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SIMPLE FRAMEWORK FOR ANALYZING MONETARY SYSTEM

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Abstract: We present a simple framework for analyzing a monetary system in which debts originated from private economic activities are repaid with fiat money, a bank is formed as risk pooling against agents' intertemporal preference shock, and a central bank alleviates the shortfall of fluctuating liquidity demand by its discount window. It is shown that a high growth rate of endowments is suitable for a bank to design incentive compatibility of deposit contracts.

Key words: payment system; deposit contract; growth.
JEL Classification Number: E52, E58.

1. INTRODUCTION

We present a simple monetary framework in which banking behavior and the discount window are systematically synthesized in a payment system. Namely, debts originated from private economic activities are repaid with fiat money, a bank is formed for risk pooling against agents' intertemporal preference shock, and a central bank alleviates the shortfall of fluctuating liquidity by its discount window.

A monetary authority usually claims that one of its main jobs is to give daily transactions full treatment by provision of a monetary base in order not to cause the shortfall of liquidity. Such an activity is subject just to private business transactions. In a sense,

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a central bank fills a passive role in the payment system, although the active (policy oriented) role such as in stability of price levels has been emphasized in previous researches. In our model, to the contrary, a central bank is described as an agent enthusiastic about alleviating the shortfall of fluctuating liquidity in the payment system. So, our model has a feature of the Real Bills Doctrine, which advocates allowing the stock of money to fluctuate to meet the needs of trade.

Our model also suggests a relationship between the design of the deposit contract and the growth rate of endowments in the above framework. The deposit contract should satisfy the incentive compatibility for the depositors, which means that all depositors will withdraw their money from a bank account at appropriate timing to attain an efficient allocation unless they are in panic. It is shown that a high growth rate of endowments is suitable for a bank to design the incentive compatibility of deposit contracts. This is one of the interesting topics for studying the economic growth through indirect finance.

To construct such a framework, we adopt as ingredients the payment model of Freeman[3),4)] and the deposit contract originated by Diamond and Dybvig.[2)] Freeman is the first to formulate a model in which debts are repaid with fiat money and there exists a shortfall of liquidity which is alleviated by a discount window. In the case of Freeman's analysis, the shortfall of liquidity is caused by the time lag of trading opportunities, and a central bank provides a monetary base in order to attain an efficient allocation. In our model with a deposit contract, a bank formulated for risk pooling needs liquidity in order to fill withdrawals attributed to intertemporal preference shock. A depositor who unexpectedly needs fiat money to buy consumption good withdraws his/her deposit from a bank account. In this process, a bank will design a deposit contract as a monetary commodity for sale to consumers, which attains an efficient allocation although a bank run equilibrium cannot be eradicated in multiple equilibria. It is a banking behavior described by Diamond and Dybvig, and a lot of subsequent research has come from it. What is different in our model from the previous works is that whenever liquidity for payment is short, the central bank buys an IOU held by a bank facing a shortfall of liquidity in order to give a monetary base to that bank in the payment system. We will show a explicit relationship between the incentive compatibility and the growth rate of endowments in a design of deposit contracts.

2. MODEL

We introduced a bank with a deposit contract as a new agent to the Freeman model[4)] constructed out of a two-period overlapping generations model. Consider an environment where many islands exist around the central island. The outer islands consist of debtor islands and creditor islands. The people born at a debtor island are called a debtor while those born at a creditor island are called a creditor. The debtor island and the creditor island are pairwised. There are indigenous goods that we call the debtor good (d good) on a debtor island and the creditor good (c good) on a creditor island respectively. The debtor (creditor) good is given a young debtor (creditor) as an initial endowment. None can get anything when they are old. Let w_t^d (W_t^C) be the initial endowments of a debtor (of a creditor) at period t. We also assume that this economy has grown as the ratio represented by the following rule of initial endowments

$$(w_{t+1}^{d}, W_{t+1}^{C}) = \left(\frac{1}{\pi} w_{t}^{d}, \frac{1}{\pi} W_{t}^{C}\right) \tag{1}$$

where $0 < \pi < 1$. Namely, π is a reciprocal of the growth rate of endowments.

Let $v(c_t, d_t)$ be the utility function of a debtor, where c_t and d_t are his/her consumption of c good and d good at period t respectively. On the other hand, the intertemporal preference shock happens for a young creditor during a period in which these creditors are separated into a group of type 1 and another group of type 2. Type 1 has the utility function $u(C_t, D_t^1)$ while type 2 has $u(C_t, D_{t+1}^2)$, where C_t is the consumption of period t, and D_t^1 and D_{t+1}^2 are the consumptions of d good at period t and period $t+1$ respectively[1,2]. The type of a creditor is private information, i.e., none knows it except the person in question. The population of debtors and creditors is represented as measure one where the ratio of creditors to the total is $\alpha \in (0, 1)$, which is public information. Both functions, v and u are increasing and concave in each argument, continuous, and continuously differentiable with indifference curves that do not cross the axes. Finally, all agents can issue unfalsifiable IOUs that identify the issuer at the beginning of each period. There is a legal authority on the central island that will be able to enforce agreement between parties (i.e., the payment of IOU) on that island.

Under this physical environment, both debtors and creditors travel around the islands. To begin with, the young debtors visit the creditors' island and return to their own island. The young debtors returning to their own island will be visited by the old creditors. All debtors and creditors go to the central island when they get old. After that, only type 2 creditors move to the debtors' island, where the young debtors have returned from the creditors' island. We assume that all agents do not consume until the end of each period. Then the financial transactions will occur as the following timing, as described by Figure 1.

(i) At the beginning of period t, a young debtor visits the creditor's island, and obtains c good by issuing B_t of IOU to a young creditor as a resident there. The amount of a debt, B_t is represented in terms of money (\$). Since young debtors and young creditors can meet again at the central island when old, inside money (IOU) can occur.

(ii) The young creditors establish a bank on a fund of IOUs in order to pool their risks. The bank offers a deposit contract that consists of the payment r_t^1 at period t and the payment r_{t+1}^2 at period $t+1$, calculating the optimal allocation for creditors. In fact, the bank maximizes a creditor's expected utility subject to the resource constraint and put such an allocation into the deposit contract. The payments (r_t^1, r_{t+1}^2) are payable in the nominal term, i.e., money (\$). This bank is supposed to pay the depositors back all assets and dissolve at the end of period $t+1$. The deposit contract must satisfy (a) the

[1] For D_j^i, superscript $i \in \{1, 2\}$ is a type of a creditor and subscript j is the time when d good is consumed by a creditor.

[2] Namely, type 1 is an early consumer and can consume the goods only when young. Type 2 is a late consumer, who can consume when both young and old.

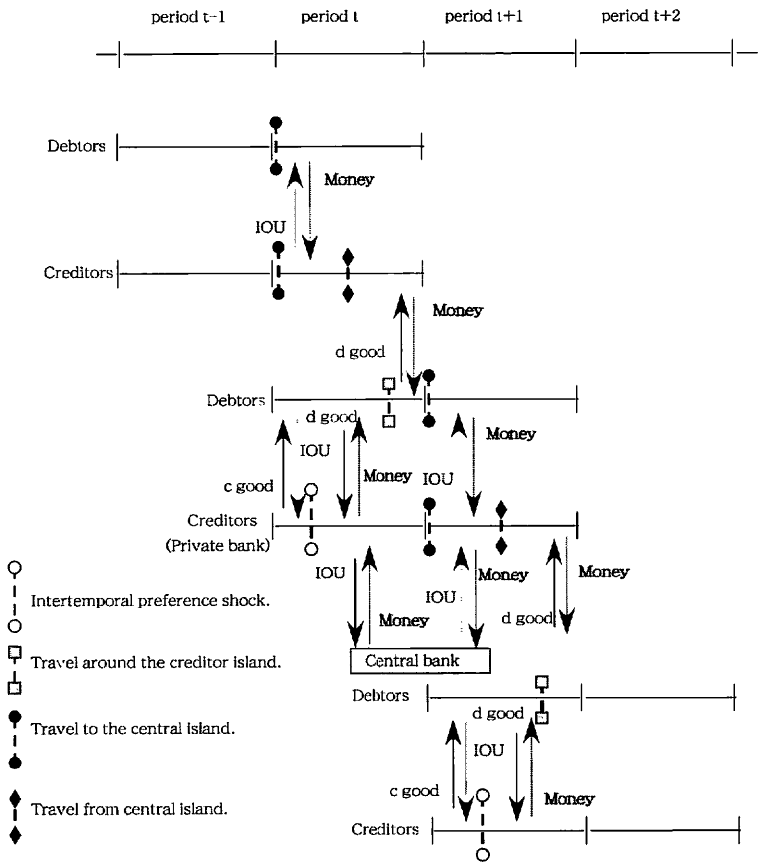

Figure 1. Model

incentive compatibility constraint and (b) the participation constraint (no side trade)[3]. Then all creditors buy the deposit contract as a monetary commodity, turning out to be depositors.

(iii) The intertemporal preference shock happens, and each young creditor realizes his own type. Type 1 creditors withdraw their deposit r_t^1 from the bank in order to buy d good from a young debtor. The bank pays back such requests by the cash $\bar{M}_t$ discounted by the central bank since the bank having only IOUs does not have any cash (fiat money). To put it another way, the central bank has to discount the IOUs so that the bank may hold $\bar{M}_t$ for the purpose of paying r_t^1 to type 1 creditors[4]. At this point,

[3] The incentive compatibility is the condition that a depositor gets more utility from the payment at period $t+1$, r_{t+1}^2 than that from the payment at period t, r_t^1. We will see this condition in detail later. The condition of no side trade is in fact a participation constraint of the deposit contract for creditors.

[4] The central bank infuses a monetary base into the current accounts of the banks.

the bank is assumed not to be able to distinguish the types of depositors. Therefore, the bank must continue taking up the requests of depositors according to first-come, first-served basis until the reserves are exhausted. Notice that type 2 creditors do not withdraw their deposit unless they get into a panic.

(iv) The young debtors return to their own land (debtor island) where the (type 2) old creditors visit. The old creditors without d good hold the currency M_t as we will see in the following step (vi). The young debtors are willing to obtain the currency M_t by selling d good to the old creditors in order to clear the IOUs when they are old at period $t+1$. At this stage, all of the currency is kept by the young debtors while the IOUs are held by the bank or the central bank.

(v) When old, the debtors and the creditors move to the central island. The debtors redeem the IOUs from the bank while the bank pays r_{t+1}^2 back to type 2 creditors by that currency. The central bank sells IOUs to the bank (absorbs fiat money from the bank) immediately before the old debtors apply money to their debts. In consequence, all the IOUs are settled.

(vi) Old type 2 creditors holding the amount of money M_t move to the debtors island, and buy d good from the young debtors.

In the above process, the step (iii) means that the central bank compensates the shortfall of liquidity that the bank needs for the urgent withdrawal of type 1 creditors[5]. This additional currency $\bar{M}_t$ is retired at the point when the bank buys back the IOUs from the central bank in the step (v). To sum up, the currency always circulating in the market is M_t, and the shortfall of the liquidity $\bar{M}_t$ caused by the deposit contract is periodically adjusted by the central bank.

The choice problems of a debtor and a bank as a proxy of creditors are given as follows.

CHOICE PROBLEM FOR A DEBTOR. Let p_t^d and p_t^c be the prices of d good and of c good at period t in terms of money respectively. The choice problem of a debtor is given as

$$\max_{c_t, d_t} v(c_t, d_t) \tag{2}$$

subject to

$$(w_t^d - d_t)p_t^d = B_t \tag{3}$$

$$B_t = p_t^c c_t = \bar{M}_t + M_{t+1}, \tag{4}$$

where

$$\bar{M}_t = \alpha r_t^1 \tag{5}$$

$$M_{t+1} = (1-\alpha) r_{t+1}^2 . \tag{6}$$

[5] On the other hand, in case of Freeman [3, 4], the shortfall of liquidity comes from the time-lag of the agents' arrivals at the trading place.

Equation (3) is the budget constraint of a debtor. B_t in l.h.s. of Equation (4) is the debt for a debtor while in r.h.s. of Equation (4) $\bar{M}_t$ is the cash obtained by selling d good to the (young) type 1 creditors and M_{t+1} is the cash obtained by selling d good to the (old) type 2 creditors. Namely, a debtor settles the debt of IOU, B_t by the cash which is the sum of $\bar{M}_t$ and M_{t+1}. In particular, Equation (6) means that all debts at period t should be cleared at period $t+1$ owing to an enforcement authority (no default) on the central island.

CHOICE PROBLEM FOR A BANK. On the other hand, the utility function of a creditor is given as

$$U(C_t, D_t^1, D_{t+1}^2) = \begin{cases} u(C_t, D_t^1) & \text{if type 1} \\ u(C_t, D_{t+1}^2) & \text{if type 2}. \end{cases} \tag{7}$$

The budget constraint of a creditor is

$$(W_t^c - C_t)p_t^c = S_t \tag{8}$$

where S_t is the returns on the deposit contract

$$S_t = \begin{cases} r_t^1 = \bar{p}_t^d D_t^1 & \text{the refund at period } t \\ r_{t+1}^2 = p_{t+1}^d D_{t+1}^2 & \text{the refund at period } t+1 \end{cases} \tag{9}$$

where $\bar{p}_t^d$ is the price of the period t payment (D_t^1) of the deposit contract (r_t^1, r_{t+1}^2) in terms of cash (outside money).

The bank established by the young creditors offers the depositors a deposit contract (r_t^1, r_{t+1}^2) as a solution of the following problem. In other words, the bank as a cooperative entity determines the optimal allocation by the way of the deposit contract.

$$\max_{C_t, D_t^1, D_{t-1}^2} \alpha u(C_t, D_t^1) + (1-\alpha)u(C_t, D_{t+1}^2) \tag{10}$$

subject to

$$(W_t^c - C_t)p_t^c = \alpha r_t^1 + (1-\alpha)r_{t+1}^2. \tag{11}$$

Substituting Equations (9) into Equation (11), we have

$$(W_t^c - C_t)p_t^c = \alpha \bar{p}_t^d D_t^1 + (1-\alpha)p_{t+1}^d D_{t+1}^2. \tag{12}$$

3. EFFICIENT EQUILIBRIUM AND BANK RUN

From the above set-up, we will define the efficient equilibrium and show that there exists not only such an efficient equilibrium but also a bank run equilibrium. By the choice problem of a debtor, let $\rho_t = p_t^d / p_t^c$ denote the price of d good in terms of c good[6]. By Equations (3) and (4), the budget constraint of a debtor is

[6] Exactly speaking, we represent the unit of the price of d good at period t, p_t^d as [\$/d good] while the one of p_t^c as [\$/c good]. So the exchange ratio of d good for c good, $\rho_t = p_t^d / p_t^c$ is defined as [c good/d good] which is the price of d good in terms of c good.

$$\rho_t w_t^d = c_t + \rho_t d_t. \tag{13}$$

Then, let also $\tilde{c}_t(\rho_t)$ and $\tilde{d}_t(\rho_t)$ denote the demand functions of c good and d good respectively, which are driven from the maximization of the utility function (2) subject to the budget constraint of Equation (13). From now on, the superscript $\tilde{}$ indicates optimality.

On the other hand, the bank offers the deposit contract, calculating the optimal allocation under the condition that the ratio of type 1, α is given as public information. This contract will be able to realize the efficient equilibrium. By the same way as the case of a debtor, the budget constraint (12) is represented in terms of c good as

$$W_t^c - C_t = \alpha \bar{\rho}_t D_t^1 + (1-\alpha)\rho_t' D_{t+1}^2. \tag{14}$$

where $\bar{\rho}_t = \bar{p}_t^d / p_t^c$ and $\rho_t' = p_{t+1}^d / p_t^c$. The bank maximizes the expected utility (10) subject to Equation (14).

$$L = \alpha u(C_t, D_t^1) + (1-\alpha)u(C_t, D_{t+1}^2) + \lambda\{W_t^c - C_t - \alpha\bar{\rho}_t D_t^1 - (1-\alpha)\rho_t' D_{t+1}^2\} \tag{15}$$

The first order necessary conditions are given as[7]

$$C_t : \alpha u_1(C_t, D_t^1) + (1-\alpha)u_1(C_t, D_{t+1}^2) - \lambda = 0 \tag{16}$$

$$D_t^1 : \alpha u_2(C_t, D_t^1) - \lambda\alpha\bar{\rho}_t = 0 \tag{17}$$

$$D_{t+1}^2 : (1-\alpha)u_2(C_t, D_{t+1}^2) - \lambda(1-\alpha)\rho_t' = 0 \tag{18}$$

$$W_t^c - C_t - \alpha\bar{\rho}_t D_t^1 - (1-\alpha)\rho_t' D_{t+1}^2 = 0. \tag{19}$$

By Equations (17) and (18),

$$\frac{u_2(C_t, D_{t+1}^2)}{u_2(C_t, D_t^1)} = \frac{\rho_t'}{\bar{\rho}_t} \tag{20}$$

holds.

By the optimal solution, $(\tilde{C}_t, \tilde{D}_t^1, \tilde{D}_{t+1}^2)$, the bank offers the deposit contract $(\tilde{r}_t^1, \tilde{r}_{t+1}^2) = (\bar{p}_t^d \tilde{D}_t^1, p_{t+1}^d \tilde{D}_{t+1}^2)$. The deposit contract must satisfy the incentive compatibility constraint $\tilde{D}_t^1 < \tilde{D}_{t+1}^2$, because without the constraint type 2 creditors are better off withdrawing when young. Namely, r.h.s. of Equation (20) should be less than unity. The allocation by the deposit contract is not realized without the incentive compatible constraint.

The money which the bank holds at period t, $\bar{M}_t$ comes from the IOUs discounted by the central bank. So,

[7] u_1 and u_2 are defined as $u_1 = \frac{\partial u(x,y)}{\partial x}$ and $u_2 = \frac{\partial u(x,y)}{\partial y}$.

$$\bar{M}_t = \alpha \bar{r}_t^1 = \alpha \bar{p}_t^d \tilde{D}_t^1 \tag{21}$$

holds. The currency circulating through the generations is

$$M_{t+1} = (1-\alpha)\bar{r}_{t+1}^2 = (1-\alpha) p_{t+1}^d \tilde{D}_{t+1}^2. \tag{22}$$

Since this amount is fixed without any particular changes, $M_t = M$ for any t holds. Equation (21) is the market clearing condition for type 1 creditors and debtors on the debtor island, and Equation (22) is the market clearing condition for type 2 creditors and debtors on the creditor island.

Then, the efficient equilibrium is defined as follows.

DEFINITION OF EQUILIBRIUM. *Let* $\{\tilde{c}_t(\rho_t), \tilde{d}_t(\rho_t)\}$ *and* $\{\tilde{C}_t(\bar{\rho}_t, \rho_t'), \tilde{D}_t^1(\bar{\rho}_t, \rho_t'), \tilde{D}_{t+1}^2(\bar{\rho}_t, \rho_t')\}$ *be the optimal solutions for a debtor and a creditor respectively. The equilibrium is defined as a triplet of prices* $(\rho_t, \bar{\rho}_t, \rho_t')$ *which satisfies the market clearing conditions Equation* (21) *and Equation* (22).

Note that the resource constraints in above economy are

$$w_t^d = d(\rho_t) + D_t^1(\bar{\rho}_t, \rho_t') + D_t^2(\bar{\rho}_{t-1}, \rho_{t-1}'), \tag{23}$$

$$W_t^c = C_t(\bar{\rho}_t, \rho_t') + c_t(\rho_t). \tag{24}$$

There is also a bank run equilibrium in this economy that belongs to the sunspot type. The bank cannot distinguish type 1 creditors from type 2 creditors, being required withdrawal. Consequently, the bank must pay back all depositors irrespective of their types according to a first-come, first-served basis. The bank goes bankrupt when their reserves are depleted. The financial assets of the banks at this point in time consist of the currency $\bar{M}_t$ that comes from IOUs discounted by the central bank and the rest of IOUs funded by the young creditors. A bank falls into the bankruptcy whenever a type 2 creditor is prepossessed with all other type 2 creditors to withdraw their deposits and dissolves his/her deposit contract.

4. EXISTENCE OF EQUILIBRIUM AND GROWTH RATE

Do the equilibria exist and can the bank actually offer an appropriate deposit contract described in our model? We adopt the method called "guess and verify" in order to show the existence of equilibrium. Then, we show that the incentive compatibility of deposit contracts is strongly related with the growth rate of endowments in our framework.

Recall that this economy has grown according to the rate of initial endowments, $1/\pi$ by the assumption of Equation (1). We guess that the equilibrium prices and the allocations are given respectively as follows:

$$(p_{t+1}^c, \bar{p}_{t+1}^d, p_{t+1}^d) = (\pi p_t^c, \pi \bar{p}_t^d, \pi p_t^d), \tag{25}$$

and

$$(\tilde{c}_{t+1}, \tilde{d}_{t+1}, \tilde{C}_{t+1}, \tilde{D}_{t+1}^1, \tilde{D}_{t+2}^2) = \left(\frac{1}{\pi}\tilde{c}_t, \frac{1}{\pi}\tilde{d}_t, \frac{1}{\pi}\tilde{C}_t, \frac{1}{\pi}\tilde{D}_t^1, \frac{1}{\pi}\tilde{D}_{t+1}^2\right). \tag{26}$$

We suppose that this equilibrium exists. Then, since

$$\rho_{t+1} = \frac{p^d_{t+1}}{p^c_{t+1}} = \frac{\pi p^d_t}{\pi p^c_t} = \rho_t \tag{27}$$

holds, we have $\rho_t = \rho$. By the similar way,

$$\bar{\rho}_{t+1} = \frac{\bar{p}^d_{t+1}}{p^c_{t+1}} = \frac{\pi \bar{p}^d_t}{\pi p^c_t} = \bar{\rho}_t \tag{28}$$

and

$$\rho'_{t+1} = \frac{p^d_{t+2}}{p^c_{t+1}} = \frac{\pi p^d_{t+1}}{p^c_{t+1}} = \pi\rho \tag{29}$$

So, we also have $\bar{\rho}_t = \bar{\rho}$ and $\rho'_{t+1} = \pi\rho$. Namely, $(\rho, \bar{\rho}, \rho')$ are all constant. For simplicity, we assume that the utility function of creditors (7) are separable.

$$U(C_t, D^1_t, D^2_{t+1}) = \begin{cases} u(C_t, D^1_t) = u(C_t) + u(D^1_t) & \text{when type 1} \\ u(C_t, D^2_{t+1}) = u(C_t) + \beta u(D^2_{t+1}) & \text{when type 2.} \end{cases} \tag{30}$$

where $\beta \in (0, 1)$ is a discount factor. Then, Equation (20) can hold as

$$\frac{\beta u'(\tilde{D}^2_{t+1})}{u'(\tilde{D}^1_t)} = \frac{\rho'_t}{\bar{\rho}_t} = \frac{\pi\rho}{\bar{\rho}} = \pi\gamma \tag{31}$$

where $\gamma \equiv \frac{\rho}{\bar{\rho}}$ which is constant. We can choose a small enough π to satisfy $\pi\gamma < \beta$ such that

$$\frac{u'(\tilde{D}^2_{t+1})}{u'(\tilde{D}^1_t)} < 1. \tag{32}$$

Therefore, we have $u'(\tilde{D}^2_{t+1}) < u'(\tilde{D}^1_t)$, and by the properties of the utility function, we can get the incentive compatibility constraint, $\tilde{D}^1_t < \tilde{D}^2_{t+1}$. Since the reciprocal of π is the growth rate of endowments, it turns out that we have obtained a relationship between an optimal deposit contract and its growth rate. It is hard for a bank to offer a deposit contract satisfying the incentive compatibility constraint under a low growth rate.

Finally, let us show such an equilibrium as an example. Here, we specify the utility function as log form to make the model tractable. The utility functions of a debtor and a creditor respectively are given as

$$v(c_t, d_t) = \ln c_t + \ln d_t, \tag{33}$$

and $u(x) = \ln x$ where $x = C_t$, D^1_t, and D^2_{t+1} in the utility function given by (30).

The solutions of the utility maximization problem are obtained as for the debtors

$$(\tilde{c}_t, \tilde{d}_t) = \left(\frac{\rho w^d_t}{2}, \frac{w^d_t}{2}\right) \tag{34}$$

and for a creditor

$$(\tilde{C}_t, \tilde{D}_t^1, \tilde{D}_{t+1}^2)$$
$$= \left[\frac{W_t^c}{(1+\alpha)+(1-\alpha)\beta}, \frac{W_t^c}{\bar{\rho}_t\{(1+\alpha)+(1-\alpha)\beta\}}, \frac{\beta}{\rho_t'}\frac{W_t^c}{\{(1+\alpha)+(1-\alpha)\beta\}} \right]. \tag{35}$$

It is easy to see that considering the assumption of (1), the solutions of (34) and (35) satisfy the property of (26). We can also verify that the solutions satisfy the property of the equilibrium price, (25) by the market clearing conditions of (21) and (22).

5. CONCLUSION

We present a simple framework for analyzing a monetary system in which banking behavior, discount window, and payment are systematically synthesized. Our model is probably the first one to combine these ingredients in a unit of an economy. In it, we present a relationship between the design of the deposit contracts and the growth rate of endowments. The growth rate of endowments is a key element to give the deposit contract a condition of incentive compatibility. One of the possible extensions of our study is to consider the topic discussed here in a more general framework such as an environment of endogenous growth[8].

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[8] Bencivenga and Smith [1] is one of the early contributions to studying endogenous growth rate through financial intermediaries although it does not include payment.