Title	Competition and Efficiency in Industry Equilibrium
Sub Title	
Author	Sengupta, Jati K.
Publisher	Keio Economic Society, Keio University
Publication year	2003
Jtitle	Keio economic studies Vol.40, No.1 (2003.) ,p.59- 73
JaLC DOI	
Abstract	
Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-20030001-0059

KEIO ECONOMIC STUDIES **40**(1), 59–73 (2003)

COMPETITION AND EFFICIENCY IN INDUSTRY EQUILIBRIUM

Jati K. SENGUPTA

Department of Economics, University of California,
Santa Barbara, USA

First version received November 2002; final version accepted June 2003

Abstract: Efficiency of competitive industry equilibrium is analyzed here at two levels: firm efficiency and market efficiency. Efficiency at the firm level is determined in a semiparametric way by extending the nonparametric methods of efficiency measurement. The competitive industry then selects the optimal number of efficient firms by minimizing total industry costs under given market demand. Conditions of convergence to the efficient industry equilibrium in terms of Walrasian price and quantity adjustments are also discussed.

1. INTRODUCTION

Competition has been most intense in recent times in the new technology-based industries such as computers and telecommunications. Declining prices and average costs, accelerating global demand and increasing innovation efficiency have intensified the competitive pressure in these industries. The key role in this competitive pressure has been played by the cost efficiency of individual firms and the increasing market share of the cost efficient firms over time. At the industry level this has also intensified the exit rate (declining market share) of firms which failed to remain on the leading edge of the cost efficiency frontier.

Our objective here is two-fold. One is to analyze the dynamics of the market selection process in this competitive environment, where cost efficient firms prosper and grow and the less efficient ones decline and fall. The second is to analyze the cost efficiency of a firm in a nonparametric way based only on the observed cost and output data of firms comprising the whole industry.

Our analysis follows a two-stage approach. In the first stage each firm minimizes costs, given the market price. The estimation of the firm's cost frontier is obtained through a nonparametric approach, which has been adopted in recent years. This nonparametric approach initially developed by Farrell (1957) and later generalized in the theory of data envelopment analysis (DEA), see e.g., Charnes et al. (1994) and Sengupta (2000) estimates the cost frontier of a firm by a convex hull method based on the

Acknowledgement. An anonymous referee's helpful comments are gratefully acknowledged.

observed cost and output data of all firms. Unlike the method of least squares it does not purport to estimate an *average* cost function, i.e., it attempts to estimate cost-specific (input specific) efficiency of each firm relative to all other firms in the industry.

In the second stage the market clearing price is determined in the industry by minimizing total industry costs. The dynamics of adjustment around the industry equilibrium is then analyzed by a Walrasian process where prices rise in response to excess demand and fall in response to excess supply and the firm's output adjusts according to profitability.

2. THE TWO-STAGE MODEL

In the DEA models of efficiency analysis the cost efficiency has been separately analyzed from market efficiency. Thus Athanassopoulos and Thanassoulis (1995), and also Norman and Stoker (1991) analyzed market efficiency in a two-stage approach, where the relative efficiency of an individual firm in capturing its share of the total market is analyzed by a linear programming (LP) version of the DEA model. We attempt here to generalize this method by explicitly allowing a nonparametric treatment of the two stages. In the first stage we estimate a cost frontier in a quadratic convex form for a firm. The second stage allows the market selection process to select the most efficient of the firms specified to be cost efficient in the first stage. This method is very similar to the economic approach of Farrell and Johansen (1972). Farrell applied the convex hull method of estimation of technical efficiency without using any market prices but mentioned allocative efficiency as the industry level when the input and output prices are assumed to be determined by demand supply equilibrium in the market. Johansen used the individual firm's production frontiers to determine the industry production frontier by maximizing total industry output under the constraints imposed by the aggregate inputs and the convex technology.

In our approach we consider first the problem of estimation of the firm-specific cost frontier. Let C_h and C_h^* be the cost function and the cost frontier (i.e., minimal cost function) of firm h, where $\varepsilon_h = C_h - C_h^* \geq 0$ indicates cost inefficiency. We assume C_h^* to be quadratic and strictly convex, e.g., $C_h^* = \gamma_0 + \gamma_1 y_h + \gamma_2 y_h^2$, where y is output and the parameters $\gamma_0, \gamma_1, \gamma_2$ are all positive. For estimation of these parameters by the DEA approach we set up the LP model

$$\begin{aligned} &\text{Min } \varepsilon_h = C_h - C_h^* \\ &\text{s.t.} \quad C_j \geq \gamma_0 + \gamma_1 y_j + \gamma_2 y_j^2;\ j = 1, 2, \ldots, n \end{aligned} \tag{1}$$

based on n observations (C_j, y_j). Here total costs C_j include both variable and fixed costs and all firms are assumed to follow a given common technology. Timmer (1971) applied a variant of this method by minimizing the sum of absolute value of errors $\sum_{h=1}^{n} \varepsilon_h$, since his interest was in *robust* estimation of the production function. Sengupta (1990) has discussed other forms of estimation including corrected ordinary least squares and the generalized method of moments. The main advantage of this type of

DEA estimation is that it is firm specific. To see this more clearly[1] one may consider the dual of the LP problem (1):

$$\begin{aligned} &\text{Min} \sum_{j=1}^{n} \lambda_j C_j \\ &\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j y_j \geq y_h; \ \sum_{j=1}^{n} \lambda_j y_j^2 \geq y_h^2 \\ &\qquad \sum\nolimits_j \lambda_j \geq 1; \ \lambda_j \geq 0; \quad j = 1, 2, \ldots, n \end{aligned} \tag{2}$$

This can also be written as

$$\begin{aligned} &\text{Min}\,\theta \quad \text{s.t.} \quad \lambda \in R \quad \text{and} \\ &R = \left\{ \sum_{j=1}^{n} \lambda_j C_j \leq \theta C_h \text{ and the constraints of (2)} \right\} \end{aligned} \tag{3}$$

and λ is the column vector with n elements (λ_j) representing nonnegative weights of the convex combination of costs for each firm. Here θ is a scalar representing a measure of inefficiency, i.e., $\theta^* = 1.0$ indicates 100% efficiency and $\theta^* < 1.0$ denotes less than full efficiency, i.e., relative inefficiency.

On using the Lagrangian function

$$\begin{aligned} L = &-\theta + \beta(\theta C_h - \textstyle\sum \lambda_j C_j) + \alpha_1(\sum \lambda_j y_j - y_h) + \alpha_2(\sum \lambda_j y_j^2 - y_h^2) \\ &+ \beta_0(\textstyle\sum \lambda_j - 1) \end{aligned}$$

and applying Kuhn–Tucker conditions with respect to θ and λ_j one obtains the cost frontier for firm j, when all the slack variables are zero

$$\begin{aligned} &\beta C_h = 1, \theta \text{ free in sign and} \\ &C_j = \gamma_0 + \gamma_1 y_j + \gamma_2 y_j^2; \ \gamma_i \geq 0, i = 0, 1, 2 \end{aligned} \tag{4}$$

where $\gamma_0 = \beta_0/\beta$, $\gamma_1 = \alpha_1/\beta$ and $\gamma_2 = \alpha_2/\beta$.

Note that the condition $\partial L/\partial\theta = 0$ yields the numeraire condition. On varying h in the index set $I_n = \{1, 2, \ldots, n\}$ the cost efficiency frontier for all the firms can be determined. Note that for any firm h which is less than 100% efficient, i.e., $\theta^* < 1.0$, one can adjust its cost C_j to $\theta^* C_j$ so that in terms of the adjusted cost firm h will be 100% efficient. Note that if we drop the constraint $\sum \lambda_j y_j^2 \geq y_h^2$ from (2) we obtain the linear cost frontier

$$C_h = C_h^* = \gamma_0 + \gamma_1 y_h \tag{5}$$

Here the constraint $\sum_j \lambda_j y_j^2 \geq y_h^2$ implies that the resulting cost function is strictly convex, which yields a unique cost minimizing output. Thus equation (4) may be viewed as a semiparametric method of determining the cost frontier, whereas (5) is usually called the nonparametric method of estimating the cost frontier. Next we consider the second

[1] The objective function of (1) may be written as: Max $C_h^* = \gamma_0 + \gamma_1 y_j + \gamma_2 y_j^2$ so that the dual problem may be written as a minimization problem.

stage of the market demand process which selects among the first stage efficiency set, so that the total industry cost (*TC*) is minimized. But since the cost frontiers of firms are not all identical, we have to assume that firms are identified by their cost structures, where each firm is assumed to belong to one of m possible types of costs, each producing a homogeneous output. Let n_j be the number of firms of type $j = 1, 2, \ldots, m$ cost structure. We now minimize *TC* for the whole industry, i.e.,

$$\begin{aligned} &\underset{\{n_j, y_j\}}{\text{Min}}\ TC = \sum_{j=1}^{m} n_j C_j(y_j) \\ &\text{s.t.} \quad \sum\nolimits_{j=1}^{m} n_j y_j \geq D; \ (n_j, y_j) \geq 0 \end{aligned} \tag{6}$$

where $C_j = C_j(y_j)$ denotes the cost frontier of firm j in terms of either (4) or (5). Total market demand D is assumed to be given. Clearly if $D > 0$, then we must have $n_j y_j > 0$ for some $j = 1, 2, \ldots, m$. On using p as the Lagrange multiplier for the market demand supply constraint and assuming the vector $n = (n_1, n_2, \ldots, n_m)$ to be given, total industry cost *TC* in (6) is minimized if and only if the following conditions hold for given $D > 0$,

$$MC_j(y_j) - p(n, D) \geq 0 \quad \text{and} \quad y_j[MC_j(y_j) - p(n, D)] = 0\,, \quad \text{for all } j \tag{7}$$

where $MC_j = MC_j(y_j)$ is the marginal cost frontier, $i \ldots$, $MC_j = \gamma_1 + 2\gamma_2 y_j$ and $p = p(n, D)$ may be interpreted as the market clearing price, i.e., the shadow price which equates total supply $S = \sum n_j y_j$ to demand D. Let $\hat{y}_j = \hat{y}_j(p)$be the optimal solution to (6). Here price p depends on n and D and hence $\hat{y}_j(p)$ may also be written as $\hat{y}_j(n, D)$. Then clearly it follows that $\hat{y}_j$ variables are uniquely determined given D and vector n. Optimal total costs can then be specified as

$$TC = \sum_{j=1}^{m} n_j C_j(\hat{y}_j(n, D))$$

Now consider the Lagrangian function L

$$L = -\sum_{j=1}^{m} n_j(\gamma_0 + \gamma_1 y_j + \gamma_2 y_j^2) + p(\sum_j n_j y_j - D)\,.$$

The first order conditions $\partial L/\partial n_j = 0 = \partial L/\partial y_j$ yield

$$p = AC_j = \gamma_0/y_j + \gamma_1 + \gamma_2 y_j \quad \text{and} \quad p = MC_j = \gamma_1 + 2\gamma_2 y_j \tag{8}$$

which implies that $p = \min AC_j$. Since n_j is an integer, the continuity of the Lagrangian function $L = L(y_j, n_j, p)$ may not hold with respect to n_j. If we ignore this integral requirement as an approximation, then the Kuhn–Tucker theorem may be easily applied to determine the optimal values $(\hat{y}_j, \hat{n}_j, p)$ by finding the saddle point $(\hat{y}, \hat{n}, \hat{p})$ of the Lagrangian L for some nonnegative vectors $\hat{y}, \hat{n}, \hat{p}$. Since the L function is strictly concave in y for a fixed $n = (n_j)$ and it is linear in n for any fixed y and p, the sufficiency conditions are also fulfilled.

In the realistic case of integral n_j however we have to adopt a different method. Now we index the firms with a continuous parameter denoted by u, which replaces the integer index j. Then

$$\tilde{C}(u) = g(u, y(u))$$

gives the total cost function of firm j. The TC function for the whole industry becomes

$$C(s) = \int_0^s \tilde{C}(u)\, F(u)\, du \quad 0 \le s \le 1$$

in place of the sum $\sum n_j C_j$, where $F(u)$ denotes the number of firms of type u and the upper limit of the integral s denotes the value of the index of the "marginal firm", i.e., the firm that it just pays to operate, given the desired total output. The industry's total output is given by $Y(s) = \int_0^s y(u)\, F(u)\, du$.

Now we order the firms with respect to their minimum average cost, so that it is a rising function of the continuous indexing parameter u. The optimal s and y is then the solution of the following problem

$$\text{Min}_{(s,y)\ge 0}\, C(s) \quad \text{s.t.} \quad Y(s) = D > 0$$

The Lagrangian is given by

$$C(s) + p\Big(D - \int_0^s y(u)\, F(u)\, du\Big)$$

If there is a minimum, then the following relations must hold:

$$\partial C(s)/\partial y - p\int_0^s F(u)\, du \ge 0 \quad \text{and} \quad \partial C(s)/\partial s - py(s)F(s) \ge 0\,.$$

Since

$$\frac{\partial C(s)}{\partial s} = \tilde{C}(s)F(s) \quad \text{and} \quad \frac{\partial C(s)}{\partial y} = \int_0^s \frac{\partial \tilde{C}}{\partial y}\, F(u)\, du$$

it follows that

$$\int_0^s \left(\frac{\partial \tilde{C}}{\partial y} - p\right) F(u)\, du \ge 0 \qquad \tilde{C}(s) - p\, y(s) \ge 0 \tag{9}$$

By hypothesis the output of the marginal form is positive, hence the necessary conditions in (9) become equalities. Moreover, without loss of generality we may assume that $F(u) > 0$ for all u with $0 < u < s$. These imply that

$$\frac{\partial \tilde{C}(u)}{\partial y} = p, \text{ for all } u \text{ with } 0 \le u \le s \quad \text{and} \quad \frac{\partial \tilde{C}(s)}{\partial y(s)} = \frac{\tilde{C}(s)}{y(s)}, \text{ i.e., } MC = AC\,.$$

The first condition means that for all active firms (i.e., firms with positive outputs) there must be a common marginal cost. The second condition means that the scale of operation of the "marginal firm" is such that its average cost is a minimum. These are the same conditions as in (7). Also, for all extra marginal firms, i.e., for all $u > s$, $y(u) = 0$.

Let us now denote by $\hat{y}$ and $\hat{n}$ the optimizing values for the total cost function $C(s)$. These may be viewed as the continuous approximation of the earlier problem with a discrete number of firms. These variables satisfy $p = AC$ and $AC = MC$. Thus the long run equilibrium price p is the minimum point of the AC curve, i.e., the minimum efficient scale (MSE). We may thus use $\hat{n}_j$ and $\hat{y}_j$ as the discrete approximation of the continuous model.

Note that the specific computation of the minimum efficient scale of output is possible here due to the quadratic cost function used here for illustration. The quadratic form of the convex function in (1) allows us to write the dual problem (2) in a linear form, as is customary in the nonparametric efficiency analysis approach of data envelopment analysis. However, log linear and other nonlinear forms can be introduced in (1) yielding a nonlinear DEA model. But the results in (9) would still hold due to the convexity of the total cost function with respect to n and y.

Several economic implications of this result may now be discussed. First of all, if the market demand function is viewed in its inverse form, i.e., $p = F(D)$ where $D = \hat{Y}$, $\hat{Y}$ being the aggregate output, then

$$F(Y) \geq LRAC(\hat{y}) \text{ as } Y \leq \hat{Y}$$

where LR denotes the long run. But since $LRAC(y_j)$ is of the form

$$LRAC(y_j) = \frac{\gamma_0}{y_j} + \gamma_1 + \gamma_2 y_j$$

its minimum is attained at

$$\hat{y}_j = (\gamma_0/\gamma_2)^{1/2} = (\gamma_{0j}/\gamma_{2j})^{1/2}$$

$$LRAC_j(\hat{y}_j) = \alpha_1 + 2\sqrt{\gamma_0\gamma_2}$$

Thus a dynamic process of entry (or increase in market share) or exit (or decrease in market share) can be specified as a Walrasian adjustment process

$$dn_j/dt = k_j(p - LRAC(\hat{y}_j)) \quad \text{and} \quad dp/dt = b(D(p) - (\hat{Y})) \tag{10}$$

where k_j and b are positive parameters. The equilibrium is then given by the steady state values $p^* = LRAC(\hat{y})$ and $D(p^*) = (\hat{Y})$. This entry (exit) rule (10) is different from the limit pricing rule developed by Gaskins (1971) and others, in that this is determined directly from the estimate of MES of cost efficient firms. Also different cost structures are allowed here, which implies that in terms of minimum average costs different firms can be ranked. For example if firms are ordered from the lowest to highest according to minimum average costs $\hat{c}_j$ as follows:

$$\hat{c}_{(1)} < \hat{c}_{(2)} < \cdots < \hat{c}_{(k)}, \; k \leq n$$

then $p_{(1)} = \hat{c}_{(1)}$ would be the lowest price, whereas $p_{(k)} = \hat{c}_{(k)}$would be the highest. Hence if due to exogenous demand shift the price p comes down to $p = p_{(1)}$, then all other firms have to exit in the long run. Likewise if demand shift raises the price to $p = p_{(k)}$ then the other firms would earn positive '*rents*' as

$$\Delta_{(j)} = p_{(j)} - p_{(1)}, \; j = 2, 3, \ldots, k \tag{11}$$

Low cost firms can produce at a lower AC than the others, because they may possess some scarce factor, such as superior technology, which is not available to others. Thus the low cost firms may earn for some time more than normal profits, i.e., excess profits. Some potential entrants, seeing the large profits made by the low cost firms, would want to adopt the superior technology, thus wiping out the extra rent. Until this happens, the low-cost firms would enjoy positive differential rent, i.e., early adopter's profit advantage. Thus in the long run the variance of $\hat{c}_{(j)}$ or of differential rent $\Delta_{(j)}$ would decline, though it may be high in the short run.

Now consider the case when each firm j has a separate cost function $C_j(y_j)$, i.e., $m = 1$. The industry model then takes a simple form

$$\begin{aligned} &\text{Min} \sum_{j=1}^{n} C_j(y_j) \\ &\text{s.t.} \quad \sum y_j \geq D;\ y_j \geq;\ j \epsilon I_n \end{aligned} \tag{12}$$

By rewriting the cost function as $C_j(y_j; k_j)$ where k_j is capital endowment, short and long run cases can be distinguished. The short run case assumes k_j to be constant so that the cost function depends on output only while in the long run case the cost function depends on output and capital inputs, which are both variable. In the long run case the Lagrangian function can be written as

$$\begin{aligned} L &= -\sum_{j=1}^{n} C_j(y_j, k_j) + p(\textstyle\sum y_j - D) \\ &= \sum_{j=1}^{n} \left[p y_j - C_j(y_j, k_j)\right] - pD \\ &= \sum_{j=1}^{n} \pi_j - pD \end{aligned} \tag{13}$$

where π_j is the profit function of firm j, if p is interpreted as the market clearing price. In a competitive industry each firm is a price taker, so market price is given as $\hat{p}$. In the short run capital inputs are also given as $\hat{k}_j$. Hence the vector $Y^* = \left(y_1^*, y_2^*, \ldots, y_n^*\right)$ is a short run industry equilibrium (SRIE) if each firm j maximizes profit π_j with respect to y_j. Since the cost function is strictly concave, this SRIE $Y^*(\hat{K})$ exists and it is unique for every given vector $\hat{K} = (\hat{k}_1, \hat{k}_2, \ldots, \hat{k}_n)$. For the long run industry equilibrium LRIE we have to modify the objective function as long run profits defined as

$$W_j = \int_0^\infty e^{-\rho t}[\pi_j(t) - h(u_j(t))]dt \tag{14}$$

where $u(t)$ is investment defined by $dk_j/dt = u_j(t) - \delta_j k_j(t), h(u_j(t))$ is a convex cost function and δ is the fixed rate of depreciation.

The LRIE is now defined by vectors K^*, Y^* if for each firm j,

$$\begin{aligned} &\text{i)} \quad y_j^* \text{ maximizes } W_j \text{for given } k_j\,, \\ &\text{ii)} \quad k_j^* \text{ maximizes } W_j \text{for given } y_j^*\,. \end{aligned} \tag{15}$$

But since investment cost function is separable, the maximization problem (i) is equivalent to max π_j. Here $\hat{p} = p(\hat{y}) = p(y^*)$ and $\hat{y} = y^* = \sum_{j=1}^{n} \hat{y}_j$. Thus the industry equilibrium price p^* clears the market and given p^* each firm maximizes long run profit with respect to y_j and k_j. Now define a *competitive industry equilibrium* by vectors Y^*, K^* and price p^* such that $D = D(p^*) = \sum y_j^*$ and the conditions (15) hold. Then one can easily prove that such a competitive industry equilibrium exists, since the cost functions are strictly convex and the profit function strictly concave; see e.g., Dreze and Sheshinski (1984).

3. DYNAMIC ADJUSTMENTS

Two types of dynamic adjustments are implicit in the two-stage model of competitive industry equilibrium outlined in earlier sections. One is the path of optimal accumulation of capital by each efficient firm j, which maximizes the discounted stream of profits. This involves continuous adjustment of existing capacity (capital) by an optimal investment program so as to reduce the current cost of using exiting capacity for producing current outputs. For example, consider a quadratic cost function $h_j = g_1 u_j + g_2 u_j^2$ for each firm j, which solves the dynamic problem

$$\begin{aligned} &\min_{u_j} \int_0^\infty e^{-\rho t} \left[\gamma_0 + \gamma_1 y_j + \gamma_2 y_j^2 + \beta_1 k_j + \beta_2 k_j^2 + h_j\right] dt \\ &\text{s.t.} \quad \dot{k}_j = u_j - \delta k_j;\ \dot{k}_j = dk_j/dt \\ &\qquad k_j(0) > 0 \text{ given} \end{aligned}$$

On using the current value Hamiltonian involving k_j and u_j:

$$H = -\beta_1 k_j - \beta_2 k_j^2 - g_1 u_j - g_2 u_j^2 + \mu\,(u_j - \delta k_j)$$

Pontryagin's maximum principle yields the optimality conditions

$$\begin{aligned} \dot{\mu} &= (\rho + \delta)\mu + \beta_1 + 2\beta_2 k_j \\ u_j &= (2g_2)^{-1}(\mu - g_1) \\ \dot{k}_j &= u_j - \delta k_j \end{aligned}$$

On eliminating μ one obtains the pair of differential equations

$$\begin{pmatrix} \dot{u}_j \\ \dot{k}_j \end{pmatrix} = \begin{bmatrix} (\rho + \delta) & \beta_2/g_2 \\ 1 & -\delta \end{bmatrix} \begin{pmatrix} u_j \\ k_j \end{pmatrix} + \begin{pmatrix} A_1 \\ 0 \end{pmatrix} \tag{16}$$

where $A_1 = (\beta_1 + (\rho + \delta)g_1)/(2g_2)$.

The characteristic equation is

$$\lambda^2 - \rho\lambda - [\delta(\delta + \rho) + \beta_2/g_2] \tag{17}$$

Since the product of two roots is negative and the sum positive and

$$\rho^2 + 4(\beta_2/g_2 + \delta(\rho + \delta))$$

is positive, the two roots are real, one positive and one negative. Hence there exists a saddle point equilibrium. The steady state levels are given by

$$\begin{aligned} \bar{k}_j &= -(\beta_1 + g_1(\rho + \delta))/2(\beta_2 + (\rho + \delta)\delta g_2) \\ \bar{u}_j &= \delta \bar{k}_j \end{aligned}$$

It is clear that as the cost coefficients g_1 or β_1 rise the steady state levels of capital and hence investments decline. The two characteristic roots imply that there is a stable manifold along which the motion of the system (16) is purely towards $(\bar{u}_j, \bar{k}_j)$ and an unstable manifold along which motion is exclusively away from $(\bar{u}_j, \bar{k}_j)$. By transversality conditions one may choose only the stable manifold. By using this stable manifold around the steady state equilibrium one may state the following proposition.

PROPOSITION 1. *For each vector K there exists a* SRIE $Y^*(K)$, *where each* y_j^* *maximizes* $\pi_j = p^* y_j - C_j(y_j, k_j)$. *There also exist a* LRIE *given by the pair* (Y^*, K^*) *where for each firm j:*

(i) y_j^* *solves max* $\pi_j = p^* y_j - C_j(y_j, k_j^*)$

and

(ii) k_j^* *and* u_j^* *solve the steady state level of profits* $\bar{\pi}_j = p^* y_j^* - C_j(y_j^*, k_j) - h(u_j)$.

The SRIE *and* LRIE *solutions are unique.*

Proof. Existence follows from the fact that the production set is convex, closed and bounded by assumption. Strict concavity of the profit function yields uniqueness. Equilibrium market price $p^* = P(y^*)$, $y^* = \sum_{j=1}^{n} y_j^*$ equalizes total demand and supply.

The adjustment of the market equilibrium may be directly shown in terms of the Walrasian process of price quantity adjustments as specified in (10) before, e.g.,

$$\begin{aligned} \dot{y} &= a[p - c(y)] \\ \dot{p} &= b[D(p) - y] \end{aligned} \qquad (18)$$

where $c(y)$ is the long run minimal average cost function and a, b are positive parameters. The minimal average cost function intersects the marginal cost function at the optimal output y^* and the marginal cost is linear in output for the quadratic cost frontier; also the demand function $D(p)$ is linear in this case. Hence (18) can be viewed as a linearized version around the optimal output level y^*. Thus the linear differential equations for (18) can be analyzed in terms of the characteristic roots. For nonlinear forms of $c(y)$ and $D(p)$ we have to consider linearized versions around the optimal point y^* in order to analyze the stability of the adjustment process. The conditions for convergence to the steady state of this linearized system are once again specified by its characteristic roots. Two important cases are:

(i) each root has a negative real part; this implies that the steady state equilibrium is stable in the sense of convergence to the steady state,

and

(ii) two real roots, one positive and one negative; this implies a saddle point equilibrium. There is a stable manifold of convergence.

Hence one can state the result.

PROPOSITION 2. *There exists a stable manifold along which the* LRIE *can be reached by a Walrasian adjustment process.*

Proof. Here one can apply the linearizing process to the cost $c(y)$ and demand $D(p)$ functions by taking their slopes c' and d', so that the characteristic equation can be derived. Thus by using the explicit cost functions assumed to be strictly convex, the two roots of the characteristic equation can be directly computed, e.g.,

$$\lambda^2 + (ac' - bD'))\lambda + ab(1 - c'D') = 0 \tag{19}$$

where $c' = \partial C/\partial y$, $D' = \partial D/\partial p$. Equation (19) has roots with negative real parts if and only if $ac' - bD' > 0$ and $(1 - c'D) > 0$ and $ab(1 - c'D') > 0$. But with $D'(p) < 0$, $c' > 0$ implies that these conditions hold. In the second case if it holds that $c' < 1/D' < 0$, then one real root is positive and the other negative. Now the two roots are

$\lambda = -1/2(ac' - bD') \pm 1/2[(ac' - bD')^2 - 4ab(1 - c'D')]^{1/2}$. Since $c' < 1/D'$, there are two real solutions in λ, one positive and one negative. The point (y^*, p^*) is now a saddle point. There is a stable manifold along which the motion converges towards (y^*, p^*) and an unstable manifold along which the motion is away from (y^*, p^*). The slopes of these manifolds at (y^*, p^*) are given by the eigenvectors of the matrix in (19a)

$$\begin{pmatrix} \dot{y} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} -a\frac{dc}{dy} & a \\ -b & b\frac{dD}{dp} \end{pmatrix} \begin{pmatrix} y \\ p \end{pmatrix} \tag{19a}$$

corresponding to the stable and the unstable roots respectively. From this one can readily verify that the unstable manifold has a negative slope at the point (y^*, p^*). Note that in the general nonlinear case the Lyapunov theory of stability has to be applied, e.g., Takayama (1988).

Two points are to be noted. One is that the concept of convergence used here is in the long run, when the system remains in its phase space within which certain properties hold and not that it actually converges to a point. Secondly, in the short run the system must be such that the initial point must be "close" to equilibrium and just right as specified by the characteristic vectors corresponding to the characteristic roots.[2]

Some important implications of the two dynamic adjustment processes have to be briefly mentioned. First of all, if individual firms do not follow these optimal paths of

[2] The author is indebted to the referee for emphasizing this point clearly.

capital accumulation, then they would not be consistent with long run industry equilibrium. Also, firms which are cost efficient in the short run may not be so in the long run unless they use an optimal investment path. Hence there is scope for analyzing inefficiency in the long run. This aspect has been analyzed in a nonparametric framework by Sengupta (1999). Secondly, the competitive industry model developed here has a decentralization interpretation in terms of firms surviving under long run equilibrium, see e.g., Gabszewicz and Michel (1991). Novshek (1980) has shown that this type of equilibrium can be extended to include the case of Cournot equilibrium, if firm size is measured by technology, market size measured by perfectly competitive demand and if firms are small relative to the overall market and free entry conditions prevail. In such a case Cournot equilibrium exists and the aggregate output is approximately competitive. Finally, one could empirically test the consistency of the cost efficiency model (4) estimated by the DEA model in respect of the industry equilibrium. So long as MES levels are different for cost efficient firms, there exists some scope for improving efficiency in the long run. This implies price changes due to entry and exit of firm in the industry with a consequent impact on individual firms through allocative efficiency.

4. ALLOCATIVE EFFICIENCY IN MARKET EQUILIBRIUM

The industry equilibrium in competitive markets may be analyzed more directly if we assume that market clearing prices are estimated by a demand function $\hat{p} = a - by$, $y = \sum_{j=1}^{n} y_j$. In this case the industry equilibrium is directly obtained from maximizing total industry profits π, where the cost function of each firm is strictly convex and quadratic:

$$\underset{y_j}{\text{Max}}\, \pi = \sum [\hat{p} y_j - C_j(y_j)] \tag{20}$$

Since $\hat{p}$ is the market clearing price, equilibrium of market demand and supply is implicit here. The industry selection process (20) selects optimal outputs y_j^* and $y^* = \sum y_j^*$ so as to meet total demand by following the rule

$$\begin{aligned} y_j^* &= A_j - B_j y^* \\ y^* &= \Big(1 + \sum_j B_j\Big)^{-1} \Big(\sum A_j\Big) \end{aligned} \tag{21}$$

where $A_j = (b + 2\gamma_{2j})^{-1}(a - \gamma_{1j})$, $B_j = (b + 2\gamma_{2j})^{-1} b$.

Let c_j^* be the optimal average cost $C_j(y_j^*)/y_j^*$, then the entry or increased market share rule can be specified as

$$\dot{y}_j = k_j(\hat{p} - c_j^*)\,, \quad k_j > 0$$

i.e., entry (market share) is positive (increasing) or negative (decreasing) according as $\hat{p}$ exceeds (falls short of) c_j^*. The price adjustment in the market can be similarly specified as

$$\dot{p} = k(D(\hat{p}) - y^*)\,, \quad k > 0$$

The equilibrium supply behavior specified in (21) implies the following comparative static consequences:

$$\partial y_j^*/\partial \gamma_{2j} < 0, \partial y_j^*/\partial \gamma_{1j} < 0, \partial y_j^*/\partial b < 0 \quad \text{and} \quad \partial y_j^*/\partial a > 0$$

If all cost functions are identical so that $A_j = A$ and $B_j = B$ for all j, then one obtains

$$\begin{aligned} y_j^* &= A - B(1 + nB)^{-1}(nA) \\ y_j^* &= (1 + nB)^{-1}(nA);\ p = a - by^* \end{aligned}$$

This shows the impact of the number of numbers in the industry on equilibrium industry output.

When firms are not alike in their cost functions but belong to a cost structure, each firm may follow one of m possible types of cost. Let n_j be the number of firms of type $j = 1, 2, \ldots, m$ cost structure. Then the allocative efficiency model (20) takes the form

$$\max_{n_j, y_j} \pi = \sum_{j=1}^{m} \hat{p} n_j y_j - \sum_{j=1}^{m} n_j C_j(y_j)$$

where $\hat{p} = a - b \sum_j n_j y_j;\ a, b > 0$. This yields the equilibrium conditions

$$\begin{aligned} p_j^* &= \hat{p}(y_j^*) = \left(1 - \tfrac{1}{|\varepsilon_p|}\right)^{-1} MC_j(y_j^*) \\ p_j^* &= AC_j(y_j^*)\ \left(1 - |\varepsilon_{n_j}|\right)^{-1} \end{aligned} \tag{22}$$

where ε_p and ε_{n_j} are the price elasticity of demand and size elasticity of price respectively, i.e.,

$$\varepsilon_{n_j} = (\partial p/p)(\partial n_j/n_j), \varepsilon_p = (\partial y_j/y_j)(\partial p/p)$$

When $|\varepsilon_p|$ tends to infinity and ε_{n_j} tends to zero then we obtain the earlier result (9), i.e.,

$$p = MC_j(y_j^*) = AC_j(y_j^*), \quad y_j^* > 0$$

Thus we can state the result:

PROPOSITION 3. *There exists an industry equilibrium specified by the pair of vectors (n^*, Y^*), where $n^* = (n_1, n_2, \ldots, n_m)$ and $Y^* = (y_1^*, y_2^*, \ldots, y_n^*)$ which satisfy the optimality conditions (22). If each cost function is strictly convex and quadratic, then this equilibrium pair (n^*, Y^*) is unique. Furthermore, there exists a stable manifold along which the equilibrium could be reached from a nearby nonequilibrium point.*

Proof. Since the profit function is closed and bounded by its continuity with respect to (n, Y), the existence of industry equilibrium is assured. By strict concavity the equilibrium is unique. Furthermore, the Walrasian adjustment process defined by the free entry (exit) rules possesses a stable manifold due to the negative slope of the demand function and strict convexity of the cost function.

Several implications of this proposition are important in economic terms. First of all, optimality of the industry equilibrium (n^*, Y^*) may be tested against the observed outputs y_j and firm sizes n_j. In cases of disequilibrium the observed values would differ from the optimum and hence the market process of adjustment through entry and exit has to be analyzed. Secondly, the impact of demand on equilibrium output and price can be directly evaluated in this framework. In particular, large demand fluctuations would tend to have some adverse reaction for the risk averse producers, e.g., their optimal output would tend to be lower. "Finally, market concentration measured by unequal firm sizes would affect the industry equilibrium, e.g., firms with the least optimal average cost would survive longer.

5. APPLICATION IN COMPUTER INDUSTRY

In order to indicate the usefulness of the cost efficiency concepts developed here, we may illustrate their application in the US computer industry. Growth and efficiency in this industry over the period 1985–2000 have been analyzed by Sengupta (2002) in some detail elsewhere. Here we select 10 out of 22 companies from the earlier study and estimate minimum efficient scale (MES) for three selected years 1987, 1990 and 1997. The data set is taken from Standard and Poor's Compustat file. The total costs here comprise the following: R & D expense, cost of goods sold and the cost of plant and equipment net of depreciation. Cost of goods sold includes manufacturing, marketing and administrative costs and the cost of change in inventory. Total combined costs comprise 75 to 80% of overall costs. For the output measure 'net sales' data are used and these are not deflated due to lack of a suitable price index.

Since the cost components are not deflated and it is difficult to separate the short run and the long run components of total costs we adopt a nonradial measure of cost efficiency. This measure differs from a radial measure in that different components may grow or decline at different proportions. Thus in order to test the cost efficiency of firm h the following LP model is set up.

$$\begin{aligned} &\text{Min} \sum_{i=1}^{3} \theta_i \\ &\text{s.t.} \quad \sum_{j=1}^{n} C_{ij}\lambda_j \le \theta_i C_{ih};\ i = 1, 2, 3 \\ &\qquad \sum_{j=1}^{n} y_j\lambda_j \ge y_h;\ \sum y_j^2\lambda_j \ge y_h^2 \\ &\qquad \sum \lambda_j = 1, \lambda_j \ge 0, j = 1, 2, \ldots, n \end{aligned} \tag{23}$$

By using $C_j = \sum_{i=1}^{3} \beta_i C_{ij}$ as the total cost measure, where β_i's are the shadow prices of the first three constraints of (23), one could derive the cost frontier specified by (4) before. The estimates of AC_j and min AC_j are then obtained and denoted by c_j and c_j^* and the gap $\varepsilon_j = c_j - c_j^*$ measures the scope of unutilized capacity.

Table 1 shows the estimates of c_j and ε_j for 10 selected companies and their rank in terms of closeness of ε_j to zero. These estimates are only illustrative, since the sample

Table 1. DEA estimates of average cost (c_j) on the frontier and its minimal value (c_j^*)

Company	1987			1990			1997		
	c_j	ε_j	rank	c_j	ε_j	rank	c_j	ε_j	rank
1. Apple	0.58	0.0	1	0.59	0.0	1	0.98	0.34	7
2. Compaq	0.76	0.18	3	0.83	0.25	6	0.82	0.18	5
3. Datapoint	0.86	0.28	6	0.69	0.11	3	0.76	0.12	3
4. Dell	0.74	0.16	2	0.73	0.15	4	0.81	0.17	4
5. Hewlett Packard	0.82	0.24	5	0.84	0.26	7	0.84	0.21	6
6. Hitachi	0.98	0.40	7	0.96	0.38	8	1.00	0.41	8
7. Silicon Graphics	4	0.56	8	0.63	0.05	2	0.73	0.09	2
8. Sun	0.78	0.28	4	0.76	0.18	5	0.65	.01	1
9. Maxwell	0.85	0.58	9	0.81	0.39	9	0.80	0.43	9
10. Encore	0.90	0.61	10	0.91	0.41	10	0.93	0.44	10
Note: optimal AC (average)		0.59			0.58			0.64	

size is very small. Nevertheless, two broad results are indicated. First, there is wide diversity in the pattern of average costs along the cost frontier, hence the levels of MES vary significantly. Thus Apple Computer seems to have the best cost structure at the end of 1980s and the beginning of 1990 but after 1995 the company had trouble in its marketing policy and its effects are evident in the ranking. By contrast Dell Computer retained its market share due to competitive pricing based on MES. Secondly, some firms like Maxwell and Encore did not perform very well from the beginning relative to the others and their market shares declined considerably. In the long run their survival is in great doubt. The R & D expenditure played a very critical role in maintaining the rank of a firm in the efficiency scale over time. The detailed study analyzes this trend elsewhere.

6. CONCLUDING REMARKS

The efficiency of competitive equilibrium is analyzed here in two stages: firm efficiency and market efficiency. At the first stage each firm seeks to attain the cost frontier given the market price. The set of firms, each on its cost frontier is then subjected to a market selection process. This selection process chooses among those firms selected in the first stage by their relative profitability, when prices help in the selection process by clearing the market.

The dynamics of entry or exit behavior of firms in this competitive industry are analyzed in this framework, so that the roles of demand and supply forces in disequilibrium can be better understood.

REFERENCES

Athanassopoulos, A., and Thanassoulis, E., "Separating market efficiency from profitability and its implications for planning," *Journal of the Operational Research Society*, Vol. 46 (1995), 20–34.

Charnes, A., Cooper, W. W., Lewin, A., and Seiford, L., *Data Envelopment Analysis: Theory, Methodology and Applications*, Dordrecht, Kluwer Academic Publishers (1994).

Dreze, J., and Sheshinski, E., "On industry equilibrium under uncertainty," *Journal of Economic Theory*, Vol. 33 (1984), 88–97.

Farrell, M. J., "The measurement of productive efficiency," *Journal of Royal Statistical Society*, Series A, Vol. 120 (1957), 253–290.

Gabszewicz, J., and Michel, P., "Capacity adjustments in a competitive industry," In *Equilibrium Theory and Applications*, New York, Cambridge University Press (1991).

Gaskins, D. W., "Dynamic limit pricing: optimal pricing under threat of entry," *Journal of Economic Theory*, Vol. 3 (1971), 306–322.

Johansen, L., *Production Functions*, Amsterdam, North Holland (1972).

Norman, M., and Stoker, B., *Data Envelopment Analysis: The Assessment of Performance*, New York, John Wiley (1991).

Novshek, W., "Cournot equilibrium with free entry," *Review of Economic Studies*, Vol. 47 (1980), 473–486.

Sengupta, J. K., "Transformations in stochastic DEA models," *Journal of Econometrics*, Vol. 46 (1990), 109–123.

Sengupta, J. K., "The measurement of dynamic productive efficiency," *Bulletin of Economic Research*, Vol. 51 (1999), 111–124.

Sengupta, J. K., *Dynamic and Stochastic Efficiency Analysis*, London, World Scientific (2000).

Sengupta, J. K., "Growth and efficiency in the computer industry." Paper sent for publication.

Takayama, A., *Mathematical Economics*, New York, Cambridge University Press (1988).

Timmer, C. P., "Using a probabilistic frontier function to measure technical efficiency," *Journal of Political Economy*, Vol. 79 (1971), 776–794.