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FACTOR INTENSITY AS EUCLIDEAN DISTANCE

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Abstract: This note extends the concept of factor intensity to include any number of productive factors and products. The proposed measure is the Euclidean distance from the unit value of each factor to the intersection of product intensity rays with the unit factor hyperplane. Factor intensity distance is a generalization of the two dimensional definition. An example of factor intensity distance in US manufacturing with 7 factors and 8 products is presented and the potential of factor intensity distance in empirical applications is discussed.

Factor intensity is a key concept in factor proportions trade theory formalized by Samuelson (1953–54) and popularized by Jones (1965). Factor intensity is defined in the model with two factors and two products but difficulties arise with as few as three of each. Ruffin (1981) shows factor intensity has a clear interpretation in the model with three factors and two products. McKenzie (1955), Leamer (1987), and Jones and Margit (1991) develop a barycentric triangle measure of factor intensity in the 3×3 model.

Studies of high dimensional models with equal numbers of factors and products have avoided defining factor intensity and focused instead on properties of the inverse of the factor input matrix. Chipman (1969) and Kemp and Wegge (1969) identify special conditions that lead to intuitive links between prices of products and factor prices. Ethier (1974), Chang (1979), Jones and Scheinkman (1977), Takayama (1982), and Thompson (1987) develop properties of higher dimensional models but include no definition of factor intensity.

In the applied literature, factor intensity has been abandoned in favor of directly modeling the factor content of trade. A definition of factor intensity that includes many products and factors will allow factor intensity to be included in tests of the determinants of trade and applications of factor proportions trade theory.

The proposed measure is the Euclidean distance from the unit value of a factor to the intersection of product intensity rays with unit factor hyperplanes. It is a direct generalization of the two dimensional definition of factor intensity that can be applied to any

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number of factors and products. For each factor, an intensity ranking across industries is derived. In empirical studies, factor intensity distance can be used to determine the general influence of factor intensity on the factor content of trade.

In models with two factors or two products, there are necessary links between factor intensity and production. In models with as few as three factors and three products, however, various patterns of production are consistent with given factor endowments. The implication is that any measure of factor intensity in a model with many factors and many products would not provide necessary theoretical links between factor intensity and the factor content of trade. Nevertheless, there is ample motivation to want to expand the generality of the concept of factor intensity. The proposed distance measure provides a generalization of the definition of factor intensity that can be used to gauge the overall success of factor proportions trade theory.

1. FACTOR INTENSITY AS EUCLIDEAN DISTANCE

Let a_{ij} represent the cost minimizing input of factor i in product j. In the 2×2 model, product 1 uses factor 1 intensively if

$$a_{11}/a_{21} > a_{12}/a_{22}\,. \tag{1}$$

Factor intensity can be defined for any number of products with only two factors. Product m uses factor 1 more intensively than product n if

$$a_{1m}/a_{2m} > a_{1n}/a_{2n}\,. \tag{2}$$

Factor intensity also can be defined for any number of factors with only two products as the ratio of factors across the two products. Beyond the simple situations with two factors or two products, however, factor intensity has no definition in the literature.

The first step toward the proposed distance measure of factor intensity is to standardize inputs by a factor. Consider the model with two factors standardized by a_{1n}. Figure 1 shows the input ratios of factors 1 and 2 in products 1 and 2. Factor intensity can also be measured by the intersection of the intensity rays with the unit line $a_{1j} = 1$. The distance from the point $a_{1j} = 1$ to the ray for product j is a_{2j}/a_{1j}. The distance for product 1 is less, $a_{2m}/a_{1m} < a_{2n}/a_{1n}$, and product 1 uses factor 1 intensively. As in (2), any number of products can be compared.

Figure 2 illustrates the distance measure of factor intensity in the model with three factors. On the left side of Figure 2, the three inputs are measured along axes a_{ij}, $i = 1, 2, 3$. The intensity ray for product m intersects the $a_{1j} = 1$ unit plane at point M. The right side of Figure 2 is this $a_{1j} = 1$ unit plane including point M. The distance from M to the origin at 1 is the factor intensity distance for product m, $d_{1j} = ((a_{2j}/a_{1j})^2 + (a_{3j}/a_{1j})^2)^{1/2}$ by the Pythagorean theorem.

For any number of factors relative to factor 1, the Euclidean distance to the intersection of a product ray with the unit hyperplane is

$$d_{1j} = ((a_{2j}/a_{1j})^2 + \cdots + (a_{rj}/a_{1j})^2)^{1/2}\,. \tag{3}$$

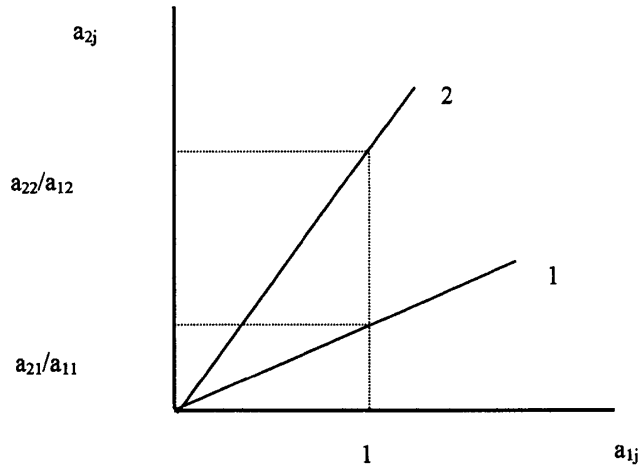

Figure 1. Factor intensity distance in the 2 × 2 model.

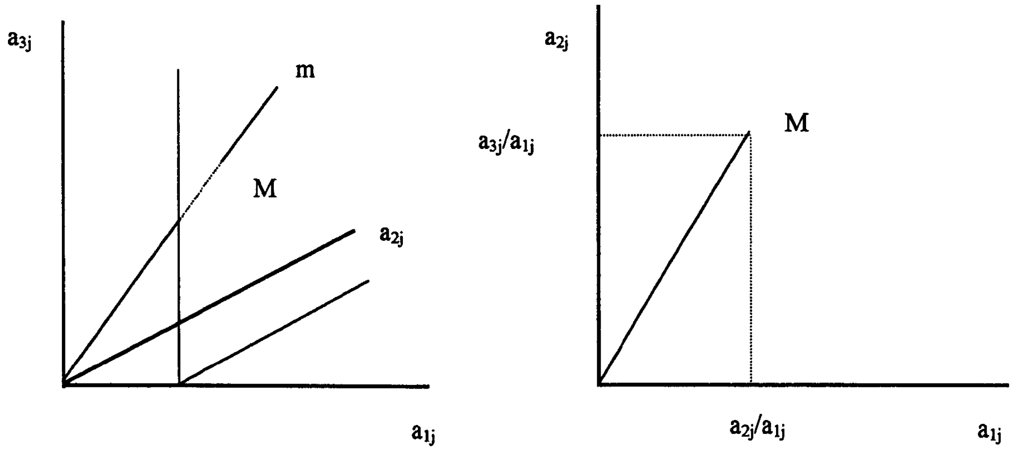

Figure 2. Factor intensity distance in the three factor model.

For any factor h, the factor intensity distance for product j is

$$d_{hj} = \left(\sum_{i \neq h} (a_{ij}/a_{hj})^2 \right)^{1/2} . \tag{4}$$

Factor intensity distance is one way to generalize factor intensity to any number of factors and products. The result is a ranking of products for each factor. Product m uses factor h intensively relative to product n if $d_{hm} < d_{hn}$.

If factors i and h are different types of labor, their ratio in (4) is a pure number. Ratios of different types of factors such as capital and labor cannot be summed directly as pointed out by Ron Jones in discussion. In practice, one option is to weight inputs by their average across industries. For a_{ij} in (4), substitute $\alpha_{ij} = a_{ij}/\mu_i$ where $\mu_i =$

$\sum_j a_{ij}/n$ and n is the number of products. The ratio of mean weighted inputs α_{ij}/α_{hj} would then be used in (4).

2. AN EXAMPLE OF DISTANCE FACTOR INTENSITY

An example of distance factor intensity is provided for 1996 US manufacturing with 8 "two digit" products and 7 skilled labor groups. The products are

F	food	N	printing & publishing
T	textiles	C	chemicals
A	apparel	O	petroleum & coal (oil)
P	paper	R	rubber & plastic

Labor groups are

m	managers & professional	s	service labor
t	technical & sales	h	handlers
c	crafts	r	transport labor
o	operators		

Table 1 reports the thousands of each type of worker in each industry. Industry output Q is value added in $billion.

Table 2 shows unit labor inputs in numbers of workers per $million of value added. Reading down the first column for the food industry F, there are more operators o per unit of output than any other type of labor. The same is true, however, for every other industry. A simple comparison of unit labor inputs suggests operators o are relatively heavy inputs in textile T and apparel A. Inputs can also be compared across industries for each factor. For instance, the input of managers m is relatively consistent across industries, while there is high variation of operators o.

Table 3 reports inputs relative to managers m for each other factor i and each industry j, a_{ij}/a_{mj}. Summing columns yields the distance intensity d_{mj} in (4). Industrial rankings are included in parentheses for reference. Printing N uses m the most intensively, followed closely by chemicals C. Textiles T and apparel A are on the low end of m intensity.

Table 1. Workers per industry and output.

Products		F	T	A	P	N	C	O	R
Labor	m	244	60	113	115	489	361	39	95
	t	214	62	115	121	576	367	29	74
	c	359	123	85	150	153	185	35	115
	o	779	418	682	361	520	371	54	415
	s	67	8	10	10	7	18	5	11
	h	255	53	46	43	62	76	23	29
	r	161	7	4	59	28	26	8	25
	Q	179	33	38	72	131	194	32	75

Table 2. Unit labor inputs.

Products		*F*	*T*	*A*	*P*	*N*	*C*	*O*	*R*
Labor	*m*	1.36	1.82	2.97	1.60	3.73	1.86	1.22	1.27
	t	1.20	1.88	3.03	1.68	4.40	1.89	0.91	0.99
	c	2.01	3.73	2.24	2.08	1.17	0.95	1.09	1.53
	o	4.35	12.7	18.0	5.01	3.97	1.91	1.69	5.53
	s	0.37	0.24	0.26	0.14	0.05	0.09	1.16	0.15
	h	1.42	1.61	1.21	0.60	0.47	0.39	0.72	0.39
	r	0.90	0.21	0.11	0.82	0.21	0.13	0.25	0.33

Table 3. Relative Management Inputs: a_{ij}/a_{mj}.

Products		*F*	*T*	*A*	*P*	*N*	*C*	*O*	*R*
Labor	*t*	0.88	1.03	1.02	1.05	1.18	1.02	0.75	0.78
	c	1.48	1.23	0.75	1.30	0.32	0.51	0.89	1.20
	o	3.20	6.98	6.06	3.13	1.06	1.03	1.39	4.35
	s	0.27	0.13	0.09	0.09	0.01	0.05	0.96	0.12
	h	1.04	0.88	0.41	0.38	0.13	0.21	0.59	0.31
	r	0.66	0.12	0.04	0.51	0.06	0.07	0.20	0.26
	d_{mj}	7.53	10.4	8.37	6.46	2.76	2.89	4.78	7.02
		(6)	(8)	(7)	(4)	(1)	(2)	(3)	(4)

All factor intensity distances are reported in Table 4. Intensities of technical labor t are similar to managers m. Craft labor c is used most intensively in paper P, food F, and textiles T. Operators o have the least variation across industries and are used most intensively in apparel A, textiles T, and rubber R. Petroleum O uses service labor s very intensively. Food F uses both handlers h and transport labor r most intensively.

Factor intensity can be summarized for each industry using the rankings. Food F uses handlers, h, transport r, crafts c, and service labor s "very intensively." Textiles T and apparel A use operators o very intensively. Paper P uses crafts c and transport labor r very intensively. Printing N is an extreme case, using managers m and technical labor t the most intensively and service labor s and handlers h the least intersively. Chemicals C uses managers m and technical labor t intensively. Petroleum O uses service labor s the most intensively, and uses handlers h, managers m, and transport workers r relatively intensively. Rubber R uses operators o and service workers r more intensively than any other labor.

Factor intensity distance offers the potential to predict general equilibrium links between prices of products and factors. For instance, NAFTA was predicted to lower prices of apparel and raise the prices of chemicals in the US. The suggestion of factor intensity distance is that the wages of operators would fall while the wages of managers and technical workers would rise. There is no necessity in this prediction, but it is based

Table 4. Distance factor intensities.

Products		F	T	A	P	N	C	O	R
Labor	d_{mj}	7.53	2.76	8.37	6.46	2.76	2.89	4.78	7.02
		(6)	(8)	(7)	(4)	(1)	(2)	(3)	(4)
	d_{tj}	8.69	10.8	7.79	6.10	2.19	2.77	6.73	8.28
		(7)	(8)	(5)	(3)	(1)	(2)	(4)	(6)
	d_{cj}	4.78	4.94	11.4	4.74	11.0	6.23	5.45	5.66
		(2)	(3)	(8)	(1)	(7)	(6)	(4)	(5)
	d_{oj}	1.68	0.75	0.55	1.28	2.52	2.79	3.17	0.85
		(5)	(2)	(1)	(4)	(6)	(7)	(8)	(3)
	d_{sj}	30.4	91.4	106	84.3	279	79.3	5.07	67.0
		(2)	(6)	(7)	(5)	(8)	(4)	(1)	(3)
	d_{hj}	7.18	12.8	22.0	18.8	28.8	17.5	8.77	25.2
		(1)	(3)	(6)	(5)	(8)	(4)	(2)	(7)
	d_{rj}	11.9	105	252	13.6	65.8	41.5	23.2	29.9
		(1)	(7)	(8)	(2)	(6)	(5)	(3)	(4)

on theory and has in fact proven true. In such a manner, factor proportions trade theory can become much more useful.

3. CONCLUSION AND THE POTENTIAL OF FACTOR INTENSITY DISTANCE

Factor proportions trade theory suggests that countries would be net exporters of products that use their abundant factors intensively. Factor content theory has provided one way to test or apply factor proportions theory assuming factor price equalization. The proposed factor intensity distance suggests a more flexible approach using a generalization of the definition of factor intensity with two factors and two products. Factor intensity distance could be combined with a similar measure of factor abundance to test whether countries tend to export products that use their abundant factors intensively.

In the model with two factors and two products given homothetic and identical production and utility functions, there is a necessary link between factor intensity and trade. The country abundant in a factor produces a higher ratio of the product using that factor intensively. With equal consumption ratios across countries, each country would export the product using its abundant factor intensively.

In models with many factors and many products, however, there are no such necessary connections. In the relatively simple model with three factors and two goods, Thompson (1985) shows there are four possible Rybczynki links between endowments and production. A country that is marginaly abundant in a factor might produce more or less of either product depending on technical conditions. The implication is that there is no necessary link between factor intensity and trade in higher dimensional models.

The literature also develops numerous theoretical models in which the link between factor intensity and trade is relaxed: imperfect competition, product differentiation, variable factor supplies, unemployment, transport costs, nonhomothetic production, joint production, and so on. The role of factor intensity in explaining trade is diminished in these models.

Ultimately, however, the importance of factor intensity is an empirical issue. The proposed distance measure of factor intensity can be used to examine the influence of factor intensity in determining trade patterns. Factor intensity is certainly expected to influence observed patterns of production and trade and the empirical issue is how far factor proportions theory can go in explaining international trade.

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