慶應義塾大学学術情報リポジトリ Keio Associated Repository of Academic resouces

Title	Optimal Pollution Tax in Cournot Oligopoly
Sub Title	
Author	Okuguchi, Koji
Publisher	Keio Economic Society, Keio University
Publication year	2003
Jtitle	Keio economic studies Vol.40, No.1 (2003.) ,p.85- 89
JaLC DOI	
Abstract	
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260 492-20030001-0085

慶應義塾大学学術情報リポジトリ(KOARA)に掲載されているコンテンツの著作権は、それぞれの著作者、学会または出版社/発行者に帰属し、その 権利は著作権法によって保護されています。引用にあたっては、著作権法を遵守してご利用ください。

The copyrights of content available on the KeiO Associated Repository of Academic resources (KOARA) belong to the respective authors, academic societies, or publishers/issuers, and these rights are protected by the Japanese Copyright Act. When quoting the content, please follow the Japanese copyright act.

OPTIMAL POLLUTION TAX IN COURNOT OLIGOPOLY

Којі ОКИБИСНІ

Department of Economics and Information, Gifu Shotoku Gakuen University, Gifu, Japan

First version received April 2003; final version accepted June 2003

Abstract: The optimal pollution tax rate is derived for Cournot oligopoly with firms which have non-constant returns technology for harmful effluents. The derivation is based on Shephard's duality theorem in production theory. The general optimal tax formula is given economic interpretations for some simple cases, including symmetric Cournot oligopoly with identical firms.

Key words: optimal pollution tax, Cournot oligopoly, harmful effluent, environmental damage **JEL Classification Number:** D43, L13

1. INTRODUCTION

The properties of the optimal pollution tax which maximizes the total social welfare of an economy are well known for two polar cases of competition in product market, i.e. perfect competition and monopoly. Under perfect competition the optimal pollution tax rate is to be set at the level equal to the marginal value of the environmental damage caused by firms' harmful effluents and under monopoly it should be lower than the marginal value of the environmental damage (see Barnett (1980), and Baumol and Oates (1980)). The properties of the optimal pollution tax for oligopoly with arbitrary number of firms are less known. Simpson (1995) has derived a formula for the optimal pollution tax for Cournot duopoly on the basis of duality theorem which relates a firm's cost function and its demand for factors of production, including harmful effluent. His analysis has shown that the optimal tax rate may not necessarily be lower than the marginal value of the environmental damage. Okuguchi and Yamazaki (1994) have analyzed the optimal pollution tax within Cournot oligopoly where firms' effluents are assumed to be proportional to their outputs. Their results relate the optimal tax rate to the marginal value of the environmental damage. In this paper I will derive the optimal pollution tax rate for Cournot oligopoly where, differently from Okuguchi and

E-mail: okuguchi@gifu.shotoku.ac.jp

Copyright@2003, by the Keio Econimic Society

Yamazaki (1994), firms are assumed to have non-constant returns technology for harmful effluents. Simpson (1995) has derived, on the basis of Shephard's duality theorem in production theory, the optimal pollution tax rate for Cournot duopoly with linear homogeneous production function. Following him, I will derive the optimal pollution tax rate for Cournot oligopoly with non-constant returns technology. I will elucidate economic implications of my general result for some special cases, including symmetric Cournot oligopoly with identical firms.

2. DUALITY APPROACH

Let there be *n* firms in Cournot oligopoly without product differentiation. Let *t* be the tax rate for a unit effluent, and $C_i(x_i, t)$ be firm *i*'s cost function, where x_i is firm *i*'s output. I assume constant prices for conventional factors of production other than effluent, hence I suppress them in cost function. Firm *i*'s profit function is given by

$$\pi_i = p x_i - C_i(x_i, t), \quad i = 1, 2, \dots, n,$$
(1)

where p is the price of the product, and

$$p = f(\sum x_j), \quad f' < 0$$

is the inverse market demand function for the product. Under the Cournot behavioristic assumption for firms, the first order condition for profit maximization for firm i is

$$f(\sum x_j) + x_i f'(\sum x_j) - C_i'(x_i, t) = 0, \quad i = 1, 2, \dots, n,$$
(2)

where I let $C_i' \equiv \partial C_i / \partial x_i$ for notational simplicity, and where I assume away a corner maximum. Let the industry output be $X \equiv \sum x_j$. I introduce the following assumption, which implies that any firm's output is a strategic substitute to any other firm's output.

ASSUMPTION 1.
$$f'(X) + x_i f''(X) < 0$$
, $i = 1, 2, ..., n$.

I need to introduce an additional assumption.

ASSUMPTION 2.

$$f' < C_i'' \equiv \frac{\partial C_i'}{\partial x_i}, \quad C_{it} \equiv \frac{\partial C_i}{\partial t} > 0, \quad i = 1, 2, \dots, n.$$

Under these two assumptions, the second order condition for profit maximization is satisfied. Solving with respect to x_i the implicit function (2) among x_i , X and t, and taking into account Assumptions 1 and 2, I get

$$x_i \equiv \varphi^i(X, t) \,, \tag{3}$$

where

$$\varphi_X^i \equiv \frac{\partial \varphi^i}{\partial X} = -\frac{f' + x_i f''}{f' - C_i''} < 0, \quad i = 1, 2, \dots, n.$$
 (4a)

$$\varphi_t^i \equiv \frac{\partial \varphi^i}{\partial t} = \frac{C_{it}}{f' - C_i} < 0, \quad i = 1, 2, \dots, n.$$
(4b)

Given t, the Cournot equilibrium industry output is characterized as the unique solution of the equation

$$X = \sum \varphi^{j}(X, t) \equiv \varphi(X, t), \qquad (5)$$

where, in light of (4a) and (4b), the partial derivatives of φ have the following signs.

$$\frac{\partial \varphi}{\partial X} < 0, \quad \frac{\partial \varphi}{\partial t} < 0.$$
 (6)

The above definition of the Cournot industry equilibrium output follows that of Szidarovszky and Yakowitz (1977), Okuguchi (1993), and Okuguchi and Yamazaki (1994). It is to be noted that the definition will remarkably simplify the analysis that follows. Let us introduce here an additional assumption.

ASSUMPTION 3.
$$f(0) > C_i'(0, t), i = 1, 2, ..., n.$$

Then (5) has a unique positive solution

$$X = X(t) \tag{7}$$

with

$$\frac{dX}{dt} = \frac{\sum \varphi_t^i}{1 - \sum \varphi_X^i} < 0.$$
(8)

Substituting (7) into (3), and differentiating it with respect to t, I derive

$$\frac{dx_i}{dt} = \varphi_X^i \frac{dX}{dt} + \varphi_t^i, \quad i = 1, 2, \dots, n,$$
(9)

the sign of which, as it stands, is indeterminate.

We are now in a position to derive the optimal pollution tax rate. Let e_i be firm *i*'s effluent level and $E \equiv \sum e_j$, and let $D \equiv D(E)$ be the value of total environmental damage caused by firms' harmful effluents. The total social welfare net of the environmental damage is defined as

$$W(t) = \int_0^{X(t)} f(x)dx - \sum C_j(x_j(t), t) + tE(t) - D(E(t)), \qquad (10)$$

of which differentiation with respect to t yields

$$\frac{dW}{dt} = \sum (f - C'_j) \frac{dx_j}{dt} - \sum \frac{\partial C_j}{\partial t} + E + t \frac{dE}{dt} - D' \frac{dE}{dt}.$$
 (11)

Shephard's theorem on duality between a firm's factor demand and its cost function leads to

$$\frac{\partial C_i}{\partial t} = e_i , \qquad (12)$$

which, combined with (11) and the first order condition, (2) makes me possible to derive

$$\frac{dW}{dt} = -f' \sum x_j \frac{dx_j}{dt} + (t - D') \frac{dE}{dt} \,. \tag{13}$$

The optimal value of t which maximizes the total net social welfare is the solution of dW/dt = 0. Hence, the optimal pollution tax rate is

$$t = D' + f' \sum x_j \frac{dx_j}{dt} \bigg/ \frac{dE}{dt} \,. \tag{14}$$

In Okuguchi and Yamazaki (1994), firm i's cost function is given by

$$C_i(x_i) = C_i^*(x_i) + t\gamma_i x_i, \quad i = 1, 2, \dots, n,$$
 (15)

where $C_i^*(x_i)$ is its conventional cost function when effluent is not a factor of production and $\gamma_i x_i$ is effluent proportional to output. Since in this case

$$E = \sum \gamma_j x_j \,, \tag{16}$$

(14) becomes

$$t = D' + f' \sum x_j \frac{dx_j}{dt} \bigg/ \sum \gamma_j \frac{dx_j}{dt}, \qquad (17)$$

which is nothing but the optimal pollution tax rate derived by Okuguchi and Yamazaki (1994) without using the duality theorem.

Some comments on (14) are in order. If the product market is perfectly competitive, f' = 0, therefore t = D'. If there is only one firm (monopoly), the second term in (14) is negative, and I get t < D'. As has been noted already, in oligopoly the sign of dx_j/dt is in general indeterminate for all j, therefore $t \le D'$ or t > D'. Assume now a symmetric case where $C_i = C$ or all i. In this case the equilibrium firms' outputs are identical, say $x_i = x$ for all i. Since X = nx, (8) implies dx/dt < 0. Hence, in this case the optimal pollution tax rate must be less than the marginal value of the environmental damage. This result is qualitatively the same as for monopoly.

3. CONCLUSION

The optimal pollution tax rate which maximizes the total net social welfare under imperfect competition in product market may be higher, lower than, or equal to the marginal value of the environmental damage caused by harmful effluents. In this paper I have derived the exact relationship between the optimal pollution tax rate and the marginal value of the environmental damage for Cournot oligopoly without product differentiation and with non-constant returns technology for harmful effluents. I have used Shephard's duality theorem to derive the optimal pollution tax rate. The result obtained in this paper contains as its special case the one derived earlier by Okuguchi and Yamazaki (1994), who have assumed that any firm's effluent is proportional to its output. I have also elucidated some implications of my fundamental results for simple cases. In symmetric Cournot oligopoly with identical firms, the optimal pollution tax rate is definitely lower than the marginal value of the environmental damage.

REFERENCES

- Barnett, A. H. (1980), "The Pigouvian Tax Rate under Monopoly", American Economic Review 70, 1034–1041.
- Baumol, W. J. and W. E. Oates (1980), *The Theory of Environmental Policy*, Second ed., Cambridge University Press.
- Okuguchi, K. (1993), "Unified Approaches to Cournot Models: Oligopoly, Taxation and Aggregate Provision of Pure Public Good", *European Journal of Political Economy* **9**, 233–245.
- Okuguchi, K. and T. Yamazaki (1994), "Ad Valorem and Specific Taxes, and Optimal Pigouvian Tax within Cournot Oligopoly", *Keio Economic Studies* **31**, 25–32.
- Simpson, R. D. (1995), "Optimal Pollution Taxation in a Cournot Duopoly", *Environmental and Resource Economics* 6, 359-369.
- Szidarovszky, F. and S. Yakowitz (1977), "A New Proof of the Existence and Uniqueness of the Cournot Equilibrium", *International Economic Review*18, 787–789.