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A TWO-COUNTRY DYNAMIC MODEL WITH ENDOGENOUS TIME PREFERENCES

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Abstract: This paper formulates a two-country by two-factor by two-good dynamic general equilibrium model of international trade with endogenous time preferences. After proving the existence, uniqueness and local saddlepoint stability of the steady state, we apply the present dynamic model to a trade issue concerning international technology transfer in order to show that the eect of once-for-all technology transfer upon the donor country may dier between the dynamic model and the standard static Heckscher-Ohlin model.

Key words: Dynamic Heckscher-Ohlin model, Endogenous time preferences, Hicks-Ikema theorem.

1. INTRODUCTION

The two-country by two-factor by two-good Heckscher–Ohlin (HO) model has been a basic general equilibrium framework of international trade for a long time. It has been developed to many directions by replacing some of the underlying assumptions with alternative ones. Based on his concept "virtual system", Wong[24)] discusses that even

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increasing returns and imperfect competition can be formally incorporated into the HO structure[1].

On the other hand, we have not had a parallel basic framework in dynamic trade theory. The straightforward extension of the HO model to a dynamic model with constant rate of time preference generates the following problems.

First, except for the "measure-zero" case such that the sum of the rates of time preference and capital depreciation is exactly the same among trading countries, it is impossible to have a steady state in which the production of each country is incompletely specialized. That is, either the country with a higher sum of the two rates is completely specialized to the production of labor-intensive goods or the country with a lower sum of the two rates is completely specialized to the production of capital-intensive goods[2].

Second and more seriously, the above complete specialization result hold *even if the international difference in the sum of the two rates is arbitrarily small.* thus, for example, if a slight change in the sum in one country reverses the ranking between the home and foreign countries, so do the production and trade structures of both countries in the long run[3].

Third, if we assume that all countries share a common sum of the two rates, the steady state is generally not unique. In fact, the contributions made by Chen[5)], Shimomura[18)] and Nishimura and Shimomura[16)] show that there exists a continuum of the steady states and it depends on the initial international distribution of physical capital which steady state the world economy converges to. As Devereux and Shi[6)] note, the existence of the continuum makes comparative statical analysis extremely difficult.

A couple of attempts have been made in order to avoid those difficulties. Shimomura[19),20)] studies trade pattern and indeterminacy in a dynamic two-country model where tradable goods consist of non-durable and durable goods. Hu and Shimomura[9)] construct a two-country dynamic model with status seeking and showed that the steady state is independent of the initial international distribution of capital stocks.

[1] For example, let us consider the model of monopolistic competition. As Kikuchi and Shimomura[15)] formulate, its closed economy can be expressed by the system of equations:

$$p\eta = a_{1w}w + a_{1r}r \quad \text{and} \quad 1 = a_{2w}w + a_{2r}r \tag{a1}$$

$$L = a_{1w}\eta Y_1 + a_{2w}Y_2 \quad \text{and} \quad K = a_{1r}\eta Y_1 + a_{2r}Y_2 \tag{a2}$$

$$Y_1 = E(p, Y_1, u), \tag{a3}$$

where (a1) are the price = average cost conditions for the monopolistically and perfectly competitive sectors, (a2) are the full employment conditions. Under the homothetic production function for the monopolistically competitive sector η is constant. a_{iy}, $i = 1, 2$ and $y = r, w$ are "virtual" input coecients. Y_1 is the total sum of the amounts of each variety produced in this country. thus, except that the expenditure function depends not only on the price and utility but also Y_1, the production structure is almost completely Heckscher–Ohlin.

[2] This complete specialization result was first obtained by Baxter[1)].

[3] A dynamic general equilibrium model with many agents and a competitive credit market has been studied by Becker[2)], Yano[22),23)] and Epstein[7)]. It is well known that the most patient agent owns the whole asset in the long run in such a dynamic model. A difference between those dynamic models and the present trade-theoretic model is in that the latter assumes away an international credit market.

While these models overcome the above difficulties, new problems come out. The model with durable consumption goods assumes away capital accumulation. The saving means the accumulation of durable consumption goods in the model. Once capital accumulation is introduced, the aforementioned problems returns. The model with status seeking, on the other hand, crucially depends on the externality term, which may deprive the model of generality.

Chen, Nishimura and Shimomura[4)] formulate a new two-country dynamic general equilibrium model of international trade by introducing the Uzawa endogenous time preference[21)] into the two-country Oniki-Uzawa model.[17)] They show that if the economic fundamentals like preferences, technologies and initial factor endowments are not internationally very dierent, there exists a unique and saddlepoint-stable steady state in which it is independent of the initial international distribution of capital and the production of both countries is incompletely specialized. Making use of the new dynamic trade model, they prove that (i) other things being equal, the country which is endowed with more labor exports the labor-intensive good and (ii) the country with higher (resp. lower) capital-labor ratio in the steady state exports the capital-intensive (resp. labor-intensive) good.

However, one may wonder whether there is any trade-specific issue for which we can obtain new results from the dynamic trade model that are not implied by static trade models. In this paper we discuss about the effect of technological progress upon the steady-state terms of trade. Specifically, we show that once-for-all uniform expansion of the production possibility set in one country does change the steady-state terms of trade even if there is no international trade before the technological progress. This comparative statical result sharply contrasts to the equilibrium in the static HO model with homothetic preferences, where an uniform expansion of the production possibility set in one country does not change the terms of trade if there is no international trade before the technological progress.

This difference in comparative statics has a serious implication for a technology-transfer issue in trade theory. As is discussed in the literature[4], in the static two-country model the technologically advanced donor country benefits from the technology transfer, irrespective of the factor-intensity ranking[5]. On the other hand in the dynamic trade model, the technology transfer reduces the steady-state welfare of the donor country.

This paper is organized as follows. Section 2 sets up the model. Section 3 and Appendix prove the existence, uniqueness and stability. Section 4 compares how once-and-for-all technology transfer from one country to the other may affect the international equilibrium price and the welfare level of the donor country between the present dynamic trade model and the standard static HO model. Section 5 concludes.

[4] See, among others, Hicks[8)], Ikema[10)], Kemp and Shimomura[14)], and Kemp, Ng, and Shimomura[13)]. See also Jones and Ruffin[12)] as a recent development of this issue.

[5] The result is sometimes called the Hicks–Ikema theorem.

2. THE MODEL

Let us describe the model. There are two countries, Home and Foreign, in the trading world where two commodities, a pure consumption good Y_1 (Good 1) and a consumable capital good Y_2 (Good 2), are produced by using a reproducible capital k and a primary and time-invariant factor of production, say labor, l. The consumable capital good can be either consumed as a non-durable good or added to the existing capital stock.

Labor is measured by efficiency unit. The Home (resp. Foreign) representative household supplies l (l^*) units of labor. The population of each country is assumed to be constant over time and normalized to be unity. Thus, the Home (resp. Foreign) household is endowed with l and k (resp. l^* and k^*) units of factors of production[6].

Following the standard trade theory, we assume away international factor movements. Moreover, in order to focus on international trade, we assume that there is no international credit market, while there is a competitive domestic credit market in each country.

The Home household maximizes the intertemporal sum of discounted utility

$$\int_0^{\infty} U(c_1, c_2) X dt \tag{1}$$

subject to

$$\dot{k} = F(p, k, l) - (pc_1 + c_2), \quad k(0) \text{ given} \tag{2}$$

$$\dot{X} = -\rho(U(c_1, c_2))X, \quad X(0) = 1, \tag{3}$$

where c_i, $i = 1, 2$, are the consumption of Good i.The first constraint (2) is the flow budget constraint the household is facing. $F(p, k, l)$ is the GDP function defined as

$$F(p, k, l) = \max_{l_i, k_i, i=1,2} pf^1(k_1, l_1) + f^2(k_2, l_2)$$

$$\text{subject to} \quad l \geq l_1 + l_2 \quad \text{and} \quad k \geq k_1 + k_2,$$

where $f^i(k_i, l_i)$, $i = 1, 2$, are the neoclassical CRS production functions[7]. Let $\Lambda^i(w, r)$, $i = 1, 2$, be the unit-cost function of Good i and $(w(p), r(p))$ is the solution to the price = unit-cost conditions

[6] An asterisk (*) is attached to each variable belonging to Foreign.

[7] It is well-known that capital accumulation process is expressed by the difference between capital good production and its consumption in a closed economy. This statement is not necessarily true in a multi-country case. Considering the definition of the GDP function, we can rewrite (2) as

$$\begin{aligned}\dot{k} &= pY_1 + Y_2 - (pc_1 + c_2)\\ &= (Y_2 - c_2) + p(Y_1 - c_1)\end{aligned}$$

Combining the corresponding Foreign flow budget constraint, $\dot{k}^* = p(Y_1^* - c_1^*) + Y_2^* - c_2^*$, and the world market-clearing condition $c_1 + c_1^* = Y_1 + Y_1^*$, we have

$$p(Y_1 - c_1) = (Y_2^* - c_2^*) - \dot{k}^*$$

Substituting this into the above Home flow budget constraint, we have

$$p = \Lambda^1(w, r) \tag{4}$$

$$1 = \Lambda^2(w, r) \tag{5}$$

It is well known that the GDP function has a portion of the straight line

$$F(p, k, l) = r(p)k + w(p)l \tag{6}$$

when the production is incompletely specialized.[8]

The second constraint (3) comes from the Uzawa formulation of endogenous time preference

$$X(t) = \exp\left[- \int_0^t \rho(U(c_1(\tau), c_2(\tau)))d\tau \right] \tag{7}$$

Following Uzawa[21)], we assume that the variable discount rate $\rho(U)$ satisfies

$$\begin{aligned} &\rho(0) > 0,\ \rho_u(u) \equiv \frac{d\rho(u)}{du} > 0,\ \rho_{uu}(u) \equiv \frac{d\rho^2(u)}{du^2} > 0 \\ &0 > \lim_{u\to\infty} \rho(u)/u < \infty\,, \\ &0 < u\rho_u(u)/\rho(u) < 1 \quad \text{for any positive } u < \infty \end{aligned} \tag{8}$$

The shape of $\rho(U)$ can be depicted as in Figure 1.

The felicity function is assumed to be homothetic

$$u = U(c_1, c_2) = h[V(c_1, c_2)]\,,$$

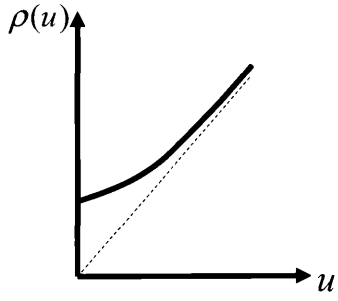

Figure 1. The Discount Rate Function.

$$\dot{k} = (Y_2 - c_2) + \{Y_2^* - (c_2^* + \dot{k}^*)\}$$

That is, the Home capital accumulation is equal to the sum of the term $(Y_2 - c_2)$ and the net import of the capital good from Foreign country.

The above equation can be rewritten as

$$\dot{k} + \dot{k}^* = (Y_2 + Y_2^*) - (c_2 + c_2^*)\,,$$

which means that the *world* capital accumulation is equal to the difference between capital good production and its consumption.

[8] A complete description of the GDP function is in the appendix of this paper.

where the function $V(c_1, c_2)$ is twice-continuously differentiable, increasing and linearly homogeneous in the two variables c_1 and c_2 and $h(V)$ satisfies

$$h(0) = 0,\ h_V \equiv \frac{dh(V)}{dV} > 0\,, \quad \text{and} \quad h_{VV}(V) \equiv \frac{dh^2(V)}{dV^2} < 0\,, \tag{9}$$

An example of $h(V)$ that satisfies (9) is $\ln(V + 1)$. The expenditure function that corresponds to this homothetic felicity function is multiplicatively separable,

$$E(p)\phi(u)\,,$$

where $\phi(u)$ is the inverse function of $h(V)$. (9) implies that

$$\phi(0) = 0,\ \phi_u \equiv \frac{d\phi(u)}{du} > 0\,, \quad \phi_{uu} \equiv \frac{d\phi^2(u)}{du^2} > 0\,, \quad \text{and}$$

$$1 < \frac{u\phi_u}{\phi} \tag{10}$$

Based on the foregoing preliminary argument, we now obtain the dynamic general equilibrium model. First, associated with the Home household's dynamic optimization problem is the Hamiltonian

$$H \equiv uX + \lambda[r(p)k + w(p)l - E(p)\phi(u)] - \theta\rho(u)X$$

The necessary conditions for optimality are

$$\frac{\partial H}{\theta u} = X\left[1 - \frac{\lambda}{X}E(p)\phi_u(u) - \theta\rho_u(u)\right] = 0$$

$$\dot{\lambda} = -\lambda r(p)$$

$$\dot{\theta} = \theta\rho(u) - u$$

and the transversality conditions are

$$\lim_{t\to\infty} \lambda(t)k(t) = \lim_{t\to\infty} \theta(t)X(t) = 0\,.$$

We can derive parallel conditions for the Foreign household.

Second, at each point in time the world market-clearing condition has to hold along an equilibrium path. That is, if the production in both countries is incompletely specialized, we have

$$\begin{aligned} 0 = r_p(p)k + r_p^*(p)k^* + w_p(p)l + w_p^*(p)l^* \\ - E_p(p)\phi(u) - E_p^*(p)\phi(u^*), \end{aligned}$$

where $r_p(p) \equiv dr(p)/dp$ and $w_p(p) \equiv dw(p)/dp$, respectively.

In what follows, we assume away factor-intensity reversal. In that case, if the pure-consumption good is more capital-intensive (resp. labor-intensive) than the consumable capital good, then for any $p > 0$ we have the Stolper–Samuelson relationships between price and factor prices,

$$pr_p(p)/r(p) > 1 > 0 > w_p(p) \ (\text{resp. } pw_p(p)/w(p) > 1 > 0 > r_p(p))$$

Following Jones[11], we shall call it the *magnification effect* that the elasticity of the factor price of a factor of production with respect to a commodity price is larger than one if the factor is intensively used in the production of the commodity.

Let us write the whole system

$$\dot{k} = r(p)k + w(p)l - E(p)\phi(u) \tag{11}$$

$$\dot{k}^* = r^*(p)k^* + w^*(p)l^* - E^*(p)\phi^*(u^*) \tag{12}$$

$$\dot{z} = z[\rho(u) - r(p)] \tag{13}$$

$$\dot{z}^* = z^*[\rho^*(u^*) - r^*(p)] \tag{14}$$

$$\dot{\theta} = \theta\rho(u) - u \tag{15}$$

$$\dot{\theta}^* = \theta^*\rho^*(u^*) - u^* \tag{16}$$

$$0 = 1 - zE(p)\phi_u(u) - \theta\rho_u(u) \tag{17}$$

$$0 = 1 - z^*E^*(p)\phi_u^*(u^*) - \theta^*\rho_u^*(u^*) \tag{18}$$

$$\begin{aligned} 0 = {} & r_p(p)k + r_p^*(p)k^* + w_p(p)l + w_p^*(p)l^* \\ & - E_p(p)\phi(u) - E_p^*(p)\phi^+(u^*), \end{aligned} \tag{19}$$

where $z \equiv \lambda/X$, and $\chi_p(p) \equiv d\chi(p)/dp$, $\chi = r, w$, and E. z^* and $\chi_p^*(p)$ is defined in a parallel way. The dynamic general equilibrium model consists of the nine unknowns, $k, k^*, z, z^*, \theta, \theta^*, u, u^*$ and p, and the nine equations.

3. THE STEADY STATE

The steady state is the solution for the system of equations

$$0 = r(p)k + w(p)l - E(p)\phi(u) \tag{20}$$

$$0 = r^*(p)k^* + w^*(p)l^* - E^*(p)\phi^*(u^*) \tag{21}$$

$$0 = \rho(u) - r(p) \tag{22}$$

$$0 = \rho^*(u^*) - r^*(p) \tag{23}$$

$$0 = \theta\rho(u) - u \tag{24}$$

$$0 = \theta^*\rho^*(u^*) - u^* \tag{25}$$

$$0 = 1 - zE(p)\phi_u(u) - \theta\rho_u(u) \tag{26}$$

$$0 = 1 - z^*E^*(p)\phi_u^*(u^*) - \theta^*\rho_u^*(u^*) \tag{27}$$

$$0 = r_p(p)k + r_p^*(p)k^* + w_p(p)l + w_p^*(p)l^*$$
$$- E_p(p)\phi(u) - E_p^*(p)\phi^*(u^*) \tag{28}$$

Let us focus on the steady state. For a given p, if

$$\rho(0) < r(p),$$

then there exists a unique and positive u such that $\rho(u) = r(p)$. Let $u(.)$ and $u^*(.)$ be the inverse functions of $\rho(.)$ and $\rho^*(.)$, respectively. Since the shadow prices, z, z^*, θ, and θ^*, are derived once the above system of equations determines p, k and k^*, we see that the main system consists of the three equations.

$$E(p)\phi(u(r(p))) = r(p)k + w(p)l \tag{29}$$

$$E^*(p)\phi^*(u^*(r^*(p))) = r^*(p)k^* + w^*(p)l^* \tag{30}$$

$$0 = r_p(p)k + r_p^*(p)k^* + w_p(p)l + w_p^*(p)l^*$$
$$- E_p(p)\phi(u(r(p))) - E_p^*(p)\phi^*(u^*(r^*(p))), \tag{31}$$

The first two equations are the budget constraints of Home and Foreign and the last one is the world market-clearing condition for the pure consumption good. Now, let us state our first main result.

PROPOSITION 1. *The steady state with incomplete specialization in both countries uniquely exists and is locally saddlepoint-stable, if the economic fundamentals are not very different between Home and Foreign. The dependence of the steady-state endogenous variables on the parameters in the model is smooth so that we can make comparative statical analysis by differentiating the steady-state endogenous variables with respect to the parameters.*

Proof. See Appendix.

Just for comparison, let us recall the standard static HO model. Under the same assumptions concerning GDP and expenditure functions, we derive the following system of equations.

$$E(p)\phi(u) = r(p)k + w(p)l \tag{32}$$

$$E^*(p)\phi^*(u^*) = r^*(p)k^* + w^*(p)l^* \tag{33}$$

$$0 = r_p(p)k + r_p^*(p)k^* + w_p(p)l + w_p^*(p)l^*$$
$$- E_p(p)\phi(u) - E_p^*(p)\phi^*(u^*), \tag{34}$$

where the unknowns are u, u^* and p.

4. THE PRICE EFFECT OF A TECHNOLOGY TRANSFER

Let us focus on a simple case in which the preferences and labor endowments are the same between the two countries, $E(.) \equiv E^*(.)$, $\phi(.) \equiv \phi^*(.)$, $\rho(.) \equiv \rho^*(.)$, $l = l^*$, and Foreign is the technologically less advanced country in the sense that the average cost functions are expressed as $\Lambda^i(r/\xi.w/\xi)$, where ξ, a positive technological parameter. It follows that

$$r^*(p) = \xi r(p) \text{ and } w^*(p) = \xi w(p)$$

4.1. The dynamic trade model

Then, the steady-state system of equations, (29)–(31), are rewritten as

$$E(p)\phi(u(r(p))) = r(p)k + w(p)l \tag{35}$$

$$E(p)\phi(u(\xi r(p))) = \xi r(p)k^* + \xi w(p)l \tag{36}$$

$$\begin{aligned} 0 = {} & r_p(p)k + \xi r_p(p)k^* + w_p(p)l + \xi w_p(p)l \\ & - E_p(p)\phi(u(r(p))) - E_p(p)\phi(u(\xi r(p))), \end{aligned} \tag{37}$$

It is clear that if ξ is initially equal to one, $k = k^*$. Let us check the relationship between ξ and p. From (35) and (36), the steady-state capital stocks are expressed as

$$k(p) \equiv \frac{E(p)\phi(u(r(p))) - w(p)l}{r(p)}$$

$$k^*(p,\xi) \equiv \frac{E(p)\phi(u(\xi r(p))) - \xi w(p)l}{\xi r(p)}$$

The substitution of them into (37) yields

$$\begin{aligned} S(p,\xi) = {} & \frac{r_p(p)[E(p)\phi(u(r(p))) - w(p)l]}{r(p)} + w_p(p)l - E_p(p)\phi(u(r(p))) \\ & + \frac{r_p(p)[E(p)\phi(u(\xi r(p))) - \xi w(p)l]}{r(p)} + \xi w_p(p)l - E_p(p)\phi(u(\xi r(p))) \\ = {} & 0 \end{aligned}$$

LEMMA 1. $\frac{\partial}{\partial p}S(p,\xi)\big|_{\xi=1} > 0$ *and*

$$sign\left[\frac{\partial}{\partial \xi}S(p,\xi)\bigg|_{\xi=1}\right] = sign\left[\frac{pr_p}{r} - \frac{pw_p}{w}\right]$$

Proof. First, it is clear from the property of the GDP and expenditure functions that

$$\begin{aligned} & r_{pp}(p)k(p) + w_{pp}(p)l - E_{pp}(p)\phi(u(r(p))) \\ & = r_{pp}(p)k^*(p,1) + w_{pp}(p)l - E_{pp}(p)\phi(u(r(p))) > 0 \end{aligned}$$

Second, considering $E_p(p)\phi(u(r(p))) = r_p(p)k + w_p(p)l$ when $\xi = 1$, we see that

$$r_p(p)\frac{dk(p)}{dp} - E_p(p)\frac{d\phi(u(r(p)))}{dp} = \frac{\phi_u u_r E}{pr} r_p\left[\frac{pr_p}{r} - \frac{pE_p}{E}\right] > 0,$$

irrespective of the factor-intensity ranking. It follows from these two inequalities that $\frac{\partial}{\partial p}ES(p,\xi)\big|_{\xi=1} > 0$, as was to be proved. (QED)

Next, let us partially differentiate $S(p,\xi)$ with respect to ξ at $\xi = 1$.

$$\frac{\partial}{\partial \xi}S(p,\xi)\bigg|_{\xi=1} = \frac{r_p E\phi_u u_r r}{r} - E_p\phi_u u_r r + l\left[w_p(p) - \frac{r_p(p)w(p)}{r(p)}\right]$$
$$= \phi_u u_r r\left[\frac{r_p E}{r} - E_p\right] + l\left[w_p(p) - \frac{r_p(p)w(p)}{r(p)}\right] \tag{38}$$

Considering

$$0 = \frac{r_p[E\phi - wl]}{r} + w_p l - E_p\phi$$
$$= \phi\left[\frac{r_p E}{r} - E_p\right] * l\left[w_p - \frac{r_p w}{r}\right],$$

we can continue calculations as follows.

$$(38) = \phi_u u_r r\left[\frac{r_p E}{r} - E_p\right] - \phi\left[\frac{r_p E}{r} - E_p\right]$$
$$= \frac{E^2\phi}{p}\left[\frac{pr_p}{r} - \frac{pE_p}{E}\right]\left[\frac{\phi_u u_r r}{\phi} - 1\right]$$
$$= \frac{E^2\phi}{p}\left[\frac{pr_p}{r} - \frac{pE_p}{E}\right]\left[\frac{u\phi_u/\phi}{u\rho_u/\rho} - 1\right],$$

where use is made of $u_r = 1/\rho_u$ and $r = \rho$. Since $u\phi_u/\phi > 1$ and $0 < u\rho_u/\rho < 1$,

$$\frac{u\phi_u/\phi}{u\rho_u/\rho} - 1 > 0,$$

It follows that

$$sign\left[\frac{\partial}{\partial \xi}S(p,\xi)\bigg|_{\xi=1}\right] = sign\left[\frac{pr_p}{r} - \frac{pE_p}{E}\right]$$
$$= sign\left[\frac{pr_p}{r} - \frac{pw_p}{w}\right]$$

(QED)

Based on this lemma, we obtain the main result of this paper.

PROPOSITION 2. *Suppose that Home and Foreign are symmetrical except that the technology parameter ξ is very close to but smaller than one. Suppose that there is a technology transfer from Home to Foreign so that ξ slightly increases toward one. The steady-state price of the pure consumption good rises (resp. declines) according as the pure consumption good is more labor (resp. capital) intensive.*

Proof. Totally differentiating $S(p,\xi) = 0$ with respect to p and ξ, we see that

$$\frac{dp}{d\xi}\bigg|_{\xi=1} = -\frac{\dfrac{\partial}{\partial \xi}S(p,\xi)\bigg|_{\xi=1}}{\dfrac{\partial}{\partial p}S(p,\xi)\bigg|_{\xi=1}}$$

It follows from Lemma 1 that

$$sign\left[\frac{dp}{d\xi}\bigg|_{\xi=1}\right] = sign\left[\frac{pw_p}{w} - \frac{pr_p}{r}\right], \tag{39}$$

as was to be proved. (QED)

From this proposition, we can check how the small change in ξ may affect the steady-state welfare of Home which is

$$\bar{U} = \int_0^\infty uXdt = u(r(p))\int_0^\infty \exp[-\rho(u(r(p))t]dt$$
$$= \frac{u(r(p))}{\rho(u(r(p))}$$

Logarithmically differentiating $\bar{U}$ with respect to ξ at $\xi = 1$, we have

$$\frac{p}{\bar{U}}\frac{d\bar{U}}{dp}\bigg|_{\xi=1} = \left[\frac{1}{u\rho_u/\rho} - 1\right]\frac{pr_p}{r}\frac{1}{p}\frac{dp}{d\xi}\bigg|_{\xi=1} \tag{40}$$

Note that $0 < u\rho_u/\rho < 1$. Combining (40) with (39), we see that

$$sign\left[\frac{p}{\bar{U}}\frac{d\bar{U}}{d\xi}\bigg|_{\xi=1}\right] = sign[r_p]sign\left[\frac{pw_p}{w} - \frac{pr_p}{r}\right] < 0$$

We now arrive at the second main result.

PROPOSITION 3. *If the two countries are sufficiently symmetrical with each other, the above small technology transfer reduces the steady-state welfare level of the donor country.*

4.2. *The static HO model*

Now let us compare the results we obtained with those based on the static HO model. The static system (32)–(34) is rewritten as

$$E(p)\phi(u) = r(p)k + w(p)l \tag{41}$$

$$E(p)\phi(u^*) = \xi r(p)k^* + \xi w(p)l^* \tag{42}$$

$$0 = r_p(p)k + \xi r_p(p)k^* + w_p(p)l + \xi w_p(p)l^*$$
$$- E_p(p)\phi(u) - E_p(p)\phi(u^*), \tag{43}$$

Substituting $\phi(u)$ and $\phi(u^*)$ in (41) and (42) into (43), we obtain

$$\bar{S}(p,\xi) \equiv r_p(p)k + w_p(p)l - \frac{E_p(p)}{E(p)}(r(p)k + w(p)l)$$
$$\xi\left[r_p(p)k^* + w_p(p)l^* - \frac{E_p(p)}{E(p)}(r(p)k^* + w(p)l^*)\right]$$
$$= 0$$

Totally differentiating $\bar{S}(p,\xi) = 0$ with respect to p and ξ at $\xi = 1$,

$$\bar{S}_p(p,\xi)\big|_{\xi=1}\,dp + \left[r_p(p)k^* + w_p(p)l^* - \frac{E_p(p)}{E(p)}(r(p)k^* + w(p)l^*)\right]d\xi = 0$$

or

$$\left.\frac{dp}{d\xi}\right|_{\xi=1} = \frac{r_p(p)k + w_p(p)l - \dfrac{E_p(p)}{E(p)}(r(p)k + w(p)l)}{\bar{S}_p(p,\xi)\big|_{\xi=1}},$$

where the denominator is positive. Whether the numerator is positive or negative does depend on, due to the Heckscher–Ohlin theorem, the international difference in the relative factor endowments, $k/l \lesseqgtr k^*/l^*$. For example, if $k/l = k^*/l^*$, then the volume of trade

$$r_p(p)k + w_p(p)l - \frac{E_p(p)}{E(p)}(r(p)k + w(p)l)$$

must be zero. So is $\frac{dp}{d\xi}\big|_{\xi=1}$. That is,

$$sign\left[\left.\frac{dp}{d\xi}\right|_{\xi=1}\right] = sign\left[r_p(p)k + w_p(p)l - \frac{E_p(p)}{E(p)}(r(p)k + w(p)l)\right]$$

The following proposition holds.

PROPOSITION 4. *In the static HO model, whether the technology transfer, i.e., a rise in the technology parameter ξ, affects the equilibrium commodity price or not depends on whether the trade volume is null or not. If the relative factor endowments in both countries have a common relative factor endowment, the equilibrium commodity price is independent of ξ.*

The Home welfare effect of a technology transfer can be easily obtained. From the budget constraint of the donor country

$$E(p)\phi_u du = \left[r_p(p)k + w_p(p)l - \frac{E_p(p)}{E(p)}(r(p)k + w(p)l)\right]dp$$

It follows that

$$\left.\frac{du}{d\xi}\right|_{\xi=1} = \frac{\left[r_p(p)k + w_p(p)l - \dfrac{E_p(p)}{E(p)}(r(p)k + w(p)l)\right]^2}{\bar{S}_p(p,\xi)\big|_{\xi=1}},$$

which is nonnegative. Particularly, it is strictly positive when the trade volume is nonzero.

PROPOSITION 5 (The Hicks-Ikema theorem). *The technology transfer generally raises the welfare of the donor country.*

5. CONCLUDING REMARKS

In this paper we formulate a two-country by two-factor by two-commodity dynamic general equilibrium model of international trade in which both commodities are consumable and the preferences over them are expressed by a homothetic utility function. After we proved the existence, uniqueness and stability of the steady state with incomplete specialization in both countries, we made comparative statical analysis focusing on the effects of a uniform expansion of the production possibility set in one country on the international commodity price and the welfare of the other country. We found that while the latter country is generally better off in the static HO model, the same expansion reduces the steady-state welfare of the donor country in the dynamic model.

Physical capital is an important reproducible factor of production in the real world. Therefore, our present exercises suggests that there may exist some trade issues for which it is difficult to justify the use of static HO model based on the "for simplicity" argument. That is, in some cases it might be difficult to say "I employ a static HO model just for simplicity, since a dynamic extension of it merely makes my analysis complicated without changing main results derived from the static mode".

Needless to say, our exercises may not necessarily mean that the Hicks-Ikema theorem is invalid in the dynamic framework. We need to take into account the welfare effect along the transitional path after the expansion of the production possibility set, which is one of our next research agenda. We would also like to apply the present dynamic trade model to other trade issues that have been studied in static trade models in order to derive new insights and understandings of those issues.

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6. APPENDIX: EXISTENCE, UNIQUENESS AND STABILITY OF THE STEADY STATE WITH INCOMPLETE SPECIALIZATION IN BOTH COUNTRIES

Here, we shall prove the existence, uniqueness and stability of the steady state with incomplete specialization in the present two-country dynamic general equilibrium model. We shall focus on the symmetric case such that preferences, technology, initial factor endowments are common between Home and Foreign. As we shall show later, the determinant of the Jacobian at symmetrical steady state is not zero, which implies that as long as the international differences in those economic fundamentals are not very large, the existence, uniqueness and stability are guaranteed.

6.1. Existence

Let us focus on the symmetric case.

$$0 = r(p)k + w(p)l - E(p)\phi(u(p)) \tag{44}$$

$$0 = \theta\rho(u(p)) - u \tag{45}$$

$$0 = 1 - zE(p)\phi_u(u(p)) - \theta\rho_u(u(p)) \tag{46}$$

$$0 = r_p(p)k + w_p(p)l - E_p(p)\phi(u(p)) \tag{47}$$

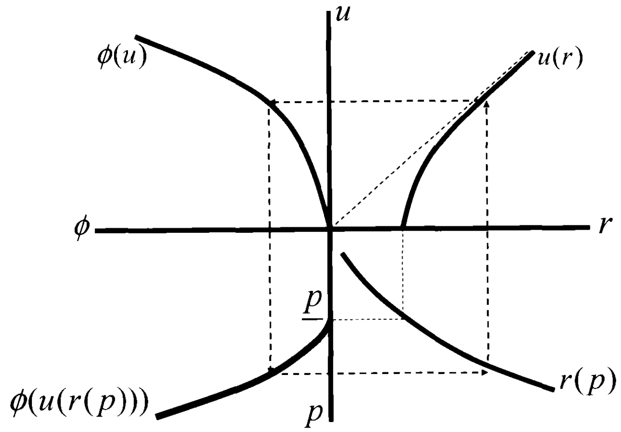

Figure A1. $\phi(u(r(p)))$ under $r_p(p) > 0$.

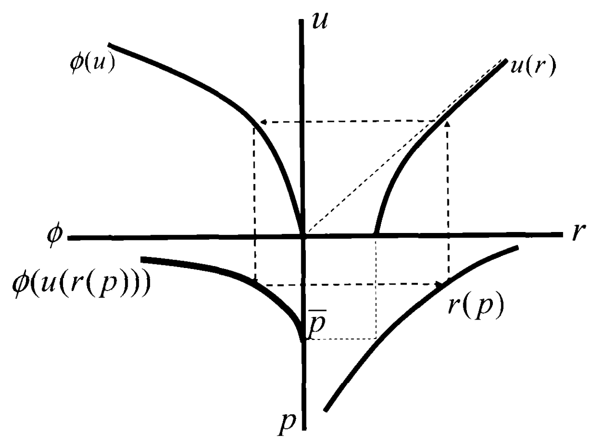

Figure A2. $\phi(u(r(p)))$ under $r_p(p) < 0$.

See Figure A1 and A2. In order that u is uniquely determined, p must be larger (resp. smaller) than $\underline{p}$ (resp. $\bar{p}$) if the pure consumption good is more capital (labor)-intensive than the consumable capital. For such a given p, however, (44), (45) and (46) uniquely determines k, θ and z. What remains to consider as to the existence of the steady state is whether (47) determines p. Making use of (44), we can rewrite (47) to

$$S(p) \equiv \frac{1}{p}\left[\left\{\frac{pr_p(p)}{r(p)} - \frac{pE_p(p)}{E(p)}\right\}E(p)\phi(u(p)) + \left\{\frac{pw_p(p)}{w(p)} - \frac{pr_p(p)}{r(p)}\right\}w(p)l\right] \tag{48}$$

First, it is clear from Figure A1 that if the pure consumption good is more capital-intensive than the consumable capital, i.e., if $w_p < 0 < r_p$,

$$S(\underline{p}) = \left\{ \frac{\underline{p} w_p(\underline{p})}{w(\underline{p})} - \frac{\underline{p} r_p(\underline{p})}{r(\underline{p})} \right\} \frac{w(\underline{p}) l}{\underline{p}} < 0 \tag{49}$$

Considering the assumed properties of $\phi(u)$ and $u(r)$, we see that $\lim_{p\to\infty} E(p)\phi(u(p)) = \infty$. Therefore, if

$$\sup_{p>\underline{p}} \frac{p r_p(p)}{r(p)} > 1 , \tag{50}$$

then for a sufficiently large p, $S(p)$ is positive. It follows that there exists a positive p^e in the open interval $(\underline{p}, \infty)$ such that $S(p^e) = 0$.

A parallel argument can be made in the case such that the pure consumption good is more labor-intensive than the consumable capital, i.e., $w_p > 0 > r_p$. We can show that under (50) $S(p) < 0$ for a sufficiently small p. It follows that there exists p^e between 0 and $\overline{p}$ such that $S(p^e) = 0$.

Second, let us show that the steady-state capital stock is positive, i.e.,

$$\begin{aligned} k^e &\equiv \frac{E(p^e)\phi(u(r(p^e))) - w(p^e)l}{r(p^e)} \\ &= \frac{E_p(p^e)\phi(u(r(p^e))) - w_p(p^e)l}{r_p(p^e)} > 0 \end{aligned} \tag{51}$$

If $w_p < 0 < r_p$, $k^e > 0$ is immediately obtained. So, let us focus on the other case such that $w_p > 0 > r_p$. Suppose that.

$$E(p^e)\phi(u(r(p^e))) - w(p^e)l < 0 < p[E_p(p^e)\phi(u(r(p^e))) - w_p(p^e)l]$$

It follows that

$$\begin{aligned} 0 &> E(p^e)\phi(u(r(p^e))) - w(p^e)l \\ &\quad - p[E_p(p^e)\phi(u(r(p^e))) - w_p(p^e)l] \\ &= \{E(p^e) - p^e E_p(p^e)\}\phi(u(r(p^e))) \\ &\quad - \{w(p^e) - p^e w_p(p^e)\}l , \end{aligned}$$

which is positive due to the concavity of $E(p)$ and the magnification effect $\frac{p^e w_p(p^e)}{w(p^e)} > 1$, a contradiction. Therefore $k^e > 0$ even if $w_p > 0 > r_p$.

(51) implies

$$\begin{aligned} &\{r(p^e) - p^e r_p(p^e)\}k^e + \{w(p^e) - p^e w_p(p^e)\} \\ &\quad = \{E(p^e) - p^e E_p(p^e)\}\phi(u(r(p^e))) > 0 \end{aligned}$$

Since $r_p(p)k + w_p(p)l = E_p(p)\phi(u(p)) > 0$, both goods are produced. That is, incomplete specialization is also guaranteed at p^e.

LEMMA A1. *Under the foregoing assumptions, there exists a steady-state price p^e such that both goods are produced.*

6.2. Uniqueness

Differentiating $S(p)$ with respect to p at p^e,

$$\left.\frac{dS(p)}{dp}\right|_{p=p^e} = [r_{pp}(p^e)k^e + w_{pp}(p^e)l]$$
$$* E(p^e)\phi_u(u(r(p^e))u_r(r(p^e))r_p(p^e)\left[\frac{p^e r_p(p^e)}{r(p^e)} - 1\right],$$

where the first term at the RHS is positive due to the convexity of the GDP function with respect to p and the second term is positive due to the magnification effect. Therefore, p^e is a unique steady-state price under incomplete specialization in both countries.

What remains to be argued concerning uniqueness is to exclude a steady state such that at least one country is completely specialized. For this purpose, let us consider the whole GDP function. Let us focus on the case such that the pure consumption good is more capital-intensive than the consumable capital good. Then the GDP function can be expressed as follows.

$$F(p,k,l) = \begin{cases} f^2(k,l) & \text{for } 0 < k < k_2(p) \\ r(p)k + w(p)l & \text{for } k_2(p) \le k \le k_1(p) \\ pf^1(k,l) & \text{for } k > k_1(p), \end{cases}$$

where

$$k_1(p) \equiv \frac{w(p) - pw_p(p)}{pr_p(p) - r(p)} \quad \text{and} \quad k_2(p) \equiv -\frac{w_p(p)}{r_p(p)}[9]$$

Using the GDP function, the Home budget constraint is

$$\dot{k} = G(p,k,l)$$
$$\equiv F(p,k,l) - E(p)\phi(u(F_k(p,k,l)))$$

Partially differentiating $G(p,k,l)$ with respect to k, we see that

$$\frac{\partial G(p,k,l)}{\partial k} = \frac{\partial F(p,k,l)}{\partial k} - E(p)\phi_u u_r \frac{\partial^2}{\partial k^2}F(p,k,l),$$

which is always positive due to the definition of $F(p,k,l)$.Therefore, there exists at most one k such that $G(p,k,l) = 0$. Since Home and Foreign are assumed to be symmetric, if there were a steady state such that at least one country is completely specialized, we would have to at least two capital stocks k^e and k^{ee} such that at the steady-state price p^e

$$G()p^e, k^e, l) = G(p^e, k^{ee}, l) = 0,$$

a contradiction.

LEMMA A2. *There exists a unique steady state where both countries are incompletely specialized. There is no steady state at which at least one country is completely specialized.*

[9] Recall that $r_p k + w_p$ is the output of the pure consumption good.

6.3. Stability

Let us consider the Jacobian matrix of the steady state,

$$\begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 & -E\phi_u & 0 & 0 \\
0 & \rho & 0 & 0 & 0 & 0 & 0 & -E\phi_u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & z\rho_u & 0 & -zr_p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & z\rho_u & -zr_p \\
0 & 0 & 0 & 0 & \rho & 0 & -Ez\phi_u & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho & 0 & -Ez\phi_u & 0 \\
0 & 0 & -E\phi_u & 0 & -\rho_u & 0 & -Ez\phi_{uu}-\theta\rho_{uu} & 0 & -E_p z\phi_u \\
0 & 0 & 0 & -E\phi_u & 0 & -\rho_u & 0 & -Ez\phi_{uu}-\theta\rho_{uu} & -E_p z\phi_u \\
r_p & r_p & 0 & 0 & 0 & 0 & -E_p\phi_u & -E_p\phi_u & \Delta
\end{bmatrix}$$

where

$$\Delta \equiv 2r_{pp}k + 2w_{pp}l - 2E_{pp}(p)\phi(u) > 0$$

The characteristic equation is

$$0 = \Omega(x) \equiv$$

$$\begin{bmatrix}
\rho-x & 0 & 0 & 0 & 0 & 0 & -E\phi_u & 0 & 0 \\
0 & \rho-x & 0 & 0 & 0 & 0 & 0 & -E\phi_u & 0 \\
0 & 0 & -x & 0 & 0 & 0 & z\rho_u & 0 & -zr_p \\
0 & 0 & 0 & -x & 0 & 0 & 0 & z\rho_u & -zr_p \\
0 & 0 & 0 & 0 & \rho-x & 0 & -Ez\phi_u & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho-x & 0 & -Ez\phi_u & 0 \\
0 & 0 & -E\phi_u & 0 & -\rho_u & 0 & -Ez\phi_{uu}-\theta\rho_{uu} & 0 & -E_p z\phi_u \\
0 & 0 & 0 & -E\phi_u & 0 & -\rho_u & 0 & -Ez\phi_{uu}-\theta\rho_{uu} & -E_p z\phi_u \\
r_p & r_p & 0 & 0 & 0 & 0 & -E_p\phi_u & -E_p\phi_u & \Delta
\end{bmatrix}$$

Let us make the following calculations to the above determinant.

- Subtract the first row multiplied by $(\frac{r_p}{\rho-x})$ from the ninth row and the second row multiplied by $(\frac{r_p}{\rho-x})$ from the ninth row.
- Subtract the third row multiplied by $(\frac{E\phi_u}{x})$ from the seventh row and the fourth row multiplied by $(\frac{E\phi_u}{x})$ from the eighth row.
- Add the fifth row multiplied by $(\frac{\rho_u}{\rho-x})$ from the seventh row and the sixth row multiplied by $(\frac{\rho_u}{\rho-x})$ from the eighth row.

Then, we see that

$$\Omega(x) = (\rho - x)^4 x^2 \times \begin{vmatrix} -Ez\phi_{uu} - \theta\rho_{uu} - \dfrac{\rho_u Ez\phi_u}{\rho - x} - \dfrac{E\phi_u z\rho_u}{x} & 0 & -E_p z\phi_u + \dfrac{E\phi_u z r_p}{x} \\ 0 & -Ez\phi_{uu} - \theta\rho_{uu} - \dfrac{\rho_u Ez\phi_u}{\rho - x} - \dfrac{E\phi_u z\rho_u}{x} & -E_p z\phi_u + \dfrac{E\phi_u z r_p}{x} \\ -E_p\phi_u + \dfrac{r_p E\phi_u}{\rho - x} & -E_p\phi_u + \dfrac{r_p E\phi_u}{\rho - x} & \Delta \end{vmatrix}$$

$$= (\rho - x)^2 \times \begin{vmatrix} -(Ez\phi_u + \theta\rho_{uu})x(\rho - x) - E\phi_u z\rho_u\rho & 0 & z\phi_u(Er_p - E_p x) \\ 0 & -(Ez\phi_u + \theta\rho_{uu})x(\rho - x) - E\phi_u z\rho_u\rho & z\phi_u(Er_p - E_p x) \\ -E_p\phi_u(\rho - x) + r_p E\phi & -E_p\phi_u(\rho - x) + r_p E\phi_u & \Delta \end{vmatrix}$$

$$= (\rho - x)^2 \times \begin{vmatrix} -(Ez\phi_{uu} + \theta\rho_{uu})x(\rho - x) - E\phi_u z\rho_u\rho & 0 & z\phi_u(Er_p - E_p x) \\ (Ez\phi_{uu} + \theta\rho_{uu})x(\rho - x) + E\phi_u z\rho_u\rho & -(Ez\phi_{uu} + \theta\rho_{uu})x(\rho - x) - E\phi_u z\rho_u\rho & z\phi_u(Er_p - E_p x) \\ 0 & \{E_p x + (r_p E - rE_p)\}\phi_u & \Delta \end{vmatrix}$$

$$= (\rho - x)^2[(Ez\phi_{uu} + \theta\rho_{uu})x^2 - \rho(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u\rho] \times \begin{vmatrix} 1 & 0 & z\phi_u(Er_p - E_p x) \\ -1 & -(Ez\phi_{uu} + \theta\rho_{uu})x(\rho - x) - E\phi_u z\rho_u\rho & z\phi_u(Er_p - E_p x) \\ 0 & \{E_p x + (r_p E - rE_p)\}\phi_u & \Delta \end{vmatrix}$$

$$= (\rho - x)^2[(Ez\phi_{uu} + \theta\rho_{uu})x^2 - \rho(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u\rho] \times \begin{vmatrix} 1 & 0 & z\phi_u(Er_p - E_p x) \\ 0 & (Ez\phi_{uu} + \theta\rho_{uu})x(x - \rho) - E\phi_u z\rho_u\rho & 2z\phi_u(Er_p - E_p x) \\ 0 & \{E_p x + (r_p E - rE_p)\}\phi_u & \Delta \end{vmatrix}$$

$$= -(\rho - x)^2[(Ez\phi_{uu} + \theta\rho_{uu})x^2 - \rho(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u\rho] \times \begin{vmatrix} (Ez\phi + \theta\rho_{uu})x(\rho - x) + E\phi_u z\rho_u\rho & -2z\phi_u(E_p x - Er_p) \\ \{E_p x + (r_p E - rE_p)\}\phi_u & \Delta \end{vmatrix}$$

$$= (\rho - x)^2[(Ez\phi_{uu} + \theta\rho_{uu})x^2 - \rho(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u\rho]$$
$$\times[\Delta\{(Ez\phi_{uu} + \theta\rho_{uu})x^2 - \rho(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u\rho\}$$
$$+2z\phi_u^2(E_p x - Er_p)\{E_p x + (r_p E - E_p\rho)\}]$$
$$= (\rho - x)^2[(Ez\phi_{uu} + \theta\rho_{uu})x^2 - r(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u r]$$
$$\times[\{\Delta(Ez\phi_{uu} + \theta\rho_{uu}) + 2E_p^2\phi_u^2 z\}x^2$$
$$-\{\Delta(Ez\phi_{uu} + \theta\rho_{uu}) - 2z\phi_u^2 E_p^2)\}rx$$
$$-E\phi_u z\{\Delta r\rho_u + 2\phi_u r_p(r_p E - E_p r)\}] \quad \because \quad r = \rho$$

Now consider the following two equations.

$$\Omega_1(x) \equiv (Ez\phi_{uu} + \theta\rho_{uu})x^2 - r(Ez\phi_{uu} + \theta\rho_{uu})x - E\phi_u z\rho_u r = 0$$

$$\Omega_2(x) \equiv \{\Delta(Ez\phi_{uu} + \theta\rho_{uu}) + 2E_p^2\phi_u^2 z\}x^2$$
$$-\{\Delta(Ez\phi_{uu} + \theta\rho_{uu}) - 2z\phi_u^2 E_p^2)\}rx$$
$$-E\phi_u z\{\Delta r\rho_u + 2\phi_u r_p(r_p E - E_p r)\}$$

First, since$Ez\phi_{uu} + \theta\rho_{uu} > 0$ and $E\phi_u z\rho_u r > 0$, the equation $\Omega_1(x) = 0$ has one positive and one negative real roots. Second, due to the Stolper–Samuelson theorem and the property of the unit expenditure function $0 < \frac{pE_p}{E} < 1$, we see that

$$r_p(r_p E - E_p r) = \frac{Err_p}{p}\left(\frac{pr_p}{r} - \frac{pE_p}{E}\right) > 0$$

Therefore, $E\phi_u z\{\Delta r\rho_u + 2\phi_u r_p(r_p E - E_p r)\}$. It follows that $\Omega_2(x) = 0$ also has one positive and one negative real roots, which implies that the characteristic equation

$$\Omega(x) = (\rho - x)^2 \Omega_1(x)\Omega_2(x) = 0$$

has two negative real roots and four positive real roots. Since the state variables are k and k^*, the steady state is locally saddlepoint-stable.

LEMMA A3. *The steady state is locally saddlepoint-stable.*

REMARK We can check that $\Omega(0) \neq 0$. It follows that the implicit function theorem ensures us that even if the economic fundamentals of Home and Foreign are slightly different with each other, the existence, uniqueness and stability of the steady state are established.