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**TRIANGLES AND TRADE:  
LIONEL MCKENZIE AS A TRADE THEORIST**

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**Abstract:** In early work in the 1950's Lionel McKenzie utilized two versions of "trade triangles" in exploring efficient patterns of specialization in Ricardian models, on the one hand, and factor price equalization possibilities with free trade, on the other. In the former triangle the corners represented commodities and in the latter factors of production. The nature of McKenzie's contribution is described and, for each case, more recent use by others of these same triangles: Roy Ruffin for the goods triangle, in which labor is the only factor of production but each country has an endowment of all types of labor, and work by Jones, Marjit and Mitra for the factor triangle to illustrate strong Stolper-Samuelson possibilities.

**Key words:** Goods triangle, factor triangle, barycentric co-ordinates.

**JEL Classification Number:** F11, B20.

1. INTRODUCTION

"Historically the development of economic theory owes much to the theory of international trade." This was Paul Samuelson's remark in his first article on trade theory (1938). Such a remark fits well the contribution that Lionel McKenzie published over 50 years ago in *Econometrica*, "On Equilibrium in Graham's Model of World Trade and

*Acknowledgements.* This paper was delivered at a conference at Kyoto University in 2004 to honor Prof. Lionel McKenzie. Many of the attendees shared the experience of being either colleagues or students of the University of Rochester, and Michihiro Ohyama is one of the stars of the Rochester family. The present special issue of *Keio Economic Studies* gives me an opportunity to acknowledge my great debt to Prof. Ohyama, who was a student both of mine and of Prof. McKenzie, not only for his generous hospitality in Japan over the years, but also for his collaboration with me in our joint paper in 1995 ("Technology Choice, Overtaking and Comparative Advantage," *Review of International Economics*), and especially for his insights in his Ph. D. dissertation, part of which appeared in an earlier volume of the *Keio Economic Studies* ("Trade and Welfare in General Equilibrium", 1972). This is a classic work, and I have made extensive use of it in succeeding Supplements to the textbook I have co-authored with Richard Caves and Jeffrey Frankel. Ohyama was able to state and provide intuitively appealing proofs of basic propositions concerning gains from trade and how to compare utilities in two trading equilibria finite differences apart. This work provides a considerable extension of earlier results of Paul Samuelson and Murray Kemp and represents the best results in this area of trade theory.

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Other Competitive Systems". Left open after the pioneering work of Abraham Wald was the question of existence and uniqueness of a general equilibrium, and McKenzie offered solid proofs for a model of general equilibrium based on Ricardian trade theory and emphasized by his Princeton professor, Frank Graham. Independent of concurrent research by Kenneth Arrow and Gerard Debreu, this paper was first presented at the Econometric Society meetings in 1952. Another quote from Samuelson (1969) seems apt: "Our subject puts its best foot forward when it speaks out on international trade".

In the present article I do not attempt fully to survey McKenzie's contributions to the theory of international trade. Instead, I concentrate on a particular geometric device used by McKenzie to illustrate a couple of major contributions. This device is a *triangle*, one devoted to illustrating in two dimensions certain properties that hold in a three-dimensional world. There are two different types of such triangles, that I label the *goods triangle* and the *factor triangle*.<sup>1</sup> However, it is important to emphasize that he used these triangles only for illustrative purposes; his proofs are always of more general dimensionality. Nonetheless, in each case there as well have been subsequent uses for the technique, and I will describe both the original application and later adaptations.

## 2. THE GOODS TRIANGLE

The year 1954 marks the 50<sup>th</sup> anniversary of McKenzie's famous article extending the Ricardian model of international trade to the case of many countries and many commodities. In this article he built upon the work of Frank Graham and utilized some of the methods being developed in the area of activity analysis (Koopmans, 1951). Although his remarks are generally directed towards the case of many commodities and many countries, (and in each of these labor is the only factor of production), he illustrated his analysis by means of a "goods triangle". This is a two dimensional representation of what in three dimensions would be a world transformation surface for the case of three commodities and four countries.<sup>2</sup> The idea is more simply illustrated in the case of two commodities (and two countries). Suppose the home country has a comparative advantage in producing the first commodity. Then the world transformation schedule consists of two linear segments. The segment coming out of the vertical (commodity 2) axis replicates the Ricardian transformation curve for the home country. That is, if both countries initially produce only the second commodity (the point where the world transformation locus hits the vertical axis) and the price of commodity 1 gets increased from very low levels, the home country is the first one to switch labor to producing the first commodity, and only after it has completely specialized in commodity 1 and the price of the first commodity has sufficiently increased would the foreign country start to produce commodity 1. This *ordering* could be illustrated in a *one*-dimensional line,

<sup>1</sup> These triangles have nothing to do with standard welfare triangles.

<sup>2</sup> Another of Frank Graham's students at Princeton, Tom Whittin, displayed a 2-country, 3-commodity model, but in a two-dimensional diagram in which degrees of difference in comparative costs are illustrated (1953, p. 533). These are ignored in the McKenzie goods triangle.

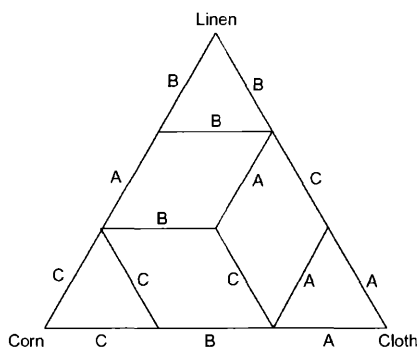


Figure 1.

as illustrated in Figure 1 along the edges of the trade triangle for the 3-commodity, 3-country case, where the information suppressed in lowering dimensionality is the extent to which one country's bilateral comparative cost ratio exceeds the other's.

In Figure 1 I have illustrated the goods triangle using the setting in Jones (1961) with three countries instead of the four used by McKenzie (1954b).<sup>3</sup> The Ricardian labor-cost per unit of output figures are those illustrated below:

Country:	A	B	C
Corn	10	10	10
Linen	5	7	3
Cloth	4	3	2

Consider, first, the ordering of countries *A*, *B*, and *C* if only corn and linen are produced. At the corn origin all countries produce corn, and the labor-cost figures in Table 1 reveal that country *C* has the greatest bilateral comparative advantage in producing linen (relative to corn), and country *B* the least. Such bilateral comparisons also suffice to yield the ordering of countries to commodities along the other two edges of the goods triangle where only two commodities are produced in the world. It is the interior assignment of countries completely specialized to different commodities that reveals potential inadequacies of purely bilateral comparisons to lead to the optimal assignment pattern of countries to commodities. The correct assignment, country *A* in corn, country *B* in cloth, and country *C* in linen, (illustrated in Figure 1) does indeed satisfy all the bilateral comparisons, but so does the inefficient candidate with *A* assigned to linen, *B* to corn

<sup>3</sup> The triangle shown in Figure 1 was actually not drawn in Jones (1961), but was illustrated in the Supplement to Chapter 5 of the Caves and Jones text, 3<sup>rd</sup> edition (1981). (It was kept in the text until the 8<sup>th</sup> edition). This diagram (along with variations on possible assignments in the triangle) was also discussed in Jones (1985).

(which would represent the optimal assignment if these were the only two countries and commodities) and country *C* to cloth. McKenzie realized the possibility that assignments that satisfied the bilateral comparisons need not be efficient in a multilateral setting when Richard Rosett pointed out to him the error he made in an internal assignment in his 4-country, 3-commodity diagram. Jones (1961) proved that the efficient world assignment in any *class* of assignments (in which it is specified how many countries are to specialize in each commodity) is the one that minimizes the product of labor input-output coefficients among all possible assignments in that class. The assignment shown in the interior of Figure 1 has the value 90 for the efficient product, and the alternative assignment in which all the bilateral cost comparisons are nonetheless satisfied yields a product of 100. And, as McKenzie emphasized in his earlier contribution, the entire world transformation surface can be mapped once knowledge of all the efficient patterns of complete specializations is obtained.

The goods triangle illustrated in Figure 1 actually suppresses two kinds of information that would be revealed in a genuine three-dimensional world transformation locus. As already mentioned, the extent of the differences in comparative costs among countries is sacrificed with only the order illustrated. As well, variations in country size (as well as absolute efficiencies of labor) are not considered. For example, consider the triangle near the corn origin in Figure 1, showing that if only small amounts of linen and cloth are to be produced in the world, it is country *C* alone that will release some labor from corn production to satisfy production of the other two commodities. Indeed, the anxiety for country *C* to escape corn production reveals that country *C* is, in a relative sense, a particularly bad choice for corn production. Similar remarks can be made for country *B* and linen and country *A* and cloth.<sup>4</sup> Now suppose that country *C* is relatively a very large country. Then in three dimensions the size of the triangle near the corn axis, where the slope of the world triangle reflects only the cost ratios for country *C*, would loom larger than the even-handed representation in Figure 1.

An alternative use for the McKenzie goods triangle can be employed to illustrate the ingenious re-configuration of the Ricardian setting devised by Roy Ruffin (1988). Suppose that labor is, indeed, the sole input required for production, with the labor input-output coefficient constant for each type of labor in producing each commodity. But now do not require each type of labor to live in just one country. That is, each country may find it has an endowment of all types of labor, each type with its own set of skills in producing each commodity. This alternative setting opens the door to a

<sup>4</sup> Triangular facets need not be found near all origins. Indeed, as illustrated in Jones (1985), a simple change in country *A*'s labor coefficient in cloth production from 4 to 1 alters the positioning of the trade triangles to reveal that parallelograms emerge from each corner while the base of each triangle is found in the middle of each edge. Whereas Figure 1 illustrates a case in which each country is relatively "bad" at producing a particular commodity, this change in numbering illustrates the situation in which each country is relatively "good" at producing a particular commodity. For example, given this numbering change, if the world initially desired a relatively balanced mixture of corn and linen, and no cloth, country *A* would produce both corn and linen. Then, if the world relative price of cloth were steadily to increase relative to the other two prices, country *A* would be the first country to start producing cloth, and only when it is completely specialized in cloth would countries *B* and *C* release labor for cloth production.

Heckscher–Ohlin type of model, in which it is the difference among countries in relative factor endowments that is key to trade patterns in a free trade world.

The multi-faceted *world* transformation surface of the pure Ricardian case is now illustrative of any particular *country's* transformation surface in the Ruffin setting, because any country has a composite endowment base consisting of all the types of labor found in the world. The difference between countries lies in the relative amount of each type of labor found there. Thus each country's multi-faceted transformation surface looks just like that of any other country *except* for the relative sizes of each facet.

Heckscher–Ohlin theory attempts to link differences in production and trade patterns to underlying differences in relative factor endowments, assuming away differences in technological knowledge between countries. The Ruffin setting makes this a particularly easy task. Now consider an arbitrary set of commodity prices. These define a price plane, and each country's production levels are determined by the “tangency” point between this price plane and that country's transformation surface. This represents a unique point for a country if it is “tangent” at a zero-dimensional facet, with each labor type specialized to a different commodity. Or, tangency may take place along a line (a one-dimensional facet), along which one type of labor is incompletely specialized to two different commodities, with all other types of labor completely specialized. In the 3-commodity case such a tangency could also take place along a planar 2-dimensional facet, with two types of labor incompletely specialized in their assignments. The important point to stress is that whatever the set of commodity prices selected, the pattern of assignments of each type of labor is the same among countries. This implies that a laborer of a specific type must earn the same return regardless of where it is located. That is, the factor-price equalization theorem, a particularly delicate result in the general Heckscher–Ohlin setting (holding only if factor-endowment proportions are fairly similar among countries), *must* hold in the Ruffin interpretation.<sup>5</sup> Furthermore, a comparison of output levels among countries is tantamount to a comparison of relative factor endowments (a strong version of the Heckscher–Ohlin theorem) with, in addition, a selection of labor assignments based on the principle of multilateral comparative advantage laid down in McKenzie's 1954b contribution.

### 3. THE FACTOR TRIANGLE

The question of factor-price equalization with trade was of central concern in McKenzie (1955). Although his proofs are quite general, not restricted by dimensionality conditions, the result is illustrated by the use of an equilateral triangle with each vertex corresponding to a particular factor of production in the 3-factor case. Each point in the factor triangle can represent the inputs of the three factors required to produce a

<sup>5</sup> The underlying rationale for such a strong result is found in the assumption that there is no “jointness” at the input level (just as there is assumed to be no joint production of outputs). Each laborer can produce commodities by itself. In typical Heckscher–Ohlin settings two or more factors must be combined to produce each commodity, so that the endowment proportions in which factors are found in one country must be fairly close to those found in other countries in order for free trade to eventuate in factor-price equalization. Ruffin's model is freed up from that requirement.

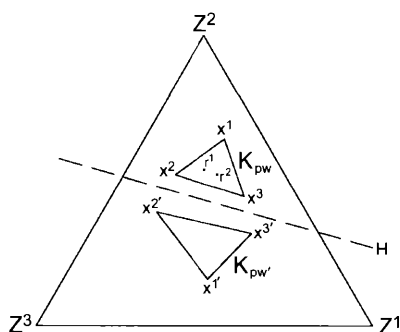


Figure 2.

particular commodity in an activity earning zero profits if factor prices are associated with the given commodity prices.

Some normalization is necessary, and for this let the sum of input quantities required for each activity be equal to unity.<sup>6</sup> One property of equilateral triangles is that at any point if perpendiculars are dropped to the three sides, the sum of the line segments is the same as that for any other point (e.g. the altitude of the triangle). Here it is convenient to let unity represent that common value. Each vertex corresponds to a particular factor, and the input of that factor for any activity (represented by a point in the triangle) is represented by the perpendicular distance to the side of the triangle away from that vertex. These are known as barycentric co-ordinates.

Figure 2 reproduces McKenzie's (1955) Figure 1. The  $Z^i$  denote the factors. Two inscribed production triangles are shown, each with vertices reflecting input requirements in a particular commodity. Suppose commodity prices are represented by  $\mathbf{p}$ , and zero profits would be earned by the three  $\mathbf{x}^i$  activities if the corresponding factor prices are given by the vector  $\mathbf{w}$ . The production triangle defined by these activities is  $\mathbf{K}_{\mathbf{p}\mathbf{w}}$ . There may exist another set of factor prices,  $\mathbf{w}'$ , consistent with zero profits being earned by a different set of three activities,  $\mathbf{x}^{i'}$ , defining a different production triangle,  $\mathbf{K}_{\mathbf{p}\mathbf{w}'}$ . McKenzie shows that these triangles cannot overlap because there must exist a separating hyperplane shown by line  $\mathbf{H}$ . The points  $\mathbf{r}^1$  and  $\mathbf{r}^2$  represent the factor endowments (also normalized) of two different countries that also lie within  $\mathbf{K}_{\mathbf{p}\mathbf{w}}$ . For any such pair of countries factor prices must be equalized (at value  $\mathbf{w}$ ) if they face the same commodity prices,  $\mathbf{p}$ . At these commodity prices the endowment point in any economy in which  $\mathbf{w}'$  is the factor price vector must lie in the separated production triangle,  $\mathbf{K}_{\mathbf{p}\mathbf{w}'}$ . Thus although the factor-price vector may not be uniquely related to the commodity price vector, all countries whose endowments lie close enough together (and that share the same technology) will have their factor prices equalized. *Inter alia*, this discussion

<sup>6</sup> McKenzie treated factor inputs as negative numbers and required the sum of absolute values to equal unity. Later we suggest a different normalization procedure, *viz.*, using factor distributive shares, whose sum will naturally be unity in any activity that earns zero profits.

helped to distinguish the question of factor price equalization in trade from the different question, perhaps of more interest to mathematicians, of the unique correspondence of factor prices to commodity prices.

Wilfred Ethier (1974) cited the four major propositions associated with the work of Heckscher and Ohlin. The first of these, the Heckscher–Ohlin theorem about the factor endowment basis for explaining the pattern of trade, is quite easily handled in the Ruffin (1988) version of McKenzie’s goods triangle discussed in the previous section. The second, the factor-price equalization theorem, is as well the subject of McKenzie’s factor triangle, as in the previous paragraphs. This factor triangle can also be used to illustrate, for the  $3 \times 3$  case, both of Ethier’s third and fourth results in Heckscher–Ohlin theory, viz. the Rybczynski (1955) theorem and the Stolper–Samuelson (1941) theorem. The use of a triangle diagram in factor space to discuss possible paths of factor growth in the  $3 \times 3$  case was discussed by Ed Leamer (1987), followed up by his remarks on the Stolper–Samuelson theorem (1994). Here I emphasize the Samuelson (1953) Reciprocity theorem that serves to link the Rybczynski theorem with the Stolper–Samuelson result, and the use of the factor triangle to explore issues in the expanded  $3 \times 3$  case. Earlier, both Murray Kemp and Leon Wegge (1969) and John Chipman (1969) proposed conditions that would allow versions of the Stolper–Samuelson theorem to be maintained in higher dimensions. They were able to do so in the  $3 \times 3$  case but their efforts for even higher dimensions were thwarted by the counter-examples they each provided. These issues were explored and illustrated with the factor triangle by Jones and Marjit (1991).<sup>7</sup>

Figure 3 re-labels the vertices of the factor triangle as labor (**L**), land (**T**) and capital (**K**). Illustrated is an inscribed production triangle defined by activities producing commodities  $x_1$ ,  $x_2$ , and  $x_3$ , activities that satisfy the competitive profit conditions with equality for a given set of commodity prices (and an assumed unique set of factor prices). Barycentric co-ordinates are adopted, as described earlier, with these co-ordinates now associated with distributive factor shares, adding to unity for each activity. Note that each activity is associated with a unique factor that it uses relatively intensively. For example, activity  $x_2$  requires a higher share of land (the distance from  $x_2$  to the **LK** axis) than it does of labor or capital (analogously shown). And in similar fashion  $x_1$  is labor-intensive and  $x_3$  is capital-intensive. Given these production techniques, the question that can be raised is whether the *strong* form of the Stolper–Samuelson theorem is satisfied, wherein the increase in *any* commodity price alone serves to raise the real return to the factor used intensively in its production as well as to reduce the return to the other two factors. (This is the version investigated by Kemp and Wegge.)

Since the factor triangle does not directly bring attention to commodity prices and factor prices, indeed they are givens in the diagram, the analysis of the Stolper–Samuelson theorem must be indirect, routed through the analysis of the Rybczynski theorem. This

<sup>7</sup> The extra conditions required to satisfy the Kemp–Wegge search for the strong form of the Stolper–Samuelson theorem were provided by Jones, Marjit and Mitra in the Becker, et. al. (eds.) 1993 festschrift volume in honor of Lionel McKenzie. Stronger conditions to satisfy the Chipman extension are found in Mitra and Jones (1999).



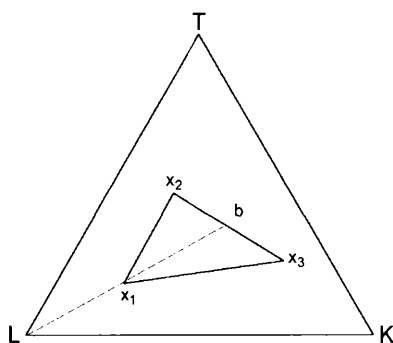


Figure 3.

latter result states that at given prices an increase in the endowment of any single factor will bring about a magnified expansion of the industry that makes intensive use of that factor while causing a reduction in all other outputs. Clearly an endowment change that would cause all outputs to expand at given prices must be shown in Figure 3 by a change that is a strict convex combination of the three activities, i.e. must lie within the production triangle. Consider, instead, an increase only in the supply of labor. To see how this affects outputs draw line segment  $Lx_1b$ . Since  $x_1$  is a convex linear combination of labor origin,  $L$ , and point  $b$ , point  $L$  must be a linear combination of  $x_1$  and  $b$ , but with a negative weight attached to  $b$ . The point  $b$  is itself a convex combination of points  $x_2$  and point  $x_3$ . The upshot is that an increase in the labor endowment by itself must be absorbed by an increase in the output of the first industry, but reductions in the outputs of the other two commodities.

In similar fashion a line segment drawn from the  $T$  origin through point  $x_2$  would hit the  $x_1x_3$  line at an interior point so that an increase in land endowment by itself would, at constant prices, eventuate in an increase in the output of the second commodity and a reduction in production of the other two commodities.

Paul Samuelson (1953) introduced the Reciprocity result, one that has been extremely useful in international trade theory, especially for situations in which there is factor growth or international factor mobility. If  $w_i$  represents the return to factor  $i$ ,  $V_i$  the endowment of factor  $i$  and  $p_j$  the price of commodity  $j$ , the reciprocity result states that:

$$\partial x_j / \partial V_i = \partial w_i / \partial p_j$$

With this in mind the preceding two exercises with endowment changes translate into strong Stolper–Samuelson results whereby an increase in the price of the first commodity raises the wage rate (by a magnified amount), but the wage rate would be reduced by a price rise in either of the other two commodities. As well, the return to land would be increased by a rise in the price of the second commodity, but lowered by an increase in the price of either commodity 1 or 3. But the strong Stolper–Samuelson results require as well that capital's return be raised by an increase in the price of the third commodity

but be depressed by an increase in the price of either of the other two commodities. In Figure 3 this is *not* the case. Following the earlier procedure, draw a ray from the **K** origin through the  $x_3$  point and observe that it does not hit the interior of the  $x_1x_2$  segment. Indeed, if this segment were extended (not shown), such a ray would hit the extension at a point north-east of  $x_2$ , and such a point would be a linear combination of  $x_1$  and  $x_2$ , with a negative weight attached to  $x_1$ . That is, an increase in **K** would actually serve to *raise* the output of the first commodity. By reciprocity, an increase in the price of the first commodity would raise the return to capital (as well as the wage rate), and this violates the strong form of the Stolper–Samuelson theorem.<sup>8</sup>

A useful construction of trade triangles that satisfy the strong Stolper–Samuelson theorem, and will do so in dimensions higher than  $3 \times 3$ , is based on the Jones and Marjit (1985, 1991) special case that they label the *Produced Mobile Factor* structure. It is based on the  $\{(n + 1) \times n\}$  specific-factor model. The difference is that the mobile factor (call it **M**) is assumed itself to be produced, with the help of all the  $n$  specific factors. For the  $3 \times 3$  case this is illustrated in Figure 4, which retains the factor constellation depicted in Figure 3. Point **M** has been chosen arbitrarily—it could be anywhere as long as land, labor, and capital (the presumed “specific” factors) are all involved in its production. The production points,  $x_i$  in Figure 4, reflect a convex combination of **M** with the appropriate factor origin. For example,  $x_1$  is any point on the **LM** line segment. And, since **M** is a convex combination of the  $x_i$ , by construction the extension of the **LM** segment must intersect the  $x_2x_3$  edge of the production triangle. The strong form of the Stolper–Samuelson theorem must be satisfied. In higher dimensions the argument remains intact since rays from each origin all pass through a common **M** point.<sup>9</sup>

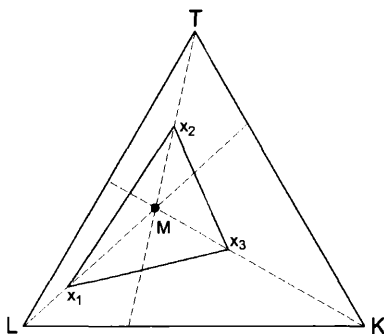


Figure 4.

<sup>8</sup> In my graduate trade seminars I encourage the students to be able to look at any inscribed production triangle and tell at a glance whether or not the strong form of the Stolper–Samuelson theorem holds for all price changes.

<sup>9</sup> In the Kemp–Wegge (1969) case rays from the origin in the  $3 \times 3$  case need not meet in a point but they would define a triangle, and production points could be selected on each ray closer to the appropriate factor origin. However, in higher dimensions these higher dimension “triangles” need not exist, and the Kemp–Wegge assumptions no longer prove sufficient. A more general sufficiency condition (than the Produced Mobile Factor case) is provided and proved in Jones, Marjit and Mitra (1993).

## 4. CONCLUDING REMARKS

The uses McKenzie made of the two types of trade triangles captures only a part of his contribution to international trade theory. As already noted, his proofs are based on a deeper analysis in higher dimensions than allowed by the triangle illustrations. In addition, in side-by-side contributions in 1967 he carefully stressed the difference between the factor-price equalization theorem that he discussed in 1955 and the technical concern with the possibility that cost functions can be inverted to yield a unique link between factor prices and commodity prices. In his 1968 contribution to the *International Encyclopedia for the Social Sciences*, McKenzie reviewed his own and other work that employed mathematical techniques in international trade theory. In the present paper I have narrowly focused on his clever use of trade triangles to allow three-dimensional properties to be captured in two-dimensional diagrams, a technique that has been extended by others to consider additional features of competitive trade theory.

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