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INDETERMINACY IN CONTINUOUS-TIME TWO-SECTOR MODELS: AN EXPOSITION

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Abstract: The aim of this paper is to discuss the role of the elasticity of capital-labor substitution on the local determinacy properties of the steady state in a two-sector economy with CES technologies and sector-specific externalities.

Key words: Sector-Specific Externalities, Constant Returns, Capital-Labor Substitution, Indeterminacy.
***Journal of Economic Literature* Classification Numbers:** C62, E32, O41.

I. INTRODUCTION

Recently, a large number of papers have established the fact that locally indeterminate equilibria and sunspots fluctuations arise within two-sector infinite-horizon growth models with sector specific external effects in production and linear utility function.[1] Benhabib and Nishimura[3)] prove within a continuous-time model with Cobb–Douglas technologies that the existence of local indeterminacy is obtained if and only if there is a reversal of the factor intensities between the private and the social levels. The consumption goods has indeed to be capital intensive from the private perspective but labor intensive from the social perspective.

When CES technologies with symmetric elasticities of capital-labor substitution across sectors are considered, Nishimura and Venditti[5)] have recently proved that local indeterminacy actually requires a reversal of the factor intensities between the private

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[1] See Benhabib and Farmer.[1),2)]

and the quasi social levels, the quasi social level being defined from the input coefficients evaluated at the equilibrium.[2]

The aim of this paper is to introduce asymmetric elasticities in a continous-time model and to give an overview of the role of factor intensities and the elasticity of capital-labor substitution on the existence of local indeterminacy.

The paper is organized as follows: Section 2 presents the basic model with the production structure, the intertemporal equilibrium, the steady state and the characteristic roots. In Section 3 we provide some results on the existence of local indeterminacy depending on the choice of the elasticities of capital-labor substitution in each sector. Section 4 finally contains some concluding comments.

2. THE MODEL

2.1. *The production structure*

We consider an economy producing a pure consumption good y_0 and a pure capital good y_1. Each good is assumed to be produced by labor x_{0j} and capital x_{1j}, $j = 0, 1$, through a CES technology which contains sector specic externalities. The representative firm in each industry indeed faces the following function:

$$y_j = (\beta_{0j}x_{0j}^{-\rho_j} + \beta_{1j}x_{1j}^{-\rho_j} + e_j(X_{0j}, X_{1j}))^{-1/\rho_j}, \quad j = 0, 1 \tag{1}$$

with $\beta_{ij} > 0, \rho_j > -1$ and $\sigma_j = 1/(1 + \rho_j) \geq 0$ the elasticity of substitution. The positive externalities are equal to

$$e_j(X_{0j}, X_{1j}) = b_{0j}X_{0j}^{-\rho_j} + b_{1j}X_{1j}^{-\rho_j}$$

with $b_{ij} \geq 0$ and X_{ij} denoting the average use of input i in sector j. We assume that these economy-wide averages are taken as given by each individual firm. At the equilibrium, since all firms of sector j are identical, we have $X_{ij} = x_{ij}$ and we may define the *social production functions* as follows

$$y_j = (\hat{\beta}_{0j}x_{0j}^{-\rho_j} + \hat{\beta}_{1j}x_{1j}^{-\rho_j})^{-1/\rho_j} \tag{2}$$

with $\hat{\beta}_{ij} = \beta_{ij} + b_{ij}$. The returns to scale are therefore constant at the social level, and decreasing at the private level. We assume that in each sector $j = 0, 1$, $\hat{\beta}_{0j} + \hat{\beta}_{1j} = 1$ so that the production functions collapse to Cobb–Douglas in the particular case $\rho_j = 0$. Labor is normalized to one, i.e. $x_{00} + x_{01} = 1$, and the total stock of capital is given by $x_1 = x_{10} + x_{11}$.

Choosing the consumption good as the numeraire, i.e. $p_0 = 1$, a firm in each industry maximizes its profit given the output price p_1, the rental rate of capital w_1 and the wage rate w_0. Its profit is:

$$\pi_j = p_j y_j - w_0 x_{0j} - w_1 x_{1j}$$

The first order conditions subject to the private technologies (1) are

[2] When Cobb–Douglas technologies are considered, the quasi social coefficients are equivalent to the social coefficients.

$$p_j \beta_{ij} (y_j / x_{ij})^{1+\sigma_j} = w_i\,, \quad i, j = 0, 1 \tag{3}$$

From (3) we have

$$x_{ij}/y_j = (p_j \beta_{ij}/w_i)^{\frac{1}{1+\rho_j}} \equiv a_{ij}(w_i, p_j)\,, \quad i, j = 0, 1 \tag{4}$$

We call a_{ij} the input coefficients from the *private* viewpoint. If the agents take account of externalities as endogenous variables in profit maximization, the first order conditions subject to the social technologies (2) are

$$p_j \hat{\beta}_{ij} (y_j / x_{ij})^{1+\rho_j} = w_i\,, \quad i, j = 0, 1$$

and the input coefficients become

$$\bar{a}_{ij}(w_i, p_j) = p_j \hat{\beta}_{ij}/w_i)^1/(1+\rho_j)\,, \quad i, j = 0, 1 \tag{5}$$

We call $\bar{a}_{ij}$ the input coefficients from the *social* viewpoint. We also define

$$\hat{a}_{ij}(w_i, p_j) \equiv (\hat{\beta}_{ij}/\beta_{ij}) a_{ij}(w_i, p_j) \tag{6}$$

as the *quasi* input coefficients from the *social* viewpoint, and it is easy to derive that

$$\hat{a}_{ij}(w_i, p_j) = \bar{a}_{ij}(w_i, p_j)(\hat{\beta}_{ij}/\beta_{ij}/)^{\rho_j/(1+\rho_j)}$$

Notice that $\hat{a}_{ij} = \bar{a}_{ij}$ if there is no externality coming from input i in sector j, i.e. $b_{ij} = 0$, or if the production function is Cobb–Douglas, i.e. $\rho_j = 0$. As we will show below, the factor-price frontier, which gives a relationship between input prices and output prices, is not expressed with the input coefficients from the social viewpoint but with the quasi input coefficients from the social viewpoint.[3]

LEMMA 1. *Denote* $p = (1, p1)'$, $w = (w_0, w_1)$ *and* $\hat{A}(w, p) = [\hat{a}_{ij}(w_i, p_j)]$. *Then* $p = \hat{A}'(w, p)w$.

The factor market clearing equation depends on the input coefficients from the private perspective.

LEMMA 2. *Denote* $x = (1, x_1)'$, $y = (y_0, y_1)'$ *and* $A(w, p) = a_{ij}(w_i, p_j)$. *Then* $A(w, p)y = x$.

These Lemmas imply that the rental rate is a function of the output price only, $w_1 = w_1(p_1)$, while outputs are functions of the capital stock and the output price, $y_j = y_j(x_1, p_1)$, $j = 0, 1$.

We now examine some comparative statics. The factor-price frontier satisfies the Stolper–Samuelson theorem:

LEMMA 3. $dw_1/dp_1 = \frac{\hat{a}_{00}}{\hat{a}_{11}\hat{a}_{00} - \hat{a}_{10}\hat{a}_{01}}$.

The factor market clearing equation finally satisfies the Rybczynski theorem:

LEMMA 4. $dy_1/dx_1 = \frac{a_{00}}{a_{11}a_{00} - a_{10}a_{01}}$.

[3] See Nishimura and Venditti.[5)]

Without external effects, i.e. $b_{ij} = 0$, we have $\hat{A}(w, p) = A(w, p)$. The Rybczynski and Stolper–Samuelson theorems are equivalent since $[\partial_y/\partial_x] = [\partial_w/\partial_p]'$. However, in presence of externalities, the Rybczynski effects depend on the input coefficients from the private perspective while the Stolper–Samuelson effects depend on the quasi input coefficients from the social perspective. The duality between these two effects is thus destroyed since true costs are not being minimized. Local indeterminacy of equilibria will be a consequence of this property.

2.2. Intertemporal equilibrium and steady state

A representative agent optimizes a linear additively separable utility function with discount rate $\delta \geq 0$. This problem can be described as:

$$\max_{\{x_{ij}(t)\}} \int_0^{+\infty} (\beta_{00}x_{00}(t)^{-\rho_0} + \beta_{10}x_{10}(t)^{-\rho_0} + e_0(X_{00}(t), X_{10}(t)))^{-\frac{1}{\rho_0}} e^{-\delta t} dt$$

$$\begin{aligned}
s.t. \quad & y_1(t) = (\beta_{01}x_{01}(t)^{-\rho_1} + \beta_{11}x_{11}(t)^{-\rho_1} + e_1(X_{01}(t), X_{11}(t)))^{-\frac{1}{\rho_1}} \\
& \dot{x}_1(t) = y_1(t) - gx_1(t) \\
& 1 = x_{00}(t) + x_{01}(t) \\
& x_1(t) = x_{10}(t) + x_{11}(t) \\
& x_1(0) \text{ given} \\
& \{e_j(X_{0j}(t), X_{1j}(t))\}_{t \geq 0}, \quad j = 0, 1, \quad \text{given}
\end{aligned}$$

where $g > 0$ is the depreciation rate of the capital stock. We can write the modified Hamiltonian in current value as:

$$\begin{aligned}
\mathcal{H} = & (\beta_{00}x_{00}(t)^{-\rho_0} + \beta_{10}x_{10}(t)^{-\rho_0} + e_0(X_{00}(t), X_{10}(t)))^{-\frac{1}{\rho_0}} \\
& + w_0(t)(1 - x_{00}(t) - x_{01}(t)) + w_1(t)(x_1(t) - x_{10}(t) - x_{11}(t)) \\
& + p_1(t)((\beta_{01}x_{01}(t)^{-\rho_1} + \beta_{11}x_{11}(t)^{-\rho_1} + e_1(X_{01}(t), X_{11}(t)))^{-\frac{1}{\rho_1}} - gx_1(t))
\end{aligned}$$

The static first order conditions are given by equations (3). The necessary conditions which describe the solution to the optimization problem are given by the following equations of motion:

$$\begin{aligned}
\dot{x}_1(t) &= y_1(x_1(t), p_1(t)) - gx_1(t) \\
\dot{p}_1(t) &= (\delta + g)p_1(t) - w_1(p_1(t))
\end{aligned} \tag{7}$$

Any solution $x_1(t), p_1(t)_{t \geq 0}$ that also satisfies the transversality condition

$$\lim_{t \to +\infty} e^{-\delta t} p_1(t)x_1(t) = 0$$

is called an equilibrium path.

A steady state is defined by a pair (x_1^*, p_1^*) solution of

$$\begin{aligned}
y_1(x_1, p_1) &= gx_1 \\
w_1(p_1) &= (\delta + g)p_1
\end{aligned} \tag{8}$$

We introduce the following restriction on parameters' values:

ASSUMPTION 1. *$\beta_{11} > \delta + g$ and $\rho_1 \in (\hat{\rho}_1, +\infty)$ with*

$$\hat{\rho}_1 \equiv \frac{ln\hat{\beta}_{11}}{ln(\frac{\beta_{11}}{\delta+g}) - ln\hat{\beta}_{11}} \in (-1, 0) \tag{9}$$

Considering the fact that, within continuous-time models, the discount rate δ and the capital depreciation rate g are quite small, the restriction $\beta_{11} > \delta + g$ does not appear to be too demanding. Assumption 1 precisely guarantees positiveness and interiority of all the steady state values for input demand functions x_{ij}. Moreover it allows to prove the following result:

PROPOSITION 1. *Under Assumption 1, there exists a unique steady state $(x_1^*, p_1^*) > 0$ such that*

$$x_1^* = \frac{\left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\frac{1}{1+\rho_0}} \left(\frac{\left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{\rho_1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{1+\rho_1}{\rho_1(1+\rho_0)}}}{1 - \left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{1}{1+\rho_1}} g\left[1 - \left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\frac{1}{1+\rho_0}} \left(\frac{\left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{\rho_1-\rho_0}{\rho_1(1+\rho_0)}}\right]}$$

$$p_1^* = \frac{\beta_{10}}{\delta + g}\left[\hat{\beta}_{00}\left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\frac{\rho_0}{1+\rho_0}} \left(\frac{\left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{\rho_0(1+\rho_1)}{\rho_1(1+\rho_0)}} + \hat{\beta}_{10}\right]^{-\frac{1+\rho_0}{\rho_0}}$$

Proof. The detailed arguments for the explicit computation of (x_1^*, p_1^*) are given in Garnier, Nishimura and Venditti.[4] To prove positivity of x_1^* consider its denominator, denoted

$$\mathcal{D} = 1 - \left(\frac{\beta_{11}}{\delta + g}\right)^{\frac{1}{1+\rho_1}} gb$$

If $b \leq 0$, $\mathcal{D}$ is obviously positive and so is x_1^*. If on the contrary $b > 0$, we necessarily have $b \in (0, 1)$ and thus:

$$\mathcal{D} > 1 - \left(\frac{\beta_{11}}{\delta + g}\right)^{\frac{1}{1+\rho_1}} g > 1 - \left(\frac{\beta_{11}}{\delta + g}\right)^{\frac{1}{1+\rho_1}} \left(\frac{\delta + g}{\beta_{11}}\right)\beta_{11} > 1 - \left(\frac{\beta_{11}}{\delta + g}\right)^{\frac{-\rho_1}{1+\rho_1}} \hat{\beta}_{11}$$
$$> \left(\frac{\beta_{11}}{\delta + g}\right)^{\frac{-\rho_1}{1+\rho_1}} \left[\left(\frac{\beta_{11}}{\delta + g}\right)^{\frac{\rho_1}{1+\rho_1}} - \hat{\beta}_{11}\right] > 0$$

□

As shown in Nishimura and Venditti,[5] a restriction concerning the admissible values of ρ_1 similar to Assumption 1 needs to be introduced in discrete-time models to guarantee the existence of a steady state. However, an upper bound instead of a lower bound has to be considered. It follows that contrary to discrete-time models, the limit case of a Leontief technology in the investment good sector can be considered in a continuous-time framework.

2.3. *Characteristic roots and factor intensity differences*

We start by linearizing the dynamical system (7) around (x_1^*, p_1^*):

$$J = \begin{pmatrix} \frac{\partial y_1}{\partial x_1}(x_1^*, p_1^*) - g & \frac{\partial y_1}{\partial p_1}(x_1^*, p_1^*) \\ 0 & -\frac{\partial w_1}{\partial p_1}(p_1^*) + \delta + g \end{pmatrix}$$

Any solution from (7) that converges to the steady state (x_1^*, p_1^*) satisfies the transversality condition and is an equilibrium. Therefore, given $x_1(0)$, if there is more than one initial price $p_1(0)$ in the stable manifold of (x_1^*, p_1^*), the equilibrium path from $x_1(0)$ will not be unique. In particular, if J has two roots with negative real parts, there will be a continuum of converging paths and thus a continuum of equilibria.

DEFINITION 1. If the locally stable manifold of the steady state (x_1^*, p_1^*) is two-dimensional, then (x_1^*, p_1^*) is said to be locally indeterminate.

The roots of J are given by the diagonal terms. We know from Lemmas 3 and 4 that $\partial y_1/\partial x_1$ corresponds to the factor intensity difference from the private viewpoint and $\partial w_1/\partial p_1$ corresponds to the quasi factor intensity difference from the social viewpoint. Using the definitions of input coefficients given in Section 2, we may indeed interpret the elements of $\partial y_1/\partial x_1$ and $\partial w_1/\partial p_1$ as follows:

DEFINITION 2. The consumption good is said to be:
i) capital intensive at the private level if and only if $a_{11}a_{00} - a_{10}a_{01} < 0$,
ii) quasi capital intensive at the social level if and only if $\hat{a}_{11}\hat{a}_{00} - \hat{a}_{10}\hat{a}_{01} < 0$,
iii) capital intensive at the social level if and only if $\bar{a}_{11}\bar{a}_{00} - \bar{a}_{10}\bar{a}_{01} < 0$.

We may thus relate the input coefficients to the CES parameters:[4]

PROPOSITION 2. *Let Assumption 1 hold. At the steady state:*

i) *the consumption good is capital (labor) intensive from the private perspective if and only if*

$$b \equiv 1 - \left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\frac{1}{1+\rho_0}} \left(\frac{\left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{1+\rho_1}{\rho_1(1+\rho_0)}} < (>)0$$

ii) *the consumption good is quasi capital (labor) intensive from the social perspective if and only if*

$$\hat{b} \equiv 1 - \frac{\hat{\beta}_{10}\hat{\beta}_{01}}{\hat{\beta}_{00}\hat{\beta}_{11}} \left(\frac{\beta_{00}\beta_{11}}{\beta_{10}\beta_{01}}\right)^{\frac{\rho_0}{1+\rho_0}} \left(\frac{\left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{1+\rho_1}{\rho_1(1+\rho_0)}} < (>)0$$

iii) *the consumption good is capital (labor) intensive from the social perspective if and only if*

$$\bar{b} \equiv 1 - \left(\frac{\hat{\beta}_{10}\hat{\beta}_{01}}{\hat{\beta}_{00}\hat{\beta}_{11}}\right)^{\frac{1}{1+\rho_0}} \left(\frac{\left(\frac{\hat{\beta}_{11}}{\delta+g}\right)^{\frac{1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{1+\rho_1}{\rho_1(1+\rho_0)}} < (>)0$$

4 See Garnier, Nishimura and Venditti.[4)]

3. LOCAL INDETERMINACY

The following Proposition establishes that local indeterminacy requires a reversal of factor intensities between the private and the quasi social levels.

PROPOSITION 3. *Let Assumption 1 hold. The steady state is locally indeterminate if and only if the consumption good is capital intensive from the private perspective (i.e. $b < 0$), but quasi labor intensive from the social perspective (i.e. $b > 0$).*

3.1. Cobb–Douglas technologies

Cobb–Douglas technologies are obtained when $\rho_0 = \rho_1 = 0$. In this case Assumption 1 holds. It follows also from Proposition 2 that the capital intensity difference at the private level depends on the sign of the following expression

$$b = 1 - \frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}$$

while the capital intensities are the social and quasi-social levels are identical (*i.e.* $\bar{b} = \hat{b}$) and depend on the sign of the following expression

$$\hat{b} = 1 - \frac{\hat{\beta}_{10}\hat{\beta}_{01}}{\hat{\beta}_{00}\hat{\beta}_{11}}$$

We easily conclude from these expressions that the consumption good is capital intensive at the private level if and only if $\beta_{10}\beta_{01} > \beta_{00}\beta_{11}$ and labor intensive at the social/quasi-social level if and only if $\hat{\beta}_{10}\hat{\beta}_{01} < \hat{\beta}_{00}\hat{\beta}_{11}$. We then derive from Proposition 3:[5]

COROLLARY 1. *The steady state is locally indeterminate if and only if $\beta_{10}\beta_{01} > \beta_{00}\beta_{11}$ and $\hat{\beta}_{10}\hat{\beta}_{01} < \hat{\beta}_{00}\hat{\beta}_{11}$.*

3.2. Symmetric CES technologies

Symmetric CES technologies are obtained when $\rho_0 = \rho_1 = \rho$. It follows also from Proposition 2 that the capital intensity difference at the private level depends on the sign of the following expression

$$b = 1 - \left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\frac{1}{1+\rho}}$$

the capital intensity at the quasi-social level depends on the sign of the following expression

$$\hat{b} = 1 - \frac{\hat{\beta}_{10}\hat{\beta}_{01}}{\hat{\beta}_{00}\hat{\beta}_{11}}\left(\frac{\beta_{00}\beta_{11}}{\beta_{10}\beta_{01}}\right)^{\frac{\rho}{1+\rho}}$$

and the capital intensity at the social level depends on the sign of the following expression

[5] See Benhabib and Nishimura.[3)]

$$\bar{b} = 1 - \left(\frac{\hat{\beta}_{10}\hat{\beta}_{01}}{\hat{\beta}_{00}\hat{\beta}_{11}}\right)^{\frac{1}{1+\rho}}$$

Now the capital intensity differences between sectors at the quasi-social levels also depend on the parameter ρ. We then derive from Proposition 3:[6]

COROLLARY 2. *Let Assumptions* 1 *hold. Then the steady state is locally indeterminate if and only if*

$$\frac{\beta_{00}\beta_{11}}{\beta_{10}\beta_{01}} < 1 < \left(\frac{\hat{\beta}_{00}\hat{\beta}_{11}}{\hat{\beta}_{10}\hat{\beta}_{01}}\right)^{1+\rho}\left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\rho}$$

As long as the condition in Corollary 2 is satisfied, both technologies may be close to Leontief functions. Note also that under this condition we have

$$\frac{\beta_{00}\beta_{11}}{\beta_{10}\beta_{01}} < \frac{\hat{\beta}_{00}\hat{\beta}_{11}}{\hat{\beta}_{10}\hat{\beta}_{01}}$$

3.3. *Asymmetric CES technologies*

If we allow for different elasticities of capital-labor substitution across industries, additional cases may be obtained.[7]

COROLLARY 3. *Under Assumption 1, the steady state is locally indeterminate if and only if*

$$\frac{\beta_{00}\beta_{11}}{\beta_{10}\beta_{01}} < \left(\frac{\left(\frac{\beta_{11}}{\delta+g}\right)^{\frac{\rho_1}{1+\rho_1}} - \hat{\beta}_{11}}{\hat{\beta}_{01}}\right)^{\frac{\rho_1-\rho_0}{\rho_1}} < \left(\frac{\hat{\beta}_{00}\hat{\beta}_{11}}{\hat{\beta}_{10}\hat{\beta}_{01}}\right)^{1+\rho_0}\left(\frac{\beta_{10}\beta_{01}}{\beta_{00}\beta_{11}}\right)^{\rho_0}$$

As long as the condition in Corollary 3 is satisfied, a Cobb–Douglas technology in the consumption good sector may be considered while the technology in the investment good sector is Leontief.

4. CONCLUDING COMMENTS

In this paper, we have considered a two-sector growth model with sector-specic externalities. Comparing three types of formulation for the technologies, depending on whether the production functions are Cobb–Douglas, CES with symmetric elasticities of capital-labor substitution or CES with asymmetric elasticities of capital-labor substitution, we have studied the role of the factor substitutability properties on the local indeterminacy. Our results show that a variety of combinations of production functions with externalities may produce indeterminacy of equilibria.

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