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**STRATEGIC ENTRY DETERRENCE THROUGH INFORMATIVE  
ADVERTISING IN A VERTICALLY  
DIFFERENTIATED-PRODUCT MARKET**

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*Abstract:* Ishigaki (2000) has shown that strategic informative advertising by an incumbent to deter entry is impossible in a duopoly game of sequential advertising followed by simultaneous price setting when the products are homogeneous. This note shows that the seemingly perverse result survives even when the products are vertically differentiated.

**JEL codes:** D43, L12, M37

**Key words:** informative advertising, strategic entry deterrence, Bertrand competition.

## 1. INTRODUCTION

In Ishigaki (2000), it has been shown that strategic informative advertising by an incumbent to deter entry is impossible in a duopoly game of sequential advertising followed by simultaneous price setting when the products are homogeneous. In this note, I will prove the seemingly perverse result survives even when the products are vertically differentiated.

## 2. MODEL FRAMEWORK

Following Ishigaki (2000), consider the following game. At Stage 1, firm  $X$  (incumbent) sinks a fixed cost of advertising,  $f$ , and sets a level of advertising,  $x$ . At Stage 2, firm  $Y$  (potential entrant) decides whether to enter the market, and if it does, it sinks  $f$  and determines a level of advertising,  $y$ . At Stage 3, the firms simultaneously choose a price.

Firms  $X$  and  $Y$  sell vertically differentiated products at zero marginal cost of production. The quality of brand  $i$  is denoted by  $v_i$  ( $i = X, Y$ ). There is an atomless continuum of homogeneous consumers, whose population is normalized to unity. The

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consumers, who are a priori uninformed, come to know of the existence and quality of brand  $i$  only through the advertisement for brand  $i$  ( $i = X, Y$ ). Their common indirect utility function is defined by  $U(v_i, p_i) = v_i - p_i$  where  $p_i$  is brand  $i$ 's price. The consumers are price takers and buy only a unit of either brand or nothing.

The advertisements are randomly distributed to the consumers (Butters 1977). Thus, for the sake of the analytical convenience, firm  $i$ 's advertising level is normalized to the fraction of all the consumers receiving the advertisements for brand  $i$  (or the probability that each consumer is informed of brand  $i$ ) ( $i = X, Y$ ). The variable cost of distributing advertisements is assumed to be zero. This simplification is not crucial to the conclusion of the present note.

Advertising creates market segmentation of the consumers. Suppose firm  $X$  has informed  $0 \leq x \leq 1$  fraction of all the consumers of the existence of brand  $X$  and firm  $Y$  has informed  $0 \leq y \leq 1$  fraction of them of the existence of brand  $Y$ . Then,  $x(1 - y)$  fraction of them know the existence of only brand  $X$ ,  $y(1 - x)$  fraction of them know the existence of only brand  $Y$ ,  $xy$  fraction of them know the existence of brands  $X$  and  $Y$ , and  $(1 - x)(1 - y)$  fraction of them know neither of the brands. The perfectly informed consumers compare the total values of brands  $X$  and  $Y$ . They choose brand  $i$  if  $v_i - p_i \geq v_j - p_j$  and  $p_i \leq v_i$ , or,  $p_i \leq \min\{p_j + (v_i - v_j), v_i\}$  ( $i, j = X, Y; i \neq j$ ). The asymmetrically informed consumers receiving one or more brand  $i$ 's advertisements will buy brand  $i$  if  $p_i \leq v_i$  ( $i, j = X, Y; i \neq j$ ), but never choose brand  $j$ . They are indeed monopolized by either of the firms.

### 3. MONOPOLY EQUILIBRIUM

For the examination of the entry equilibrium, one needs to know the monopoly outcome. Let firm  $X$  be the monopolist. Suppose firm  $X$  has just informed  $0 \leq x \leq 1$  fraction of the potential consumers of its product. Then, firm  $X$ 's monopoly profit function in the post-advertising stage is  $x p_X$ . The optimal price and the post-advertising profit are  $v_X$  and  $x v_X$ , respectively. Firm  $X$ 's optimal advertising level as a monopolist is  $x^M = 1$ , and the post-entry monopoly profit is  $\Pi_X^M := v_X$ . Similarly, firm  $Y$ 's post-entry monopoly profit is  $\Pi_Y^M := v_Y$ .

### 4. POST-ADVERTISING BERTRAND-NASH EQUILIBRIUM

To examine the subgame perfect equilibrium of the game, one first needs to find the Nash equilibrium of the post-advertising price-setting game. It is now well-known that the price-setting games of this type have no pure-strategy Nash equilibrium but have a mixed-strategy Nash equilibrium (See Dasgupta and Maskin (1986)). In particular, Narasimhan (1988) has already characterized the Nash equilibrium of an virtually identical game to this post-advertising price-setting game.<sup>1</sup> Therefore, I simply state the mixed-strategy Bertrand-Nash equilibrium profits for the later analysis.

<sup>1</sup> He investigated the Nash equilibrium of Bertrand duopolistic games in which consumers are imperfectly informed of either of two brands, or perfectly informed.

PROPOSITION 1 (Post-advertising Bertrand-Nash Equilibrium: Narasimhan (1988)). *The simultaneous-move price-setting game in the post-advertising stage has a unique mixed-strategy Bertrand-Nash equilibrium. If  $yv_X < xv_Y$ , the equilibrium expected profits are  $\pi_X^* = x(1 - y)v_X$  and  $\pi_Y^* = y(v_Y - v_X y)$ . If  $yv_X \geq xv_Y$ , the expected equilibrium profits can be obtained from the above with arguments and subscripts transposed.*

## 5. ADVERTISING EQUILIBRIUM AND STRATEGIC ENTRY DETERRENCE

To find the subgame perfect equilibrium of the advertising game, one needs to derive the advertising best-response correspondence. From Proposition 1, firm X's profit function in the advertising stage is

$$\Pi_X(x, y) = \begin{cases} x(1 - y)v_X & \text{if } yv_X < xv_Y \\ x(v_X - v_Y x) & \text{if } yv_X \geq xv_Y \end{cases}. \quad (1)$$

The first sub-profit function is maximized at  $x = 1$  and its maximum is  $(1 - y)v_X$ . The second sub-profit function is maximized at  $x = v_X/2v_Y$  and its maximum is  $v_X^2/4v_Y$  if  $v_X/2v_Y \leq 1$ ; it is maximized at  $x = 1$  and its maximum is  $v_X - v_Y$  if  $v_X/2v_Y > 1$ . Comparing the two local maximal profits given  $v_X/2v_Y \leq 1$  or  $v_X/2v_Y > 1$ , one can have firm X's advertising best-response correspondence,

$$x^*(y) = \begin{cases} 1 & \text{if } 0 \leq y \leq 1 - v_X/(4v_Y) \\ v_X/(2v_Y) & \text{if } 1 - v_X/(4v_Y) \leq y \leq 1 \end{cases} \quad \text{Given } v_X/2v_Y \leq 1 \quad (2)$$

$$x^*(y) = 1 \quad \text{if } 0 \leq y \leq 1 \quad \text{Given } v_X/2v_Y > 1. \quad (3)$$

Firm Y's advertising best-response correspondence is analogous.

The profit functions in the sequential advertising stage can be obtained from the advertising best-response correspondences.<sup>2</sup> If  $2v_X \leq v_Y$ ,

$$\Pi_X(x, y^*(x)) = \begin{cases} x(v_X - v_Y x) & \text{if } 0 \leq x < v_X/v_Y \\ 0 & \text{if } v_X/v_Y \leq x \leq 1 \end{cases}; \quad (4)$$

$$\Pi_Y(x, y^*(x)) = \begin{cases} (1 - x)v_Y & \text{if } 0 \leq x < v_X/v_Y \\ v_Y - v_X & \text{if } v_X/v_Y \leq x \leq 1 \end{cases}; \quad (5)$$

if  $2v_X > v_Y$ ,

$$\Pi_X(x, y^*(x)) = \begin{cases} x(v_X - v_Y x) & \text{if } 0 \leq x < 1 - v_Y/(4v_X) \\ (v_X - v_Y/2)x & \text{if } 1 - v_Y/(4v_X) \leq x \leq 1 \end{cases}; \quad (6)$$

$$\Pi_Y(x, y^*(x)) = \begin{cases} (1 - x)v_Y & \text{if } 0 \leq x < 1 - v_Y/(4v_X) \\ v_Y^2/(4v_X) & \text{if } 1 - v_Y/(4v_X) \leq x \leq 1 \end{cases}. \quad (7)$$

Figures 1 and 2 have the above profit functions of  $x$  when  $2v_X \leq v_Y$  and  $2v_X > v_Y$ , respectively.

<sup>2</sup> To derive the profit functions, it is required to consider three patterns of the combination of the advertising best-response correspondences. When  $v_Y \geq 2v_X$ , the advertising best-response correspondences are (2) and firm Y's version of (3). When  $v_X/2 \leq v_Y < 2v_X$ , they are (2) and firm Y's version of (2). When  $0 < v_Y \leq v_X/2$ , they are (3) and firm Y's version of (2).

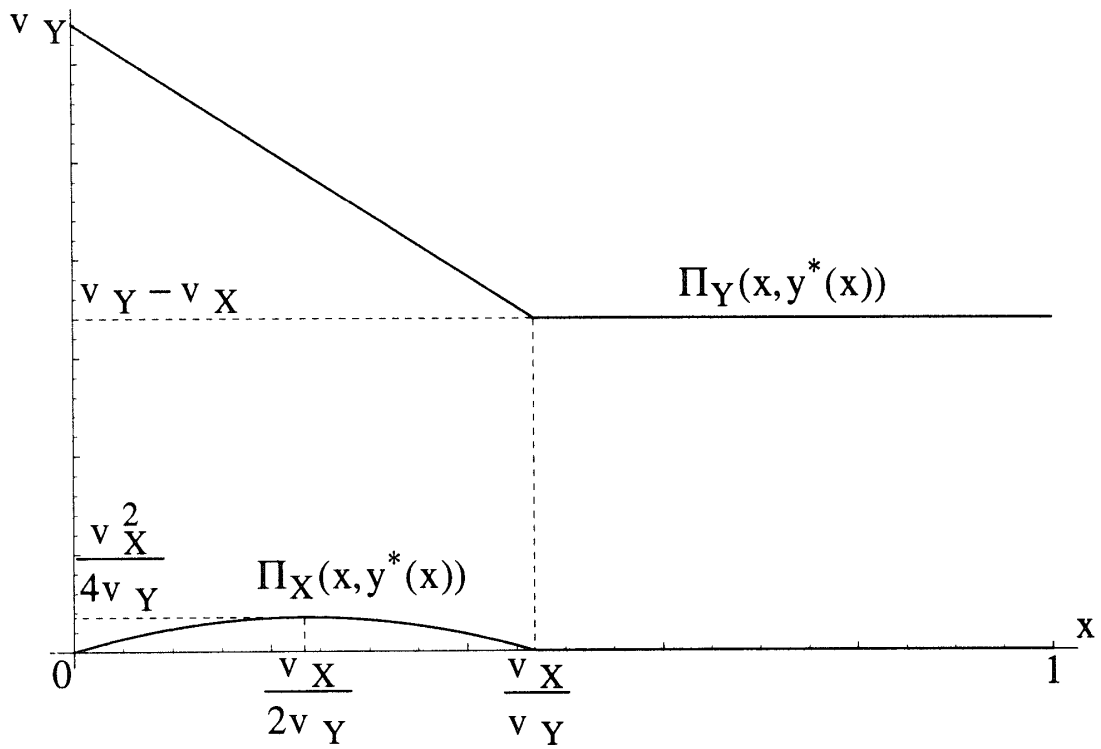


Figure 1. The Profit Functions of the Sequential-move Informative Advertising Game:  $2v_X \leq v_Y$ .

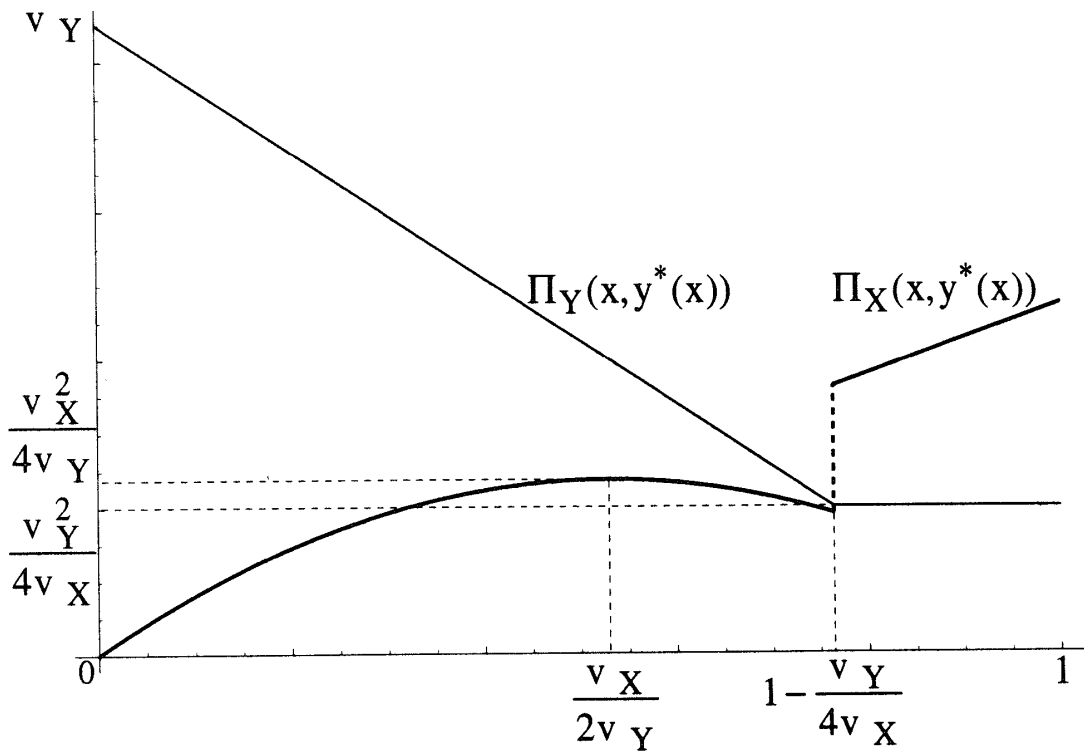


Figure 2. The Profit Functions of the Sequential-move Informative Advertising Game:  $2v_X > v_Y$ .

In the sequential-move advertising game, the subgame perfect equilibrium is identified by the maximizer of firm X's profit. From (4) and (6), it is easy to find the maximizer of firm X's profit in each case, which can be graphically verified in Figures 1 and 2.

**PROPOSITION 2** (Sequential-move informative advertising subgame perfect equilibrium). *The sequential-move advertising subgame in which the firms expect the Bertrand-Nash equilibrium shown in Proposition 1 has the following equilibrium. If  $2v_X \leq v_Y$ ,  $(x^S, y^*(x^S)) = (v_X/2v_Y, 1)$  and  $(\Pi_X^S, \Pi_Y^S) = (v_X^2/4v_Y, v_Y - v_X/2)$ ; If  $0 < v_Y \leq 2v_X$ ,  $(x^S, y^*(x^S)) = (1, v_Y/2v_X)$  and  $(\Pi_X^S, \Pi_Y^S) = (v_X - v_Y/2, v_Y^2/4v_X)$  where  $x^S := \arg\max \Pi_X(x, y^*(x))$  and  $\Pi_i^S := \Pi_i(x^S, y^*(x^S))$  ( $i = X, Y$ ).*

Now, consider the incumbent's strategic advertising and the entry decision by the potential entrant. The "entry" subgame perfect equilibrium is completely characterized by the values of  $v_X$ ,  $v_Y$ , and  $f$ . Figure 3 illustrates the combinations of them. Suppose  $2v_X \leq v_Y$  holds. If  $\Pi_Y^M \leq f$ , which corresponds to Region A1 in Figure 3, no firms enter the market since  $\Pi_X^M < \Pi_Y^M \leq f$ . If  $\Pi_X^S \leq f < \Pi_Y^M$ , which corresponds to Region B1 in Figure 3, firm X does not enter the market, but firm Y enters

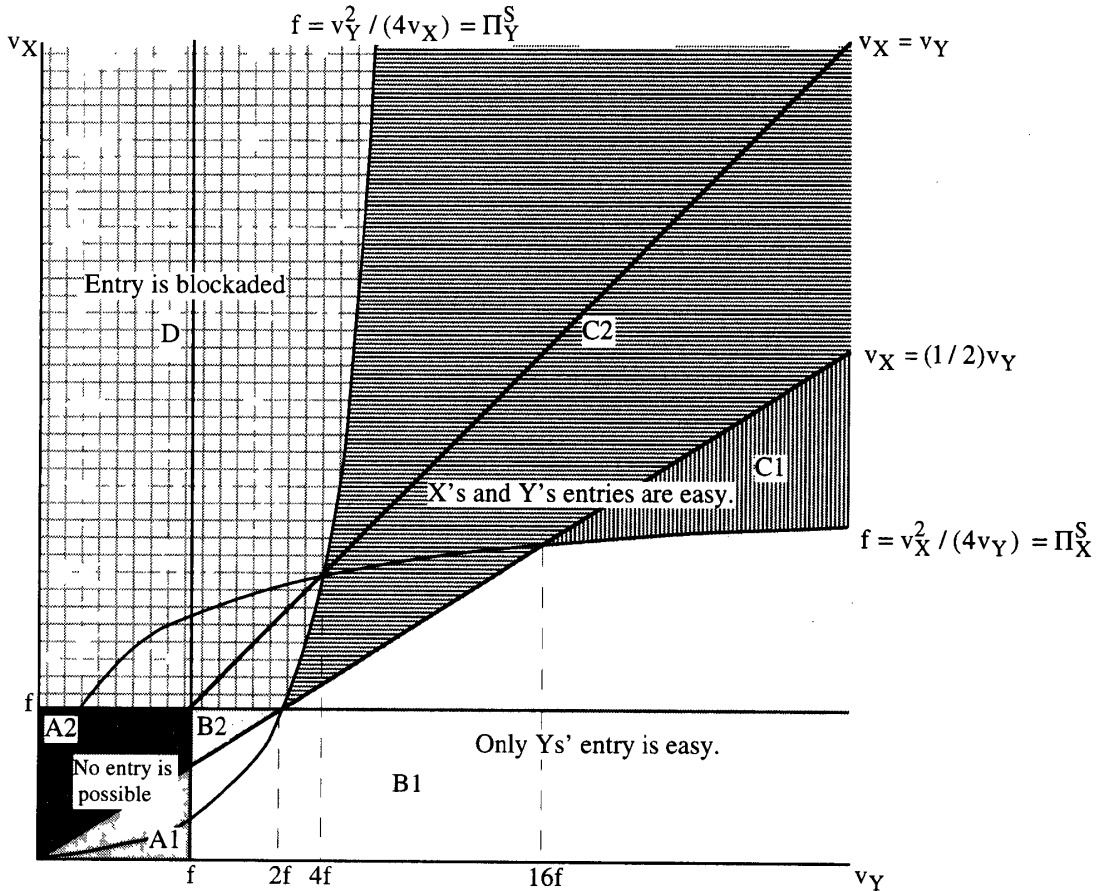


Figure 3. The Entry Subgame Perfect Equilibrium.

the market to be the monopolist with setting the monopoly level of advertising since firm  $X$ 's maximal profit in the presence of firm  $Y$ 's entry is less than  $f$ . If  $0 \leq f < \Pi_X^S$ , which corresponds to Region C1 in Figure 3, firm  $X$  profitably enters the market by choosing  $x^S$  level of advertising, and firm  $Y$  profitably enters the market with setting  $y^*(x^S)$  level of advertising since  $\Pi_Y(x^S, y^*(x^S)) > \Pi_X(x^S, y^*(x^S)) \geq f$ . The last two situations could be referred to as "entry is easy" in Bain's (1956) terminology.

Similar analyses can be applied to the case of  $2v_X > v_Y$ .<sup>3</sup> If  $\max\{\Pi_X^M, \Pi_Y^M\} \leq f$ , which corresponds to Region A2 in Figure 3, no firms enter the market. If  $\Pi_X^M \leq f < \Pi_Y^M$ , which corresponds to Region B2 in Figure 3, only firm  $Y$ 's entry occurs as in the above case of  $\Pi_X^S \leq f < \Pi_Y^M$  for  $2v_X \leq v_Y$ . If  $\Pi_Y(x^M, y^*(x^M)) \leq f < \Pi_X^M$ , which corresponds to Region D in Figure 3, firm  $Y$ 's entry is blockaded (Bain 1956): firm  $Y$ 's post-entry profit is less than the fixed cost even if firm  $X$  keeps the monopolist's advertising policy; thus, firm  $X$  can operate as a monopolist without firm  $Y$ 's entry. If  $0 \leq f < \Pi_Y(x^M, y^*(x^M))$ , which corresponds to Region C2 in Figure 3, both firms' entries are easy as in the above case of  $\Pi_X^S > f \geq 0$  for  $2v_X \leq v_Y$ .

The considerations so far suggest that this game has no possibility of strategic entry deterrence via the incumbent's pre-commitment to investment in informative advertising even if the incumbent's product's quality is much superior to the potential entrant's.<sup>4</sup> This result depends on a property that since  $\Pi_Y(x, y^*(x))$  is minimized at  $x^S$ , firm  $X$  can not find  $x$  such that  $\Pi_Y(x, y^*(x)) \leq f$ . In the post-advertising equilibrium, when  $yv_X < xv_Y$ , firm  $X$  essentially earns the profit as if it sold only to its brand loyal consumer segment,  $x(1 - y)$ , at its monopoly price,  $v_X$ . On the other hand, firm  $Y$  does as if it sold to all the consumers knowing brand  $Y$ ,  $y$ , at a price,  $v_Y - yv_X$ , below its monopoly price,  $v_Y$ . Hence, even in the presence of firm  $X$ 's heavier investment in advertising, firm  $Y$  can always ensure a positive profit independent of  $x$  by limiting  $y$  such that  $yv_X < xv_Y$  holds.

The summary of the considerations so far is given in the form of proposition.

**PROPOSITION 3** (Entry subgame perfect equilibrium). *In the entry-deterrence game developed, strategic entry deterrence via informative advertising is impossible. The game has four possible entry equilibria, depending on  $v_X$ ,  $v_Y$ , and  $f$ : (1) no firm enters the market; (2) entry is blockaded; (3) entry is easy in the sense that only the second mover enters the market to be the monopolist; (4) entry is easy in the sense that both firms profitably enter the market.*

This "strategic entry deterrence impossibility" result is not an artifact resulting from the assumption of no variable cost of advertising. Suppose the marginal cost of advertising is positive and not decreasing. The introduction of such a variable cost of

<sup>3</sup> To find the equilibrium, it is advised to analyze the game separately (1) when  $2v_X > v_Y > v_X$  and (2) when  $v_X \geq v_Y$ .

<sup>4</sup> It should be noticed that if the incumbent surpasses the entrant in product quality, the entrant always has a disadvantage in earning profits over the incumbent. Thus, it is true that the difference between  $v_X$  and  $v_Y$  is an important factor in determining the market structure (the number of active firms in an industry) even if it is not necessarily be the source of strategic entry deterrence by means of heavy advertising.

advertising essentially only shifts down all the post-entry profit functions and moves the critical level (at which the post-entry profit functions change) toward the origin. In particular, when  $2v_X > v_Y$ , firm  $X$ 's optimal advertising level in the sequential-move informative advertising game and firm  $X$ 's monopoly advertising level would be no longer 1. However, either of the advertising levels of firm  $X$  would be still more than the critical value at which the profit functions change. Thus, the introduction of variable costs of advertising would not qualitatively change the results obtained.

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