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RYBCZYNSKI THEOREM AND ASYMMETRIC ADJUSTMENT COSTS

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Abstract: While the role of adjustment costs in general equilibrium models is widely recognized there is no compelling reason to believe that such adjustment costs will be uniform across sectors. This paper demonstrates the implications of asymmetric adjustment costs for the Rybczynski theorem. The Rybczynski path is shown to be a special case of a more general expansion path.

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Key words: International trade, Heckscher-Ohlin-Samuelson theory, Rybczynski theorem, adjustment cost, general equilibrium

1. INTRODUCTION

Any process of investment is incomplete till new assets are absorbed into a firm's productive capacity. When capital (equipment, machinery, tools etc.) is added or moved from one sector to another a fraction of the services of the factor(s) of production (particularly labor) must be devoted to (i) the reconfiguration (assimilation, integration, etc.) of incoming capital and (ii) the adaptation of labor to incoming capital. This is embodied in the "adjustment cost" that generates a wedge between the purchase price of capital and the cost of its installation and implementation. While the role of adjustment costs in general equilibrium models is widely recognized,[1] there is no compelling reason to believe that such adjustment costs will be uniform across sectors. Chakrabarti (1999) presents evidence from U.S. industries that indicate systematic differences in costs of adjustment associated with the introduction of new capital across sectors differing in factor-intensities.[2]

[1] Businesses change their demand for inputs more slowly than the shocks to input demand warrant. The standard explanation for this slow adjustment is that, because the firm must incur adjustment costs that are inherent in the act of changing the amount of the input used, the response to shocks will not be instantaneous... what those costs look like should concern economists of many stripes", Hamermesh and Pfann (1996).

[2] A panel of annual observations on 457 industries by 4-digit SIC in the U.S. economy over the period 1958–1994 indicates that, on an average, the estimated cost of adjustment is significantly higher in the capital-intensive sectors than it is in the labor-intensive sectors. A panel of annual observations on 521 firms from the U.S. economy over the period 1984–1992 leads to a similar conclusion: the estimated cost of adjustment is significantly higher in the relatively capital-intensive group of firms than it is in the labor-intensive group.

The Rybczynski theorem, a cornerstone of the Heckscher-Ohlin-Samuelson (H-O-S) theory of international trade, identifies a mapping of exogenous factor supplies into output levels. This paper discusses the implications of asymmetric adjustment costs for the Rybczynski theorem.

The rest of the paper is organized as follows. Section 2 analyzes the implications of asymmetric adjustment costs for the Rybczynski theorem. Section 3 concludes.

2. ANALYSIS

The two-factor two-good version of the Rybczynski theorem states that with product prices unchanged, an increase in the quantity of one factor of production results in a more than proportionate increase in the output of the good which uses that factor relatively intensively and a reduction in the output of the other good. The positive derivative is not a surprise; the negative derivative is: at least one negative derivative for each factor occurs for higher dimensional models.[3]

The negative Rybczynski derivative requires a reallocation of resources across sectors. This triggers the role of adjustment costs in transition. If the act of investment itself is costly[4] it is hard to imagine that any movement of capital between sectors would be cost-less. Capital, in the form used in the textile industry, is certainly not appropriate for the production of semi-conductors. Shifting capital from one such production to another will require a reconfiguration of capital into a form suitable to its new use and the adaptation of labor to incoming capital. Greater the dissimilarity between sectors higher will be the cost (adjustment cost) of reconfiguration and adaptation. Intuitively, when an economy experiences an increase in the endowment of one of its factors of production it will be rational to move resources away from one sector to another only when the marginal benefit from the movement outweighs the marginal cost.[5]

Visualize a small open economy that uses two factors (in fixed supply), capital (K) and labor (L) to produce two goods, X (labor-intensive) and Y (capital-intensive), with constant returns to scale technology. Let x and y denote the quantities of X and Y, respectively. Let w_i denote the purchase price of factor i, K_j and L_j denote the capital and labor employed in sector j, k_j denote capital-intensity of sector j, P_j denote the world price of good j, where $i = K, L$ and $j = X, Y$. Labor is freely mobile between sectors. Capital is a quasi-fixed factor in the sense that adding capital and/or moving capital from one sector to another entail an adjustment cost. For simplicity, let labor be the only resource used up in the installation and implementation of capital.

Consider an expansion in the endowment of capital. Let the marginal cost of adjustment be higher in the capital-intensive sector (Y) relative to the labor-intensive sector

[3] See Leamer and Levinsohn (1995) for insightful discussions on related issues.

[4] For an early treatment of adjustment cost of investment see Lucas (1967) where he suggested that this cost behavior can be thought of as a sum of purchase costs (with perfect or imperfect factor markets) and installation costs.

[5] Mussa (1978) was the first to recognize the use of economic resources in the movement of capital from one sector to another. He demonstrated how a balance between expectations of future returns and costs of capital movement determine the efficiency of the adjustment process.

(X). Let a_j denote a constant marginal cost of adjustment in sector j. In transition, the factor cost ratios faced by the producers of X and Y goods will then bear the following relationship:

$$(w_L/w_K) > [w_L/(w_K + a_X)]_X > [w_L/(w_K + a_Y)]_Y \qquad (1)$$

Effects the expansion of capital will have on the composition of output are shown using Lerner-Pearce diagrams in Figures 1 through 5. The solid lines indicate unit-value isoquants, unit isocost lines, and allocation of factor endowment (E) for capital and labor before the expansion of capital. The broken lines represent the situation after the expansion of capital forming the new endowment (E′).

Figure 1 captures the standard Rybczynski effect in the absence of any adjustment cost. An increase in capital moves endowment from E to E′. Relative factor prices and capital-intensities remain unchanged. The new output composition is represented by the rays OC and OD replacing OA and OB for goods Y and X respectively: Y expands and X contracts. Figures 2 through 5 demonstrate the effects that the same expansion of capital can have in the presence of asymmetric adjustment costs. In Figure 2, the two broken lines indicate the different factor cost ratios (as in inequality (2)) to be faced by each sector in transition. Clearly, neither sector has any incentive to expand or contract. This results in an excess supply of capital (measured by the broken line EE′). This excess supply pressure will lower the purchase price of capital (w_K). When w_K falls enough to offset the adjustment cost of capital in sector X (i.e. when the broken iso-cost line for sector X swings to right of the solid iso-cost line) firms in sector X will find it profitable to expand production.[6] Wage (w_L) will rise in the face of expansion of the labor-intensive sector resulting in a rise in the purchase price of labor relative to capital. At a new factor purchase price ratio (w'_L/w'_K) and the corresponding factor cost ratios ($w'_L/(w'_K + a_X)$ for X and $w'_L/(w'_K + a_Y)$ for Y) full-employment will be achieved. Capital-intensities will rise in both sectors. Figures 3 through 5 depict three possible

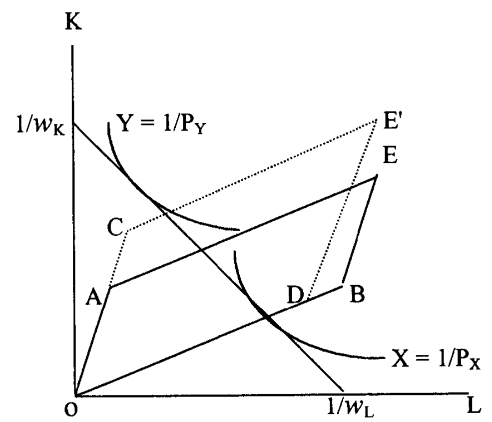

Figure 1. The Rybczynski Effect.

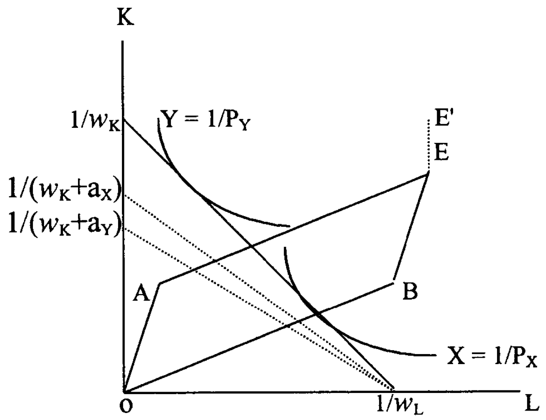

Figure 2. Excess Supply of Capital.

[6] Note that as w_K keeps falling sector X will have the incentive to expand first since the adjustment cost is higher in sector Y.

effects on output composition. The new output composition is represented by the rays OF and OG for goods Y and X respectively. A thick isoquant is drawn to indicate the initial level of output in each sector. The effective endowment (net of the labor used up exclusively for installation and implementation of capital, measured by the broken line $E'e'$) is represented by e'. In Figure 3, X expands and Y contracts. In Figure 4, X expands and the output of Y remains unchanged. In Figure 5, X and Y both expand.

In general which of the two sectors expand, contract or remain unaffected following a change in endowment depends on two effects, namely, an *endowment effect* and a *substitution effect*. At unchanged relative factor prices the endowment effect of an expansion in capital would induce an expansion in the capital-intensive sector and a contraction in the labor-intensive sector generating the familiar Rybczynski effect. When there are costs of adjustment neither sector would have an incentive to expand or contract. This results in an excess supply of capital that lowers the purchase price of capital relative to that of labor. The decline in the relative purchase price of capital induces each sector to substitute away from labor. This substitution effect operates against the endowment effect. The change in the output of each sector depends on the relative strength of the endowment effect and the substitution effect.

A formal analysis may be built on the structure of simple general equilibrium models.[7] Let sector Y be assumed, without loss of generality, to be relatively capital-intensive. The notations used in this section are consistent with those used in Section 2. In addition, let a_{ij} represent the amount of factor i used to produce one unit of good j; $\bar{L}$ and

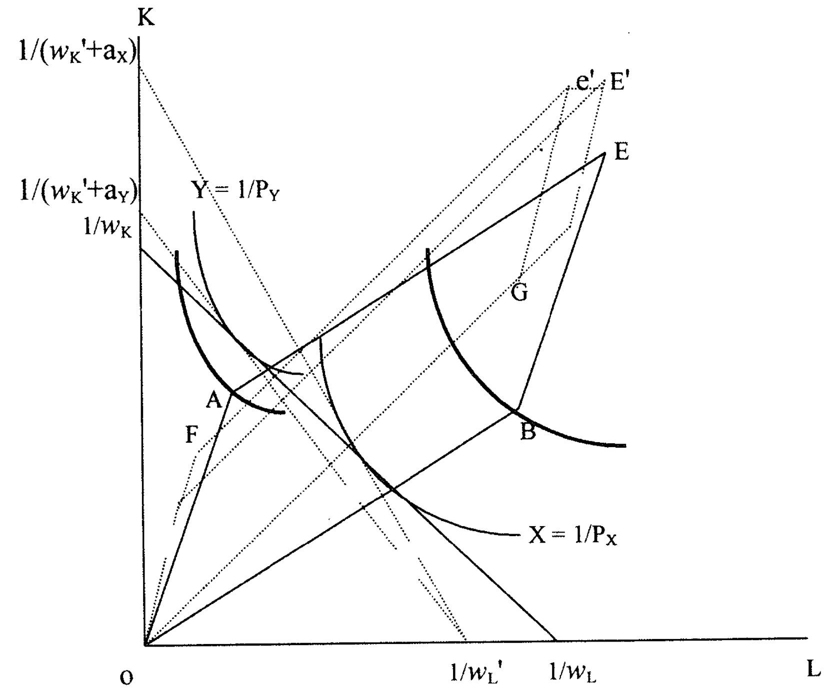

Figure 3. Labor-intensive sector expands; Capital-intensive sector contracts.

[7] See Jones (1965), Marjit (1987, 1989, 1990, 1993), and Marjit and Beladi (1993).

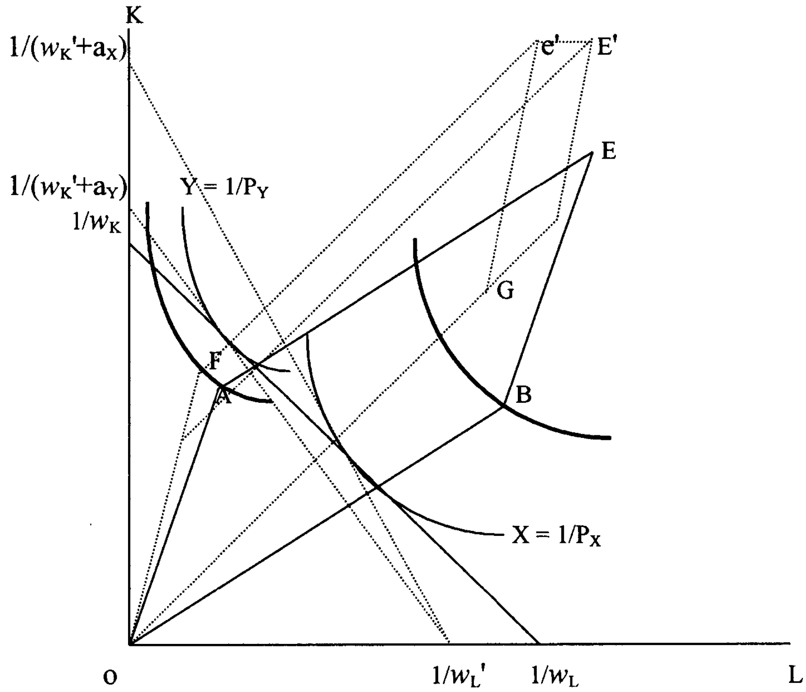

Figure 4. Labor-intensive sector expands; No change in the output of the Capital-intensive sector.

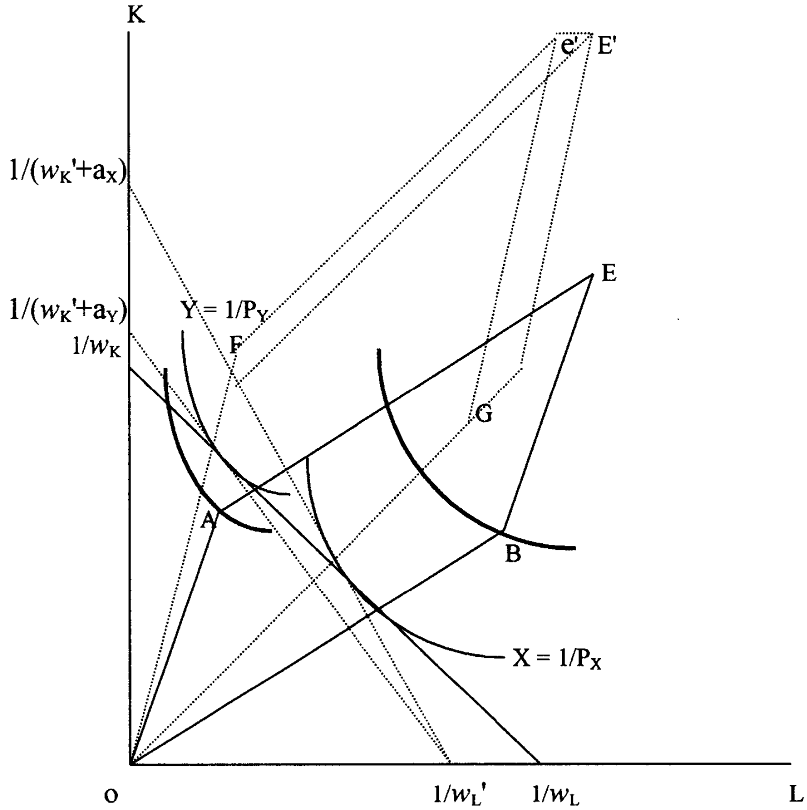

Figure 5. Both (Labor-intensive and Capital-intensive) sectors expand.

$\bar{K}$ represent the supply of capital and labor; K_{Aj} represent the amount of capital used exclusively for adjustment in sector j. A variable with ^ denotes the percentage change in that variable. The adjustment technology is specified as:

$$K_{Aj} = \alpha_j dj \tag{2}$$

where $\alpha_j = \begin{cases} +\alpha^j \text{ for } dj \geq 0 \\ -\alpha^j \text{ for } dj \leq 0 \end{cases}$ and $\alpha^j \geq 0$ are constants.

In a competitive equilibrium with both goods being produced the zero-profit condition requires:

$$w_L L_X + w_K K_X + w_K K_{AX} = P_X x \tag{3A}$$

$$w_L L_Y + w_K K_Y + w_K K_{AY} = P_Y x \tag{3B}$$

Full employment of resources ensures:

$$L_X + L_Y = \bar{L} \tag{4A}$$

$$K_X + K_Y + K_{AX} + K_{AY} = \bar{K} \tag{4B}$$

Conditions (3A) through (4B) reduce to the following equations of change:

$$\theta_{LX}\hat{w} + \theta_{KX}\hat{r} = \hat{P}_X - \theta_{AX}\hat{x} \tag{5A}$$

$$\theta_{LY}\hat{w} + \theta_{KY}\hat{r} = \hat{P}_Y - \theta_{AY}\hat{y} \tag{5B}$$

$$\lambda_{LX}\hat{x} + \lambda_{LY}\hat{y} = \hat{L} + \delta_L(\hat{w} - \hat{r}) \tag{5C}$$

$$\tilde{\lambda}_{KX}\hat{x} + \tilde{\lambda}_{KY}\hat{y} = \hat{K} - \delta_K(\hat{w} - \hat{r}) \tag{5D}$$

where $\theta_{ij} = \left(\frac{w_i a_{ij}}{P_j}\right)$ is the distributive share of factor i in industry j; $\lambda_{Lj} = \frac{L_j}{L}$ and $\lambda_{Kj} = \frac{K_j}{K}$ are fractions of the economy's labor and capital employed in the production of good j; $\theta_{Aj} = \left(\frac{\alpha_j w_K}{P_j}\right)$; $\lambda^K_{Aj} = \frac{\alpha_j j}{K}$; and $\tilde{\lambda}_{Kj} = (\lambda_{Kj} + \lambda^K_{Aj})$.

Let $\hat{K} = \hat{L} = \hat{P}_X = \hat{P}_Y = 0$ in the initial equilibrium. Consider the effect that a subsequent expansion in capital stock (i.e. $\hat{K} > 0$) has on the composition of output. Let commodity prices and labor supply remain unchanged i.e. $\hat{L} = \hat{P}_X = \hat{P}_Y = 0$. Under these simplifications, the relative change in the factor prices is given by:

$$(\hat{w} - \hat{r}) = \frac{\Delta_Y \hat{y} - \Delta_X \hat{x}}{|\theta|} \tag{6}$$

where $|\theta| = [(\theta_{KY} - \theta_{KX}) - (\theta_{AX}\theta_{KY} - \theta_{AY}\theta_{KX})]$; $\Delta_X = (\theta_{KY} + \theta_{LY})\theta_{AX}$ and $\Delta_Y = (\theta_{KX} + \theta_{LX})\theta_{AY}$. It may be noted that absent any cost of adjustment (i.e. $\alpha^j = 0$) relative factor rewards will not change in response to the rise in capital stock.

Let

$$\delta_L = (\lambda_{LX}\theta_{KX}\sigma_X + \lambda_{LY}\theta_{KY}\sigma_Y) > 0 \tag{7A}$$

and

$$\delta_K = (\lambda_{KX}\theta_{LX}\sigma_X + \lambda_{KY}\theta_{LY}\sigma_Y) > 0 \tag{7B}$$

where σ_j is the elasticity of substitution between labor and capital in industry j.

From (5C), (5D) and (6), the change in the output of each sector can be expressed as:

$$\hat{x} = \frac{\Psi_Y}{|\Phi|}\hat{K} \tag{8A}$$

$$\hat{y} = \frac{-\Psi_X}{|\Phi|}\hat{K} \tag{8B}$$

where $\Psi_X = \left(\lambda_{LX} + \frac{\delta_L \Delta_X}{|\theta|}\right)$; $\Psi_Y = \left(\lambda_{LY} - \frac{\delta_L \Delta_Y}{|\theta|}\right)$; $\Phi_X = \left(\tilde{\lambda}_{KX} - \frac{\delta_K \Delta_X}{|\theta|}\right)$; $\Phi_Y = \left(\tilde{\lambda}_{KY} + \frac{\delta_K \Delta_Y}{|\theta|}\right)$; and $|\Phi| = (\Phi_X \Psi_Y - \Phi_Y \Psi_X)$. $\frac{\lambda_{Lj}}{|\Phi|}\hat{K}$ captures the endowment effect and $\frac{\delta_L \Delta_j}{|\theta||\Phi|}\hat{K}$ the substitution effect. The endowment effect isolates the direct effect that an expansion in capital (net of the capital used in adjustment) has on the output of each sector at any given factor price ratio. The substitution effect arises from any change in the relative factor rewards following the excess supply of capital at the initial factor price ratio.

Absent any cost of adjustment (i.e. $\alpha^j = 0$) equations (8A) and (8B) yield the familiar Jones-Rybczynski (JR) effect: $(\hat{x} - \hat{y}) = -\frac{\hat{K}}{|\lambda|}$ where $|\lambda| = (\lambda_{KY} - \lambda_{KX}) \in (0, 1)$. The capital-intensive sector expands proportionately more than the rise in the capital stock (Jones' magnification effect) and the labor-intensive sector contracts.

From (8A) and (8B),

$$\frac{\hat{x}}{\hat{y}} = -\frac{\Psi_Y}{\Psi_X}. \tag{9}$$

Equation (9) characterizes a generalized (non-linear) path of expansion when there are costs of adjustment. If Ψ_X and Ψ_Y have the same sign then one of the sectors expands at the cost of another. A condition sufficient to induce an expansion in both sectors is: $\frac{\theta_{KY}}{(1-\theta_{AY})} < \frac{\theta_{KX}}{(1-\theta_{AX})}$ and $\frac{\Delta_X}{\lambda_{LX}} < \frac{\|\theta\|}{\delta_L}$. If $\frac{|\Delta_X|}{\lambda_{LX}} > \frac{|\theta|}{\delta_L}$, $\frac{\Delta_Y}{\lambda_{LY}} < \frac{|\theta|}{\delta_L}$, and $\alpha^X > a_{KX}$, then X will contract and Y will expand. If, on the other hand, $\frac{\theta_{KY}}{(1-\theta_{AY})} > \frac{\theta_{KX}}{(1-\theta_{AX})}$ and $\alpha^Y > a_{KY}$, then X will expand and Y will contract.

The introduction of asymmetric adjustment costs in a simple general equilibrium framework thus breaks the analytically convenient dichotomy between the price sub-system (3A and 3B) and the quantity sub-system (4A and 4B) of the Heckscher-Ohlin-Samuelson (H-O-S) model. In the H-O-S model, absent any asymmetry in costs of adjustment, the dichotomy allows reallocation of resources between industries without affecting factor prices, following any change in factor endowments within the cone of diversification.[8] The presence of asymmetric adjustment costs generates an excess supply of the expanding factor inducing a decline in the purchase price of that factor

[8] See Chakrabarti (1998) for a generalized factor price equalization (FPE) theorem which identifies a condition, stated in terms of the allocation of factor endowments across countries relative to the demand for and the factor intensities of goods, that is *necessary* and *sufficient* for FPE in a world with arbitrary number

relative to the other factors which in turn sets in motion a substitution away from that factor across sectors.

By establishing a meaningful link between factor price and output determination the analysis yields interesting implications for patterns of trade. For instance, let two small economies (foreign and home) be characterized by identical endowments and preferences, share identical production technologies, and be both initially in autarky. Let the free trade price be equal to the initial autarkic price. Consider the following configuration of adjustment technologies in the two countries:

$$\frac{\theta_{KY}}{(1-\theta_{AY})} > \frac{\theta_{KX}}{(1-\theta_{AX})}, \quad \alpha^{Y} > a_{KY},$$
$$\frac{|\Delta_X^*|}{\lambda_{LX}^*} > \frac{|\theta^*|}{\delta_L^*}, \quad \frac{\Delta_Y^*}{\lambda_{LY}^*} > \frac{|\theta^*|}{\delta_L^*}, \quad \text{and} \quad \alpha^{X^*} > \alpha_{KX}^*.$$

Let both these economies experience an identical expansion in capital. The endowment effect for both these economies will be identical. The substitution effect for the foreign economy will reinforce the endowment effect and it will experience the JR effect: the capital-intensive sector will expand and the labor-intensive sector will contract. For the home country the substitution effect will operate against the endowment effect. If the home substitution effect is strong enough to more than offset the endowment effect then the JR effect will be reversed: the labor-intensive sector will expand and the capital-intensive sector will contract. This can result in a trading equilibrium between two economies that differ in their adjustment technologies but are otherwise identical, the home country exporting its labor-intensive good to the foreign country and the foreign country exporting its capital-intensive good to the home country.

3. CONCLUSION

This paper demonstrates how the Rybczynski theorem can be sensitive to the existence of asymmetry in adjustment costs. Innumerous attempts, in a vast pool of empirical literature, are continuously made to test (weak) implications of the H-O-S theory. Most studies report that the implications are spectacularly at odds with the description of international data.[9] By bringing out the implications of asymmetric adjustment costs for the Rybczynski theorem the paper highlights the importance of allowing sector-specific adjustment costs in testing the implications of H-O-S theory.

REFERENCES

Chakrabarti, A., "Proof of Sufficiency of the Lens Condition for Factor Price Equalization", *mimeo*, University of Michigan, Ann Arbor, July 1994.

———, "A Generalized Factor Price Equalization Theorem", *mimeo*, University of Wisconsin-Milwaukee, October 1998.

of countries, goods and factors. See Chakrabarti (1994) for a proof that Deardorff's (1994) lens condition is necessary and sufficient for FPE *iff* the world has at least as many goods as factors.

[9] See Helpman (1999) for a comprehensive review of the relevant literature.

———, "Estimation of Internal Adjustment Costs: Capital-intensive and Labor-intensive Industries", *mimeo*, University of Wisconsin-Milwaukee, December 1999.

Deardorff, A., "The Possibility of Factor Price Equalization Revisited", *Journal of International Economics*, 36, November 1994.

Hamermesh, D. S. and G. A. Pfann, "Adjustment Costs in Factor Demand", *Journal of Economic Literature*, 34(3), 1996.

Helpman, E., "The Structure of Foreign Trade", *Journal of Economic Perspectives*, 13, 1999.

Jones, R. W., "The Structure of Simple General Equilibrium Models", *Journal of Political Economy*, 73, 1965.

Leamer, E. E. and J. A. Levinsohn, "International Trade Theory: The Evidence", in: G. M. Grossman and K. Rogoff, eds., Handbook of International Economics, vol. 3 (North-Holland, Amsterdam), 1995.

Lucas, R., "Optimal Investment Policy and the Flexible Accelerator", *International Economic Review*, 8, 1967.

Marjit, S., "Trade in Intermediates and the Colonial Pattern of Trade", *Economica*, 1987.

———, "The Product-Cycle Hypothesis and the H-O-S Theory of International Trade", *Journal of Economic Integration*, 1989.

———, "A Simple Production Model in Trade and its Applications", *Economics Letters*, 1990.

———, "Uniform Tariffs in General Equilibrium: A Simple Model", *Zeitschrift fur Nationalokonomie*, 1993.

——— and H. Beladi, "Factor Underutilization and the Theory of International Trade", *Keio Economic Studies*, 1990.

Mussa, M., "Dynamic Adjustment in the Heckscher-Ohlin-Samuelson Model", *Journal of Political Economy*, 1978.