' = [\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2}\}$$
$$(+)$$
$$\times \{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}]$$
$$(-)$$
$$- [\theta_{23}\{T'(\cdot)Z'X_2 - T(\cdot)\}/(P_2X_2)][\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2}$$
$$+ (\theta_{L1}\sigma_3/(1 - \theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})] \tag{26.1}$$
$$(+)$$

APPENDIX III: DERIVATION OF EQUATION (28)

The effective emission tax rate is given by

$$F = \frac{T(Z(X_2) - \bar{Z})}{X_3}$$

Differentiating and using (18), we get

$$F\hat{F}X_3 = T'(\cdot)Z'X_2\hat{X}_2 - T(\cdot)\hat{X}_2 - T'(\cdot)\bar{Z}\hat{\bar{Z}}$$
$$= \hat{X}_2\{T'(\cdot)Z'X_2 - T(\cdot)\} - T'(\cdot)\bar{Z}\hat{\bar{Z}}$$

Now using (22) one gets

$$F\hat{F}X_3 = \{T'(\cdot)Z'X_2 - T(\cdot)\}[-(1/\Delta|\theta|)\lambda_{L1}\hat{K}\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}$$
$$- \hat{\bar{Z}}\{(\theta_{23}T'(\cdot)\bar{Z})/(\Delta|\theta|P_2X_2)\}C] - T'(\cdot)\bar{Z}\hat{\bar{Z}}$$

where,

$$C = [\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2})$$
$$+ (\theta_{L1}\sigma_3/(1 - \theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})] > 0$$

Use of (24) and simplification yield

$$F\hat{F}X_3 = -[\{T'(\cdot)Z'X_2 - T(\cdot)\}(\lambda_{L1}/\Delta|\theta|)\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}]\hat{K}$$
$$(+)$$
$$- \{(\theta_{23}T'(\cdot)\bar{Z})/(\Delta|\theta|P_2X_2)\}[\{T'(\cdot)Z'X_2 - T(\cdot)\}C + \{(\Delta|\theta|P_2X_2)/\theta_{23}\}]\hat{\bar{Z}}$$

Further simplification gives

$$F\hat{F}X_3 = \{T'(\cdot)Z'X_2 - T(\cdot)\}[(\lambda_{L1}/\Delta|\theta|)\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}]\hat{K}$$
$$- \{(T'(\cdot)\bar{Z})/(\Delta|\theta|)\}[\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1} - \theta_{L2})\}$$
$$\times \{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\}]\hat{\bar{Z}} \qquad (28)$$

APPENDIX IV: DERIVATION OF EQUATION (29)

Differentiation of equation (12) yields

$$dY = h(\cdot)dW + Wh'(\cdot)Z'dX_2 + (W^* - W)a_{L3}dX_3 + (W^* - W)X_3da_{L3}$$
$$- a_{L3}X_3dW + K_Ddr + T'(\cdot)Z'dX_2 - T(\cdot)d\bar{Z} - mP_3dX_3$$

or,

$$Y\hat{Y} = Wh(\cdot)\hat{W} + Wh'(\cdot)Z'X_2\hat{X}_2 + (W^* - W)a_{L3}X_3\hat{X}_3$$
$$+ (W^* - W)a_{L3}X_3(\theta_{K3}\sigma_3/(1 - \theta_{23}))\hat{r} + rK_D\hat{r}$$
$$- Wa_{L3}X_3\hat{W} + T'(\cdot)Z'X_2\hat{X}_2 - T'(\cdot)\bar{Z}\hat{\bar{Z}} - mP_3X_3\hat{X}_3$$

(note that$\hat{a}_{L3} = (\theta_{K3}\sigma_3/(1 - \theta_{23}))\hat{r}$; $(\theta_{L3} + \theta_{K3}) = (1 - \theta_{23})$.).

Now substitution of $\hat{X}_2$ in place of $\hat{X}_3$ into the above equation yields

$$Y\hat{Y} = \{h(\cdot)W(1 - \lambda_{L3})\}\hat{W} + \{(W^* - W)a_{L3}X_3(\theta_{K3}\sigma_3/(1 - \theta_{23})) + rK_D\}\hat{r}$$
$$+ [\{Wh'(\cdot) + T'(\cdot)\}Z'X_2 + (W^* - W)h(\cdot)\lambda_{L3} - mP_3X_3]\hat{X}_2 - T'(\cdot)\hat{Z}\hat{\bar{Z}}$$

With the help of (15) and (16) the above expression becomes

$$Y\hat{Y} = [-Wh(\cdot)\theta_{K1}(1 - \lambda_{L3}) + (W^* - W)h(\cdot)\lambda_{L3}(\theta_{K3}\sigma_3\theta_{L1}/(1 - \theta_{23})) + rK_D\theta_{L1}]$$
$$\cdot(\hat{P}_2/|\theta|) - T'(\cdot)\hat{Z}\hat{\bar{Z}} + [\{T'(\cdot) + Wh'(\cdot)\}Z'X_2$$
$$+ (W^* - W)h(\cdot)\lambda_{L3} - mP_3X_3]\hat{X}_2$$

Using (22) and (23) we can write

$$\begin{aligned} Y\hat{Y} = \hat{\bar{Z}}[\{A\theta_{23}T'(\cdot)\hat{Z})/(|\theta|\Delta P_2X_2)\}\{\lambda_{K1}(\lambda_{L2}+\lambda_{L3}-(h'(\cdot)Z'X_2/h(\cdot)) \\ -\lambda_{L1}(\lambda_{K2}+\lambda_{K3})\}] + \hat{K}[\{A\lambda_{L1}\theta_{23}/|\theta|\Delta P_2X_2\}\{T'(\cdot)Z'X_2 - T(\cdot)\} \\ - T'(\cdot)\hat{Z}\hat{\bar{Z}} - \hat{K}[(B\lambda_{L1}/\Delta|\theta|)\{\theta_{K3}\theta_{L1}+\theta_{23}(\theta_{L1}-\theta_{L2})\}] \\ - \hat{\bar{Z}}[\{(B\theta_{23}T'(\cdot)\bar{Z})/(\Delta|\theta|P_2X_2)\}C] \end{aligned}$$

where

$$A = [\{W^* - W)h(\cdot)\lambda_{L3}\theta_{K3}\theta_{L1}\sigma_3/(1-\theta_{23})\} - Wh(\cdot)\theta_{K1}(1-\lambda_{L3}) + rK_D\theta_{L1}],$$

$$B = [(W^* - W)h(\cdot)\lambda_{L3} + \{T'(\cdot) + h'(\cdot)W\}Z'X_2 - mP_3X_3]; \quad \text{and},$$

$$C = [\lambda_{K1}\lambda_{L1}\sigma_1 + \sigma_2(\lambda_{K1}\lambda_{L2}\theta_{K2} + \lambda_{L1}\lambda_{K2}\theta_{L2}) + (\theta_{L1}\sigma_3/(1-\theta_{23}))(\lambda_{K1}\lambda_{L3}\theta_{K3} + \lambda_{L1}\lambda_{K3}\theta_{L3})] > 0.$$

Further simplification gives

$$\begin{aligned} Y\hat{Y} = \hat{\bar{Z}}\{(T'(\cdot)\bar{Z})/(\Delta|\theta|P_2X_2)\}[A\theta_{23}\{\lambda_{K1}(\lambda_{L2}+\lambda_{L3} \\ -(h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2}+\lambda_{K3}\} - B\theta_{23}C - (|\theta|\Delta P_2X_2)] \\ \qquad\qquad (-) \qquad\qquad (+) \\ + \hat{K}(\lambda_{L1}/|\theta|\Delta)[(A\theta_{23}/P_2X_2)\{T'(\cdot)Z'X_2 - T(\cdot)\} \\ - B\{\theta_{K3}\theta_{L1} + \theta_{23}(\theta_{L1}-\theta_{L2})\}] \qquad (29) \\ \qquad\qquad (+) \end{aligned}$$

APPENDIX V: TWO POSSIBLE CASES

Depending on the different signs and values of $|\theta|$ the following two cases are possible.

CASE I. $|\theta| > 0$. From (26.2)–(26.4) it follows that $|\lambda| > 0$. Then, from the stability condition in the market for commodity 2 (given by (25)) one obtains: $\Delta > 0$. We have stated in the text that $\Delta > 0$ only if $\{T'(\cdot)Z'X_2 - T(\cdot)\} < 0$ i.e. $E_T < (1 - \hat{Z}/Z)$. Using (10.1) it is then easy to check from (22) that $\hat{X}_2 < 0$ when $\hat{K} > 0$ and $\hat{X}_2 > 0$ when $\hat{\bar{Z}} < 0$.

Again with the help of (10.1) from (29) we find that $\hat{Y} < 0$ when $\hat{K} > 0$ if (i) $A > 0$; and, (ii) $B \geq 0$. On the other hand, $\hat{Y} > 0$ when $\hat{\bar{Z}} < 0$ under the same two sufficient conditions.

CASE II. Let us now concentrate on the case where the agricultural sector is more capital (less labour) intensive than the informal manufacturing sector. In this case, we have $(a_{K1}/a_{L1} > (a_{K2}/a_{L2})$. This implies that $(\lambda_{L1}\lambda_{K2} - \lambda_{K1}\lambda_{L2}) < 0$ and $|\theta| < 0$. There can be two sub-cases depending on the sign of $|\lambda|$, given by (26.2).

SUB-CASE 1. $|\theta| < 0$ and $|\lambda| > 0$. From (25) and (26.1) we find that $\Delta > 0$ and $\Delta' < 0$ (as$|\theta| < 0$). A sufficient condition for $\Delta' < 0$ is that $\{T'(\cdot)Z'X_2 - T(\cdot)\} \geq 0$ i.e. $E_T \geq (1 - \bar{Z}/Z)$.

From (22) one finds that $\hat{X}_2 > 0$ when $\hat{K} > 0$ and $\hat{X}_2 < 0$ when $\hat{\bar{Z}} < 0$. This from (27), in turn, implies that $\hat{Z} > 0$ when $\hat{K} > 0$ and $\hat{Z} < 0$ when $\hat{\bar{Z}} < 0$.

Then from (29) it follows that

$\hat{Y} < 0$ when $\hat{K} > 0$ if (i) $B < 0$; and, (ii) $A(T'(\cdot)Z'X_2 - T(\cdot)) \geq 0$.

On the contrary, $\hat{Y} > 0$ when $\hat{\bar{Z}} < 0$ if

$$[A\theta_{23}\{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\} - B\theta_{23}C)] \geq 0\,.$$

or, if

$$[A\theta_{23}\{\lambda_{K1}(\lambda_{L2} + \lambda_{L3} - h'(\cdot)Z'X_2/h(\cdot)) - \lambda_{L1}(\lambda_{K2} + \lambda_{K3})\} - (|\theta|\Delta P_2X_2)] \geq (>)0\,; \quad \text{and}, \quad B < (\leq)0\,.$$

SUB-CASE 2. $|\theta| < 0$; and, $|\lambda| < 0$. This implies that for stability of equilibrium in the market for commodity 2 we need: $\Delta < 0$, a necessary condition for which is: $\{T'(\cdot)Z'X_2 - T(\cdot)\} < 0$ i.e. $E_T < (1 - \bar{Z}/Z)$.

From (22) it then follows that $\hat{X}_2 < 0$ when $\hat{K} > 0$; and, $\hat{X}_2 > 0$ when $\hat{\bar{Z}} < 0$.

On the other hand, from (29) it follows that

$\hat{Y} < 0$ when $\hat{K} > 0$ if (i) $A \geq 0$; (ii) $B \geq 0$; and, (iii) either A or B is non-zero.

Also $\hat{Y} > 0$ when $\hat{\bar{Z}} < 0$ if (i) $A \geq 0$; and, (ii) $B \geq 0$.