<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Growing challenges : a Ricardo-Vanek model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sub Title</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Author</strong></td>
<td>Uchupalanun, Jakrapong Wan, Henry, Jr.</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>Keio Economic Society, Keio University</td>
</tr>
<tr>
<td><strong>Publication year</strong></td>
<td>2005</td>
</tr>
<tr>
<td><strong>Jtitle</strong></td>
<td>Keio economic studies Vol.42, No.1/2 (2005. ) ,p.51- 68</td>
</tr>
<tr>
<td><strong>Abstract</strong></td>
<td>This paper presents a Ricardo-Vanek model of international trade, where the presence of a &quot;home good bias&quot; in demand implies the possible existence of a multiplicity of real equilibrium exchange rates, when successful growth makes developing economies &quot;graduated&quot; from their small country status. They emerge to become relevant in the determination of the world market prices. Observation in Crisis 1997 is consistent with such an explanation. The currently rapid growth of both China and India makes the potential upheaval of the world economy a matter of serious challenge.</td>
</tr>
<tr>
<td><strong>Notes</strong></td>
<td>Special issue in honor of Professor Michihiro Ohyama</td>
</tr>
<tr>
<td><strong>Genre</strong></td>
<td>Journal Article</td>
</tr>
</tbody>
</table>

The copyrights of content available on the KeiO Associated Repository of Academic resources (KOARA) belong to the respective authors, academic societies, or publishers/issuers, and these rights are protected by the Japanese Copyright Act. When quoting the content, please follow the Japanese copyright act.
GROWING CHALLENGES—A RICARDO–VANEK MODEL

Jakrapong UCHUPALANUN

Faculty of Economics, Thammasat University, Bangkok, Thailand

and

Henry WAN, Jr.

Department of Economics, Cornell University, Ithaca, USA

and IEAS, Nankang, Taiwan

First version received October 2005; final version accepted February 2006

Abstract: This paper presents a Ricardo-Vanek model of international trade, where the presence of a ‘home good bias’ in demand implies the possible existence of a multiplicity of real equilibrium exchange rates, when successful growth makes developing economies ‘graduated’ from their small country status. They emerge to become relevant in the determination of the world market prices. Observation in Crisis 1997 is consistent with such an explanation. The currently rapid growth of both China and India makes the potential upheaval of the world economy a matter of serious challenge.

Key words: Real Exchange Rates, Economic Crisis, Multiple Equilibrium, Home Good Bias.

JEL Classification Number: F100 F110 F310.

1. INTRODUCTION

It is our honor to write a paper for the festschrift issue for Professor Michihiro Ohyama. From the beginning of his eminent career, he contributed on both the trading gains for the rich and poor countries (Kemp and Ohyama, 1978) and the non-traded goods (Ohyama and Suzuki, 1980). Written a quarter of a century later, this paper studies how these same elements operate in a current concern, as a complement to his work.

This paper extends the factor-content model of international trade so that technology and preference may differ across countries\(^1\). The extended model is then applied to

Acknowledgements. In his work, the second author thanks IEAS for partial support, also the contributions by a referee, C. H. Lu and A. C. Tung, also Andrea Wan.

\(^1\) For some use and discussion of of this important model, see Davis and Weinstein (1996), and Trefler and Zhu (2000).

Copyright©2005, by the Keio Economic Society
three major current issues in international economics. The study is solidly within the tradition of Neoclassical trade models. Yet, surprises may lurk within a terrain familiar to us all.

It is the conventional wisdom that (a) international financial crises arise, mostly because of unsound ‘fundamentals’ in some economies (typically, the debtor), (b) the growth in the less developed economies is an unmixed blessing, and (c) the roots of debt crisis are not in the real sector, and the Keynes–Ohlin debate on unilateral transfers has scarcely any relevance for the 21st Century. We argue all these are false.

We claim that rapid growth in very large economies (India and China, for example) may make the world economic system ‘fundamentally unstable’. Goodwill, prudence and foresight of all the States will be needed to make sure that the world economy will not suffer financial crisis under external shocks, when the income effect exceeds the substitution effect near an equilibrium.

To illuminate the above claims, this paper supplies an example under the simplest possible assumptions. The broad ideas are sketched out in Section 2, followed by the formulation and analysis of the model. We then return to the above claims before the concluding remarks.

2. DESCRIPTION OF OUR MODEL

Most of the final goods consumed today are not traded and this implies the ‘home good bias’. Much of what is traded are intermediate inputs, some produced with hired inputs abroad, then shipped home. Both these facts are studied in a model here: tradable goods are exchanged as proxies for the endowed inputs. Non-traded goods are included by making utility indices country-specific. This extension of the ‘factor content’ model of Vanek (1968) makes it possible to use it to analyze both the transfer problem of Keynes and Ohlin, and later studied by Samuelson (1952) and Jones (1970), and the possible presence of multiple equilibria. All these arise when the income effect dominates the substitution effect at some equilibrium. Hence, instability of the world economy may follow growth. Under the simplified assumption that all outputs are produced by labor under constant returns, this model may be called Ricardo–Vanek. To bring out the essence of this model in the sharpest relief, it is assumed in addition that the utility indices are the fixed coefficient type (Leontief). The interested readers are invited to make further extensions in admitting more general utility indices and non-labor inputs.

3. A SIMPLE MODEL

There are two countries (Home, and Foreign), each of which is endowed respectively with $N$ and $N^*$ units of labor, which is the only primary input. Denote the labor-augmenting efficiency indices at time $t$ as $a(t)$ and $a^*(t)$ for the two countries, and represent labor in efficiency units as,

---

2 For a recent discussion, see Mundell (2002).
3 This theme was first studied in Uchupalanun (1999).
\[ L(t) = a(t)N ; \quad L^*(t) = a^*(t)N^* . \] (3.1)

In these two countries, denote further the wage rates for each unit of efficiency labor as \( w(t) \) and \( w^*(t) \), respectively. The double factorial exchange rate at \( t \) is then,

\[ \omega(t) = w^*(t)/w(t) . \] (3.2)

In each country, all residents are identical, sharing the same labor endowment and the utility index. For each instant \( t \), the utility indices are of the Leontief type:

\[
\begin{align*}
    u &= \min\{x, z\} \quad \text{for the home country} \\
    u^* &= \min\{x^*, z^*\} \quad \text{for the foreign country}
\end{align*}
\] (3.3)

where for the home and foreign countries at time \( t \),

\( z^* \) and \( z^* \) are the local non-tradable goods (say, houses) consumed, and

\( x \) and \( x^* \) are the tradable final goods (say, computers) consumed.

Each unit of the tradable final goods is causelessly assembled from two separate tradable inputs, of one unit each:

\( \eta \) (say, hardware) and \( \sigma \) (say, software).

Assume now the unit output of efficiency labor \( L \) and \( L^* \) are as follows:

<table>
<thead>
<tr>
<th>Productivity for</th>
<th>( z )</th>
<th>( z^* )</th>
<th>( \eta )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>1</td>
<td>0</td>
<td>( b )</td>
<td>( c )</td>
</tr>
<tr>
<td>( L^* )</td>
<td>0</td>
<td>1</td>
<td>( c^* )</td>
<td>( b^* )</td>
</tr>
</tbody>
</table>

Adopt the following assumptions for simplicity,

Assumption 3.1. (Symmetry)

\[ L^* = L \quad \text{(Equal size of labor force in efficiency units)} \] (3.4)

\[ b^* = b ; \quad c^* = c . \quad \text{(Symmetrical structure)} \]

Assumption 3.2. (Comparative advantage): Home Country excels in hardware; Foreign Country excels in software.

\[ bb^* > cc^* , \] (3.5)

which implies, in view of (3.4):

\[ b > c > 0 . \] (3.6)

Assumption 3.3. (Home good bias)

\[ 1 + (1/b) > (1/c) ; \quad 1 + (1/b^*) > (1/c^*) . \] (3.7)

In terms of \( x \) and \( z \), the indifference locus for the Home (Foreign) Country is Leontief.

Next, we focus on the decision problem of the representative resident in the Home Country. The Foreign Country is merely its mirror image.

First, note that under autarky, the unit cost of the final goods are:
Given the national income of $wL$, and the form of the utility index in (3.3), it is easy to compute the equilibrium levels of consumption of $z$ and $x$, also the utility level, stated below as:

Observation 3.1. In an Autarkic equilibrium,

- $z = L/[1 + (1/b) + (1/c)]$ which costs $wL/[1 + (1/b) + (1/c)]$
- $x = L/[1 + (1/b) + (1/c)]$ which costs $[(1/b) + (1/c)]wL/[1 + (1/b) + (1/c)]$
- $u = L/[1 + (1/b) + (1/c)]$.

Moreover, in autarky, domestic costs decide the unit prices of the intermediary inputs. Denote their output levels and the unit prices in the two countries as follows:

<table>
<thead>
<tr>
<th>Intermediary input</th>
<th>$\eta$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home output</td>
<td>$H$</td>
<td>$S$</td>
</tr>
<tr>
<td>Foreign output</td>
<td>$H^*$</td>
<td>$S^*$</td>
</tr>
<tr>
<td>Unit price</td>
<td>$h$</td>
<td>$s$</td>
</tr>
</tbody>
</table>

Adopt now,

Assumption 3.4. Labor market in both countries are competitive.

This implies:

Observation 3.2 (Market equilibrium). In any equilibrium, the following four triplets of the Kuhn–Tucker complementary slackness conditions always hold:

<table>
<thead>
<tr>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive profit</td>
</tr>
<tr>
<td>(No profit is earned in perfect competition) $w/b - h \geq 0$</td>
</tr>
<tr>
<td>Non-negative output</td>
</tr>
<tr>
<td>(Any output must be positive or zero) $H \geq 0$</td>
</tr>
<tr>
<td>Complementary slack</td>
</tr>
<tr>
<td>(No output is produced at a loss) $H(w/b - h) = 0$</td>
</tr>
<tr>
<td>(3.9)</td>
</tr>
<tr>
<td>(3.10)</td>
</tr>
<tr>
<td>$w/c - s \geq 0$</td>
</tr>
<tr>
<td>$S \geq 0$</td>
</tr>
<tr>
<td>$S(w/c - s) = 0$</td>
</tr>
<tr>
<td>(3.11)</td>
</tr>
<tr>
<td>$w^*/c - h \geq 0$</td>
</tr>
<tr>
<td>$H^* \geq 0$</td>
</tr>
<tr>
<td>$H^<em>(w^</em>/c - h) = 0$</td>
</tr>
<tr>
<td>(3.12)</td>
</tr>
<tr>
<td>$w^*/b - s \geq 0$</td>
</tr>
<tr>
<td>$S^* \geq 0$</td>
</tr>
<tr>
<td>$S^<em>(w^</em>/b - s) = 0$</td>
</tr>
</tbody>
</table>
Next consider the equilibrium trading behavior of the home country. Let $H^0$ and $S^0$ be the amounts of hardware and software traded: import from the foreign country carries a positive sign; export to the foreign country carries a negative sign.

Thus, the representative agent of the home country solves the problem:

$$\begin{align*}
Max u(z, x) &= Max \min (z, x) \\
&= Max\min (z, Min(H + H^0, S + S^0)) \tag{3.13}
\end{align*}$$

Subject to:

$$\begin{align*}
w(z + H/b + S/c) + (hH^0 + sS^0) &\leq wL. \quad \text{(Balance of budget)} \tag{3.14} \\
hH^0 + sS^0 &= 0. \quad \text{(Balance of trade)} \tag{3.15} \\
z &\geq 0, \quad H + H^0 \geq 0, \quad S + S^0 \geq 0. \quad \text{(Non-negative consumption)} \tag{3.16}
\end{align*}$$

In view of (3.8), the solution for problem (3.13–16) of the Home Country can be characterized as follows:

$$\begin{align*}
H + H^0 &= S^0 + S = z = u \quad \text{(Implication for the maximized minimum)} \\
H/b + S/c &= L - z \quad \text{(Rearranging (3.14) by using (3.15))} \\
hH^0 + sS^0 &= 0 \quad \text{(Restating (3.15))} \\
z &\geq 0, \quad H + H^0 \geq 0, \quad S + S^0 \geq 0.
\end{align*}$$

Now note that one can never import $z$, the local good, nor will one import the hardware, on which the Home Country has comparative advantage. The only issue is a make-or-buy decision for software, and to what extent.

First, there is the locus for consumption proportions, as implied by (3.13):

$$\begin{align*}
(z + H + H^0) &= 2(S + S^0). \tag{3.17}
\end{align*}$$

Second, all budget lines pass through the production point of buying-all-software:

$$\begin{align*}
S &= 0; \quad H = b(L - z). \tag{3.18} \\
H^0 &= -(s/h)S^0; \quad z + H + H^0 = z + H - (s/h)S^0 = 2S^0. \tag{3.19}
\end{align*}$$

Charts 1 and 2 illustrate the above discussion with the following numerical values:

$$\begin{align*}
L &= 7/2, \quad b = 1 \quad \text{and} \quad c = 2/3.
\end{align*}$$

Under autarky, the labor allocation will be as follows:

<table>
<thead>
<tr>
<th>Goods</th>
<th>Local good (housing) $z$</th>
<th>Intermediate input</th>
<th>Sum $x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta$ (hardware)</td>
<td>$\sigma$ (software)</td>
<td></td>
</tr>
<tr>
<td>Unit productivity</td>
<td>1</td>
<td>1</td>
<td>2/3</td>
</tr>
<tr>
<td>Labor allocation</td>
<td>1</td>
<td>1</td>
<td>3/2</td>
</tr>
<tr>
<td>Output</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Under autarky, out of 3.5 units of labor, one unit each is devoted to the local good (housing) and the hardware, $\eta$ (the intermediate input where the country enjoys comparative advantage), while the balance of 1.5 units is assigned to software $\sigma$.

In Chart 2, the consumption of $z$ and $\eta$ are lumped together to fit into the two dimensional diagram. $H$ and $H^o$ represent production and trade respectively. Thus $H + H^o$ denotes consumption. Likewise, software consumption is represented as $S + S^o$.

Point C is the production-cum-consumption point under autarky. The line OC has a slope of $1/2 < 1$, showing ‘home good bias’.

What happens when trade opens depends upon $h/s$, the market price ratio of hardware to software. This is of course the negative of the slope of the price line, AB.

Next, one can characterize the response behavior of an individual in the market.

Observation 3.3. Equilibrium response to the market
(a) Facing any price ratios less than $2/3 = c/b$, the home country would prefer autarky. (In reality, such ratios cannot be observed in equilibrium.)
(b) If the price ratio is exactly $2/3$, the home country may either stay at autarky, or use all its labor, to produce (i) the needed local good and (ii) as much hardware as possible, up to point A, then trade for any needed software. Or, it is ready to choose anything within the interval in between and facilitate the choice of others. That interval is denoted ‘The Facilitating Interval’. The consumption remains the same at point C, which is associated with the autarkic level of utility. So there is no gain from trade.
(c) More favorable terms of trade are represented by any line AB', which is steeper than AB. Here, it is assumed that the slopes of AB and AB' are reciprocals to each other. The consumption will then be at C'. There is now a gain from trade.
Formally, for each vector of consumption response, $v$, one can define a facilitating interval, $I(v)$, associated with it. In Chart 2, for $v$ at $C$, $I(v)$ is the AC interval; for $v$ at $C'$, $I(v)$ is just $v$ itself.

Because the Leontief utility index is homothetic, the price-consumption curve, OCC', is an *Expenditure Ray* from the origin and shares a segment with the offer curve.

The entire offer curve has two arms, the lower arm along the 'production frontier' of the home country, and the upper arm along OCC'. This forms the classical 'hook' form of Becker (1952). The slope of OCC' is less than 45 degrees, reflecting the 'home good bias'.

For the ease of depiction, we reproduce the ACC' part in Chart 3. We shall superimpose on it a similar diagram of the foreign country. For simplicity, we have assumed the counterpart of AC for the foreign country is actually AC'.

Taking note of the symmetry assumption adopted as Assumption 3.1 (Equation (3.4)), one can make another useful re-interpretation. On any point of the offer curve of the home country, the vertical coordinate represents that intermediate input produced by the foreign country with its comparative advantage. Since the value of $b$ is assumed to be unity, that is also the amount of foreign labor needed. One can interpret this as the home country entering the foreign labor market, hiring labor there to produce the needed input, and shipping back for assembly. As a consequence, we can re-label the side of the box in terms of country-specific labor, L and L*. Actually, since each country only exports one unique intermediate input, produced by its labor under constant returns, the value of the parameter $b$ can be chosen without losing generality.

The heavy lined offer loci in the diagram clearly indicate where the equilibrium positions are located.

![Chart 3. The box diagram.](chart3.png)
It must be said that in identifying the number of equilibria, the resemblance of the above diagram to the Edgeworth box diagram and its offer curves is only suggestive. It takes either analysis, or detailed accounting as in the numerical example below, to confirm the conclusion.

Next, consider two countries together. One takes notes of (3.9–3.12), and obtains:

**Proposition 3.1 (Characterization).** At any equilibrium,

(a) Each intermediate input must be produced either in one country, or both,
(b) Each country must produce one intermediate input, or both,
(c) For any country producing both intermediate inputs, the world input price ratio must agree with its cost ratio,
(d) Both countries would not simultaneously produce both intermediate inputs,
(e) The world intermediate input price ratio can agree with the cost ratio of either no country or one country, but never both countries, and
(f) Whether any intermediate input is produced in both countries, the following combination can never happen: that is, the home country produces \( \sigma \), the software; the foreign country produces \( \eta \), the hardware.

**Proof.**

The following table lists all 16 logical possibilities. A check mark is assigned to those compatible with the equilibrium concept, and cross-references are supplied for all the rest to those specific numbered paragraphs below, about why they are incompatible with the equilibrium concept.

<table>
<thead>
<tr>
<th>Production of ( \sigma ), the software</th>
<th>Production of ( \eta ), the hardware</th>
</tr>
</thead>
<tbody>
<tr>
<td>No country</td>
<td>No country</td>
</tr>
<tr>
<td>Home country</td>
<td>(i)</td>
</tr>
<tr>
<td>Foreign country</td>
<td>(i)</td>
</tr>
<tr>
<td>Both</td>
<td>(i)</td>
</tr>
<tr>
<td></td>
<td>(i)</td>
</tr>
</tbody>
</table>

(i) Both the hardware and the software must be produced in some country, or else no computer can be produced, and that is not an equilibrium outcome, since either country can do better by producing both intermediate inputs and then assemble computers.

(ii) Neither the Home Country, nor the Foreign Country can be the only place to produce both intermediate inputs, for that means the other country produces nothing, and that outcome is dominated: the country left out can do better by producing both intermediate inputs, and then assembling computers.

(iii) When one country produces both intermediate inputs, then the world input price ratio must equal to the cost ratio of that country, by (3.9–3.10) and (3.11–3.12). But the cost ratios differ between these two countries, and the world input price ratio can never equal both. So both countries cannot produce both goods.
(iv) If both intermediate inputs are produced by a single country, and the hardware is produced by the Foreign Country and the software is produced by the Home Country, then the cost of any computer assembled this way is more expensive than a computer assembled from intermediate inputs produced the other way round. So the result is out of equilibrium, by (3.5).

(v) If some intermediate input is produced in both countries, and either the Foreign country alone produces the hardware, or the Home country alone produces the software, there must be some computer assembled with intermediate inputs assigned to producers in an unnecessarily costly way.

By the process of elimination, it is straightforward to derive from the above, Corollary. There are three types of equilibrium:

First, the home cost ratio equilibrium:

\[ s/h = b/c = 1/c; \quad H > 0, \quad S > 0, \quad H^* = 0, \quad S^* > 0; \quad (3.20) \]

Second, the foreign cost ratio equilibrium:

\[ s/h = c/b = c; \quad H^* > 0, \quad S^* > 0, \quad H > 0, \quad S = 0; \quad (3.21) \]

Third, the limbo ratio equilibrium:

\[ 1/c > s/h > c; \quad H > 0, \quad S = 0, \quad H^* = 0, \quad S^* > 0. \quad (3.22) \]

For the symmetric case considered here, the limbo equilibrium ratio \( s/h = 1 \).

Observation 3.4. The ratio \( s/h \) is the same as the ratio:

\[ w^*/w = \omega \]

the double factorial terms of trade.

In Chart 3, there are three equilibria, circled for emphasis:

1. Home Country cost ratio equilibrium, CC*:
   Home Country consumes at C: Response is type (b), without gains from trade;
   Foreign Country consumes at C*: Response is type (c), with gains from trade;
   The foreign consumption vector matches the facilitating interval of the Home Country.

2. Foreign Country cost ratio equilibrium, C*C:
   Home Country consumes at C': Response is type (c), with gains from trade;
   Foreign Country consumes at C*: Response is type (b), without gains from trade.
   The home consumption vector matches the facilitating interval of the Foreign Country.

3. Limbo ratio equilibrium: C°:
   Home Country consumes at C°: Response is type (c), with gains from trade
   Foreign Country consumes at C°: Response is type (c), with gains from trade
Thus, the graphic approach in Chart 3 has identified three distinct types of equilibria, that is all the three types listed in the Corollary to Proposition 3.1. The necessary and sufficiency condition for the existence of three equilibria is the presence of the limbo ratio equilibrium in the interior of the multiplicity cone. The latter is formed by the national budget lines AC for the Home Country and AC* for the Foreign Country.

Observation 3.4.

The graphic method in Chart 3 has done what it is supposed to do, in motivating the analysis and identifying all the equilibria for the example. But its proper interpretation has to differ somewhat from the Edgeworth box diagram, used by Dixit and Norman (1980), as well as Helpman and Krugman (1985). There, the analysis is limited to the zone of factor price equalization.

Here the wages of efficient labor differ from country to country most of the time. The balance of trade is achieved in the markets of intermediate inputs. When intermediate inputs are imported, these are produced by labor from countries enjoying comparative advantage. Yet for both the ‘home cost ratio equilibrium’ and the ‘foreign cost ratio equilibrium’, the intermediate inputs which are consumed include partly imports but also units produced by local labor.

Thus, in representing each country’s consumption vectors in labor by this manner, one finds that both for the case of the ‘home cost ratio equilibrium’ and for the case of the ‘foreign cost ratio equilibrium’, equilibrium should not be defined only when the vector sum equals the sizes of ‘the box’ (a criterion which holds for the ‘limbo ratio equilibrium’). Instead, a more general definition is called for: an equilibrium should be defined as a pair of equilibrium consumption vectors chosen by the individuals, each of which is attached with its ‘facilitation interval’, (which includes itself) such that one can take a vector from each such interval and sum to the dimension of the box. This is the market clearance condition, at the factor content level.

For this symmetric case of our model, all that is left is to illustrate numerically how it works. This is now done for the equilibria corresponding to (3.21) and (3.22): (3.20) is just the mirror image of (3.21).

**Numerical examples**

For (3.21),

- **Home Country**
  Consumption: \( z = H + H^o = S + S^o = u = 1.3125 \).
  Production: \( z = 1.3125; H = 2.1875 \).
  Trade
  Export: \(-H^o = 0.875\); Import: \(S^o = 1.3125\).

- **Foreign Country**
  Consumption: \( z^* = H^* + H^{o*} = S^* + S^{o*} = u^* = 1 \)
  Production: \( z^* = 1; H^* = 0.125; S^* = 2.3125 \)
  Trade:
  Export: \(-S^{o*} = 1.3125\); Import: \(H^{o*} = 0.875\)
For (3.22),

- **Home Country**
  
  **Consumption:** \( z = H + H^o = S + S^o = u = 1.166 \).
  
  **Production:** \( z = 1.166 \); \( H = 2.333 \).
  
  **Trade**
  
  Export: \(-H^o = 1.166\); Import: \(S^o = 1.166\).

- **Foreign Country**
  
  **Consumption:** \( z^* = H^* + H^{o*} = S^* + S^{o*} = u^* = 1.166 \).
  
  **Production:** \( z^* = 1.166 \); \( S^* = 2.333 \).
  
  **Trade**
  
  Export: \(-S^{o*} = 1.166\); Import: \(H^{o*} = 1.166\).

For easy checking, numerical values are given in computing the above example. Clearly all these can be replaced with symbols by readers so interested. Thus, by construction, we can present:

**Proposition 3.2 (Existence and Multiplicity).**

(a) For the symmetric case, there exists a competitive equilibrium for our example.

(b) There will be exactly three distinct equilibria, if and only if Assumption 3.3

(Equation (3.7)) holds.

**Observation 3.5.**

Like the classical ‘Battle of the Sexes’ game, in this case, there is no criterion to discriminate among these two ‘symmetric but distinct’ solutions, and regard one solution as the most natural.

It is time to consider the situation where the two countries are of different sizes. Without losing generality, consider the Home Country is smaller than the other, that is, \( L < L^* \). This may be due to either a smaller population, \( N \), or a less efficient labor force, namely, a lower labor-augmenting efficiency index, \( a(t) < a^*(t) = 1 \), say. In our model, the effect of a smaller value for \( L \) simply reduces Chart 2 in proportion. Since for our diagram, the width is decided by \( L \) and the height by \( L^* \), the gradual reduction of the former alone will reach a critical stage as depicted in Chart 4, where there exist exactly two equilibria.

By comparison with Chart 3, it is clear that the critical value of the supply of efficiency labor must be such that the Home Expenditure Ray in Chart 4 must be parallel to the old one, but passes through \( C^* \), where the Foreign Expenditure ray intersects the border of the Multiplicity Cone. This ray marks off on the \( L \) axis that critical size of efficiency labor, namely, \( L^o = 3.0625 \).

It is time to isolate the impact of the three factors—technology, preference and relative size—in placing point \( C^0 \) of Chart 3 inside the multiplicity cone, and thus causing multiplicity. Here is a dichotomy. The borders of the cone are decided only by technology: the country-specific cost ratios, \( c/b \) and \( b/c \) (or \( c \) and \( 1/c \), by the simplifying assumption of \( b = 1 = b^* \)). In contrast, the location of the point \( C^0 \) is jointly determined by preference and relative size, so that equilibrium terms of trade, \( h/s \), can achieve the balance of payment.
We can now tabulate both the fourteen variables, and those ten relationships among them which define the limbo ratio.

A recapitulation of the 14 relevant variables for the limbo ratio:

<table>
<thead>
<tr>
<th>Home country</th>
<th>Foreign country</th>
<th>Unit price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>$z^*$</td>
<td></td>
</tr>
<tr>
<td><strong>Hardware</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td>$H$</td>
<td>$H^*$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$H + H^o$</td>
<td>$H^* + H^{os}$</td>
</tr>
<tr>
<td>Trade</td>
<td>$H^o$</td>
<td>$H^{os}$</td>
</tr>
<tr>
<td><strong>Software</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td>$S$</td>
<td>$S^*$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$S + S^o$</td>
<td>$S^* + S^*$</td>
</tr>
<tr>
<td>Trade</td>
<td>$S^o$</td>
<td>$S^{os}$</td>
</tr>
<tr>
<td><strong>Labor</strong></td>
<td>$L$</td>
<td>$L^*$</td>
</tr>
</tbody>
</table>

We then tabulate the ten relevant relationships among these variables for the limbo ratio equilibrium, and summarize the information in Chart 5.
### Country-specific

<table>
<thead>
<tr>
<th></th>
<th>Home country</th>
<th>Foreign country</th>
<th>Cross country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor allocation</td>
<td>$H + z = L$</td>
<td>$S^* + z^* = L^*$</td>
<td></td>
</tr>
<tr>
<td>Leontief relation</td>
<td>$z = H + H^o + S^*$</td>
<td>$z^* = S^* + S^{o*} = H^{o*}$</td>
<td></td>
</tr>
<tr>
<td>Derived proportions</td>
<td>$(H + H^o + z)/S^o = 2$</td>
<td>$(S^* + S^{o*} + z^*)/H^o = 2$</td>
<td></td>
</tr>
<tr>
<td>Fixed input relation</td>
<td></td>
<td></td>
<td>$H = S^*$</td>
</tr>
<tr>
<td>Market clearance</td>
<td></td>
<td></td>
<td>$H^o + H^{o*} = 0 = S^o + S^{o*}$</td>
</tr>
<tr>
<td>Balance of payment</td>
<td></td>
<td></td>
<td>$hH^o = sS^{o*}$</td>
</tr>
<tr>
<td>Population size</td>
<td></td>
<td></td>
<td>$L/L^* = m$</td>
</tr>
</tbody>
</table>

#### Chart 5. Anatomy for the limbo ratio.

By routine calculation, the limbo terms of trade are,

$$h/s = (2m - 1)/(2 - m).$$

Utilizing this information, one can construct Chart 6, showing the correspondence of the market-clearing ratios $h/s$ for various values of $m$, the parameter for relative population size. Specifically, write,

$$h/s = \psi(m),$$
Note that near the limbo ratio $h/s$, if one considers another exchange rate which is more advantageous to the Home Country than to the Foreign Country, like the ray through $p'p''$, then, proportionally, the Home Country would like to trade less (at $p'$) while the Foreign Country would like to trade more (at $p''$). This is what is said that ‘the income effect dominates the substitution effect’. Regardless of how an economist today views the neoclassic notions of stability of an equilibrium, this phenomenon described above signifies the presence of multiple equilibria, which concerns us. This is studied in the next section.

4. CAN SUCCESS IN THE CATCHING-UP OF SOME NATIONS CAUSE CRISIS IN THE WORLD?

In this section, we return to the discussions in the introductory section. First, although no model offered here encompasses financial operations, we argue that the presence of multiple equilibria in a ‘real’ model is relevant for the emergence of financial crisis. Next, in reviewing evidence from Crisis 1997, we surmise that the observed facts are consistent with the presence of multiple equilibria in the real sector of the world economy. Following that, we claim that the current trade expansion of China and India makes the present model relevant. Finally, we compare our approach against the literature on financial crisis.
A. Our example in the last section shows that:

(a) Within a world of two countries, one developed and the other less developed, when the former has far more income than the latter, then world prices are likely to be determined by the conditions in the larger country.

(b) Since people normally spend relatively heavily on non-traded local goods (like real estate), a ‘home good bias’ exists. Consequently, with the successful catching up of the less developed country, multiple equilibria may develop, each with its distinct relative prices.

(c) In contrast with the model we studied, individuals borrow and lend in real life, trading across time. But by Horioka and Feldstein (1980), the differences between national savings and investments are usually relatively modest. Supposing that the real sector is such that multiple equilibria tend to develop, there is no convincing evidence that, in the actual amounts involved, international borrowing and lending would render our analysis of the real sector irrelevant.

(d) Next suppose in our model that a person borrows from abroad during a period of high market price ratio, $h/s$, to invest in hardware production at home, with both principal and interest denominated in terms of software (or, more likely, the currency of the country exporting software). Then any large, unexpected fall of the $h/s$ ratio may make debt servicing prohibitively expensive, in terms of the export revenue. Thus, a financial crisis occurs.

(e) Suppose we live in a world with multiple equilibria. There is no conceptual basis to predict how Nature must select which market equilibrium. Then financial crises cannot be completely avoided even by better prediction.

B. Our previous discussion is not purely conjectural. Among all wide-spread financial crises, Crisis 1997 is the most recent the world has witnessed. In several aspects, events in that traumatic episode are consistent with our analysis.

(a) The virulence of the crisis had victimized some of the High Performance Asian Economies (HPAEs). These were specifically praised by the economics profession right up to the eve of the Crisis for their sound macroeconomic management, like Thailand and, Malaysia (see respectively, Chistenson et al. (1993) and Mahathir (1999)). Nor did the financial market know any better than before the fall. If sound fundamentals of individual economy could forestall the crisis, they were not lacking for those fell victim.

(b) The suddenness of the onset of Crisis 1997 was sometimes spearheaded by sharp changes in commodity prices. The collapse of Korea’s export prices was legendary. The unit price of the largest export item, memory chips, went from nearly $50 to under $4 in one year (227, Chang 1998). This sounds like simply outrageous misfortune. But the need to depreciate from 910 won to $1 in September 1997 to 1,484 won to $1 in December 1997 (25, Yanagita 2000) implies the limited demand elasticity of Korea’s other export staples: cars, ships, and steel. This evidence is not inconsistent with a switch among multiple equilibria in the real terms of trade. In fact, it suggests to the open-minded that even some of the ‘real estate bubbles’ might be considered in an entirely different light.
There appear to exist different equilibrium real exchange rates, say, between the American dollar and the Thai baht. Records show there was little inflation in either America or Thailand. There are also two distinct, stable levels of exchange rates before and after the Crisis.

C. What has been presented so far is not just the bygone concerns of Keynes and Ohlin, juxtaposed to the a-mouldering record of an economic crisis in a defunct millennium. Currently, the economies of both China and India, with a population of two billion plus, are growing at unprecedented rates, far outdistancing the industrialized America, European Union, and Japan. The impending change in the relative total incomes among the nations is precisely the sort of mechanism that can trigger the economic instabilities discussed above. The mounting American deficits in the balance of payment may well play some part in such future upheaval.

D. Having successfully demonstrated that, apart from the real factors, the financial system can have a profound impact on the performance of an economy, we find the current literature in international finance has paid relatively little attention to issues pertaining to the terms of trade. This is especially true, regarding its endogenous dynamics, as a byproduct of internationally unbalanced growth, due to the process of convergence (or catching up). This is understandable. After all, in growth theory, there is little concern about how terms of trade upheavals can be caused by successful economic development. The present study is intended as a small step to strike a more even balance.

E. Economists seek efficiency to explain past events and to predict future development. We, as members of the profession, focus attention on the single theory we believe in most. Occam's razor has been wielded with confidence. In contrast, makers of public policy enjoy no such luxury. The Center for Disease Control (CDC) in Atlanta cannot dismiss a newly discovered viral strain as a potential threat, simply because the chance of an outbreak is less than the chance that there is no outbreak. It is against this perspective that we introduce our approach to interpret Crisis 1997 and to chart the probable consequences of successful growth of those billion-person economies in Asia, namely China and India.

5. CONCLUDING REMARKS

We have introduced the Ricardo–Vanek model to analyze the possible turbulence of the world economy, caused by successful economic development. The model is an outgrowth of the theory of trade. It is only natural to add some remarks relating our model to that literature.

Samuelson studied factor price equalization within the Heckscher–Ohlin paradigm, where technology is shared across countries. For him, it is a parable both to highlight the technology—factor price nexus, and to spur researchers in explaining wage differentials from causes beyond endowment differences. The factor-content theory further postulates that countries share homothetic preference. These assumptions turn out to
be expedient in studying both monopolistic competition and increasing returns, in the 1980s. The introduction of the Ricardo–Vanek model in this paper suggests that if tractability is the principal concern, the valuable results might be accessible based on a far less contrafactual basis.

APPENDIX

The parametric payoff function under the Ricardo–Vanek approach:

To complete the description of the Ricardo–Vanek model, one can proceed to define the felicity index of an individual (in the Home Country, say) as the amounts of labor service in both countries at one's disposal, namely, $L$ and $L^*$. These latter magnitudes may be viewed as parameters in the problem of parametric linear programming. Consider the problem:

Find $W(L, L^*) = \text{Max} \text{ Min} \{L_z, (L_H/b) + (L^*_H/c), (L_S/c) + (L^*_S/b)\}$

subject to:

$L_z + L_H + L_S \leq L$,

$L^*_H + L^*_S \leq L^*$,

$L_z \geq 0, \quad L_H \geq 0, \quad L_S \geq 0, \quad L^*_H \geq 0, \quad L^*_S \geq 0$.

Clearly, $W$ is an increasing, quasi-concave function, homogeneous of the first degree. It is straightforward to construct one of its piece-wise linear niveau line, as shown in Chart 7, where there are three subsets:

![Chart 7. The Niveau Line W.](image-url)
\[ \begin{align*}
L &= \{(L, L^*) : L^* \geq (5/2)L\}, \quad \text{over which } W = L, \\
A_2 &= \{(L, L^*) : (5/2)L > L^* > L/2\}, \quad \text{over which } W = \frac{2(L + L^*)}{7} \\
A_3 &= \{(L, L^*) : L/2 > L^* \}, \quad \text{over which } W = \frac{2L + 3L^*}{7}. 
\end{align*} \]

For the Foreign Country, \( W^*(L, L^*) \) can be similarly defined. It is then possible to analyze the trading behavior of all individuals, given the equilibrium wage rate, 
\[ \omega = \frac{w^*}{w}. \]

REFERENCES

bin Mohamad, Mahathir (1999), A New Deal for Asia Selangor Darul Ehsan: Pelanduk Publications.