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JOINT PRODUCT DEVELOPMENT: SOME DYNAMIC ISSUES

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Abstract: We examine the impact of an increase in technological asymmetry on the success of joint product development ventures, with a view to exploring the free-riding problems inherent in cooperative R&D. In a static model we find that if technological diversity increases then the chances that a joint venture forms at all is reduced. Given that a joint venture forms however, an increase in technological diversity increases the probability of success. We then examine how the results extend to a dynamic context. In a two period model the earlier sufficient conditions no longer ensure that the effort stream increases as the firms become more technologically diverse. In the infinite horizon game we find that for linear cost and return functions all the earlier results still hold qualitatively. Moreover, the above results are independent of the nature of the sharing rule and are therefore consistent with any endogenous sharing rule.

1. INTRODUCTION

In this paper we pose the following question. Consider cooperative R&D (more specifically joint product development) in a duopoly setting. What determines whether the firms will be successful in developing the product? We want to examine the question whether, in a *dynamic* joint venture, collaboration among technologically similar firms are more likely to succeed compared to collaboration among technologically dissimilar firms.

Mowery (1989) suggests that technologically dissimilar firms are usually more successful in developing the product. Product development between technological equals, like that between Rolls Royce and Pratt and Whitney in the JT 10D jet engine, Fokker and Mcdonell Douglas in the MDF 100 commercial aircraft project, and Saab and Fairchild in the SF 340 commuter aircraft project, has frequently failed to develop the product or to market it. The AT & T-Phillips venture in telecommunications also

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ran into problems. (Business Week, 1/18/8 p. 62.) Product development ventures between dissimilar firms e.g. the CFM International venture between General Electric and SNECMA of France and that between Boeing and the Japan Commercial Transport Development Corporation appeared to be doing better. (See Mowery (1980) for a more detailed discussion of these issues.)

We seek to explain this phenomenon through a dynamic analysis of the free-rider problem involved in joint research. In order to focus on the simplest possible model in which the free-rider problem can be posed, some of the usual questions addressed in the R&D literature are abstracted from. These include the problem of market interaction following the product development stage as well as the spill-over effect. D'Aspremont and Jacquemin (1988), Katz (1986) and Killing (1983) are some of the papers (among others) which examine these issues.

We begin by examining a static model of joint product development before going on to analyse the dynamic game.

Consider a duopoly model of R&D where the probability of success depends on the joint effort stream of the two firms. In the event of success the firms share equally in the resulting profits. The technological dissimilarity pertains to the marginal costs of making the effort. Efforts are either unobservable or nonverifiable by courts and hence cannot be contracted upon. Both the firms therefore have an incentive to free-ride on the other firm.

To begin with we examine, in an one period setting, whether the firms would find it individually rational to opt for joint product development rather than pursue competitive R&D. We find that cooperative R&D is individually rational if the technology levels are not too dissimilar and the cost levels are not too high compared to the gross payoff. We also find that an increase in technological dissimilarity (mean preserving spread of marginal costs) increases the probability of success. Since the incentive to free-ride increases for the inefficient firm and decreases for the efficient firm, we find that the efficient firm increases its effort level and the inefficient firm decreases its effort level. The increase in effort by the efficient firm, however, more than compensates for the decrease in effort by the other firm. Since the efficient firm has more to lose from a fall in effort, it tries to more than make up for the decrease in efforts by the inefficient firm. We also identify sufficient conditions for this to happen.

We then examine how the results extend to a dynamic context where the firms interact over two periods. The earlier sufficient conditions no longer ensure that the effort stream increases as the firms become more technologically diverse. The reason is as follows. As a result of greater technological diversity the second period payoff of the firms may increase. This would have a negative effect on the first period efforts because the consequences of a failure in the first period is reduced. The net effect on the effort stream could therefore go either way.

The preceding result, however, is driven by the sharp asymmetry between the first and the second stages of the game. This asymmetry is somewhat artificial, and would disappear in an infinite horizon framework. We subsequently examine the infinite horizon formulation of the joint venture game where we restrict attention to stationary Markov equilibria. We find that for linear costs our earlier theme reemerges i.e. while an increase in technological diversity increases the success probability in absolute terms, it also leads to an increase in the divergence between the efficient and the equilibrium effort levels. The interesting fact is that the above results are independent of the nature of the sharing rule and are therefore consistent with any endogenous sharing rule. Moreover, if the return function is also linear then we find that for symmetric firms, both the firms are likely to prefer joint product development to competitive R & D.

We can summarise our basic findings as follows. We find that technological diversity decreases the possibility of a joint venture forming at all. If, however, a joint venture does form an increase in technological diversity increases the success probability in absolute terms. This essay therefore suggests the testable hypothesis that the proportion of observed successes among heterogenous firms would be higher. Another implication of our analysis is that the amalgamation of the joint venture firms makes greater sense (in terms of the probability of success) if the firms are technologically far apart.

The paper closest to this one is by Ray Chaudhuri (1995). Ray Chaudhuri (1995) considers the same problem but in a completely static framework. Moreover, he assumes that the sharing rule is endogenously determined. He finds that if the sharing rule obeys some reasonable regularity conditions, an increase in technological asymmetry leads to an increase in the probability of success. He also identifies some situations when the sharing rule would obey these restrictions. The present paper differs from Ray Chaudhuri (1995) in focussing on the dynamic issues involved in the problem. In the process we gain two additional insights that were not clear from the static analysis. First, we show that in a finite horizon game an increase in technological asymmetry might have a negative effect on R & D, as it increases the profit levels of the firms and thus decreases the incentive to do R & D. Second, in an infinite horizon framework, this effect becomes less important and the qualitative results obtained in the static analysis continue to hold. Moreover, we demonstrate that the actual sharing rule may be immaterial for the R & D efforts. This demonstrates that the perception of the partner firms regarding the longevity of the joint venture is an important factor in determining the success of the project.

The rest of the paper is organised as follows. The basic one period model is examined in Section 2. Section 3 briefly examines the two period game. The infinite horizon game is explored in Section 4. Section 5 concludes.

2. THE BASIC MODEL

Two firms, firm 1 and firm 2, are jointly trying to develop a product denoted X. If they succeed they jointly receive a gross payoff of R, which they split equally. One way to interpret R is to think of the joint venture as a purely research venture which, in the event of success, is going to sell the product to a third firm for a fixed price R. Alternatively one can think of the joint venture firms as cooperating in the product market as well. In this case R can be interpreted as the payoff accruing to the firms from jointly marketing the product. If they fail to develop the product they receive nothing, despite incurring development costs. (In fact we can assume that in case of failure also they receive a positive payoff, which is smaller than what they would receive in case of success. This will not affect the analysis in any way.)

In this section we briefly describe a one period game where the firms simultaneously decide on their level of efforts. Efforts are either unobservable or non-verifiable by courts and hence cannot be contracted upon. This leads to the well known free-rider problem and implies that we must solve for a Nash equilibrium of the game.

The cost functions of the two firms are given by $C_i(e_i) = h_i \int_0^{e_i} c(\tilde{e}i)d\tilde{e}_i$, $i = 1, 2$, where e_i is the amount of effort put in by the ith firm, $h_i c(e_i)$ is the marginal cost of effort of the ith firm and h_i^{-1} is a productivity index. If $h_1 < h_2$ we say that firm 1 is technologically superior to firm 2. If $h_1 = h_2$ the firms are considered to be technologically identical. The probability of success is given by the return function $\lambda(e_1 + e_2)$. We define $E = e_1 + e_2$.

We make the following assumptions of $c(e_i)$ and $\lambda(E)$,

(A) $c(e_i)$ and $\lambda(E)$ are twice continuously differentiable.

(B) Marginal costs are positive and strictly increasing in the effort level i.e. $c(e_i) \geq 0$ and $c'(e_i) > 0$, $\forall e_i$.

(C) Marginal productivity of effort is positive but decreasing in the effort level i.e. $\lambda(E) \in [0, 1]$, $\lambda'(E) > 0$, $\lambda''(E) \leq 0$.

(D) $c(0) = 0$.

Assumption D is a simplifying assumption which ensures that for all $h_1, h_2 \neq 0$ the equilibrium effort levels for the joint product development game are strictly positive.

To begin with we examine whether the firms find it individually rational to opt for joint product development rather than pursue competitive R&D.[1] Note that under joint product development the profit of the i-th firm is given by

$$P_i(e_1, e_2) = \lambda(e_1 + e_2)\frac{R}{2} - h_i \int_0^{e_i} c(\tilde{e}_i)d\tilde{e}_i \,. \tag{1}$$

We can use a standard reaction function approach to solve for the equilibrium effort levels.

We assume that the competitive R&D payoffs, when both the firms succeed in developing the product, are zero. This can be justified by assuming that the product market involves price competition. Denoting the disagreement profits by $D_i(e_1, e_2)$

$$D_i(e_1, e_2) = \lambda(e_i)(1 - \lambda(e_j))R - h_i \int_0^{e_i} c(\tilde{e}_i)d\tilde{e}_i \,. \tag{2}$$

We can again use a standard reaction function approach to solve for the equilibrium effort levels.

Proposition 1 below demonstrates that if h_1 and h_2 are equal and the payoff R high enough relative to the cost parameter, it is individually rational for both firms to opt

[1] Marjit (1991) also examines, in a model with exogenous probabilities of success, the individual rationality of pursuing joint research. He finds that joint research in profitable provided the exogenous probability of success is either very high or very low.

for joint product development.[2] The result is quite intuitive. For large values of R and similar technological levels, free-riding problems are not very severe. The joint venture venture firms therefore have a sufficient incentive to invest and the probability of success is high. In competitive R&D however, a large R implies that both the firms overinvest. Not only does this reduce the individual success probabilities but increases the cost levels as well. Hence the result follows.

PROPOSITION 1. *Assume that* $h_1 = h_2 = h$ *and* $\lambda'(e_i)(1 - \lambda(e_j)) \geq \lambda'(e_i + e_j)$, $\forall e_i, e_j, i \neq j$. *We also assume that under competitive R&D a unique solution exists and there exists* e^* *such that* $\lambda(2e^*) \geq \frac{1}{2}$. *If the payoff R is high enough compared to the cost parameter h, the profit from joint product development exceeds that from competitive R&D.*

The condition that $\lambda'(e_i)(1 - \lambda(e_j)) \geq \lambda'(e_i + e_j)$, $\forall e_i, e_j, i \neq j$ is not necessary for the result. Consider the case where the return function is linear viz. $\lambda(e_1 + e_2) = \min(1, e_1 + e_2)$ and hence does not satisfy the above condition. Even in this case we can show that if R is high enough relative to the cost parameter h, it is individually rational for the firms to opt for joint product development.

In the next proposition we prove that the technology levels of the two firms cannot be too far apart if joint product development is to take place. The intuition is as follows. As technological asymmetry increases the level of effort put in by the inefficient firm becomes negligible. In the event of success however it stil obtains half the payoff. There is too much free-riding by the inefficient firm and it is therefore better for the efficient firm to pursue competitive R&D.

PROPOSITION 2. *Assume that* $\lambda(0) < \frac{1}{2}$. *For any given level of* h_1, *it is not individually rational for the first firm to opt for joint product development if the second firm is inefficient enough i.e. if* h_2 *is large enough.*

Propositions 1 and 2 are concerned with properties that hold in the limit. The spirit of the results however, seems to be that joint product development is individually rational, provided the technology levels are not too far apart. We can use a simple example where the return function is linear viz. $\lambda(e_1 + e_2) = \min(1, e_1 + e_2)$ and the cost function is quadratic viz. $C_i(e_i) = \frac{h_i e_i^2}{2}$ to show that this is indeed true.

We then examine the impact of an increase in technological asymmetry on the probability of success in a joint venture. We also compare the equilibrium outcome with the efficient outcome and examine the effect of an increase in technological diversity on the gap between the efficient and the equilibrium effort stream.

The next proposition provides two sufficient conditions for the effort stream to increase for a mean preserving spread of the technology levels.

PROPOSITION 3. (A) *A unique and interior Nash equilibrium exists.*

[2] Given Ray Chaudhuri (1995) we have omitted the proofs of Propositions 1 to 3. These are, however, available from the author on request.

(B) *Either of the following is a sufficient condition for the joint effort stream* $(e_1 + e_2)$ *to increase for a mean preserving spread of* h_1 *and* h_2:

(i) $c(e_i)$ *be concave,*

(ii) c^{-1} *be homogeneous of degree* k, $k \geq 0$.

The intuition is as follows. As technological diversity increases the incentive to free-ride increases for the inefficient firm and decreases for the efficient firm. This follows because the efficient firm now has more to gain from any success (his costs having fallen), while the inefficient firm has less to gain. This implies that the amount of effort put in by the efficient firm increases whereas the amount put in by the inefficient firm decreases. The increase in efforts by the efficient firm however exceeds the decrease in efforts by the other firm. This is because the efficient firm, having more to lose from a decrease in efforts, tries to more than make up for the decrease in effort by the inefficient firm. The sufficient conditions essentially ensure that doing this does not prove too costly for the efficient firm. Symmetrically they also ensure that decreasing the level of efforts is not too attractive for the inefficient firm.

Part B(ii) of Proposition 3 enables us to provide a definite answer for some cost curves with constant elasticity of the marginal cost (in fact these have convex marginal costs) e.g. $C_i(e_i) = h_i e_i^m$, $m > 2$.

The above result pertains to the case where the sharing rule is exogenous. We next briefly consider the case where the sharing rule is endogenously determined. We find that, for any sharing rule obeying some reasonable regularity conditions, an increase in technological asymmetry leads to an increase in the probability of success. We also identify some situations when the sharing rule would obey the stipulated restrictions. Such restrictions may be satisfied either when the sharing rule is determined through a Nash bargaining process or when it is determined so as to maximise aggregate profits. We refer the readers to Ray Chaudhuri (1995) for a detailed discussion of these results.

3. TWO PERIOD MODEL

Joint ventures are, in reality, spread over many periods. It is therefore of interest to examine whether in a dynamic context, the earlier conditions still ensure that a mean preserving spread of the technological levels leads to an increase in the effort stream. We find that for a two period model the previous conditions are no longer sufficient.

We analyse a two period game where in both period 1 and period 2 the firms simultaneously decide on their effort levels. They have a one-shot payoff of R, which they split equally. Of course if they succeed in developing the product in period 1 then the game stops after period 1 itself. For the sake of simplicity, we assume that the firms do not discount the future and that there is no 'learning' i.e. spill-overs of first period effort into the second period is absent. The probability of success in period 2 therefore depends solely on the effort level in that period.

Let e_{ij} denote the amount of effort put in by firm i in period j. In general we let the first subscript denote the firm and the second subscript denote the period.

We solve for the subgame perfect Nash equilibrium of this game. We use a standard backwards induction argument to solve for the second period game. In the second period game the Nash equilibrium conditions (i.e. the reaction functions) are given by

$$\lambda'(e_{12}+e_{22})\frac{R}{2} = h_1 c(e_{12}), \tag{3}$$

$$\lambda'(e_{12}+e_{22})\frac{R}{2} = h_2 c(e_{22}). \tag{4}$$

It is entirely standard to argue that the period 2 reaction functions, are negatively sloped and that a unique, interior solution exists.

Since the first period payoff functions are strictly concave in their own effort levels (this follows from Assumptions B and C), the equilibrium is given by

$$\frac{\partial P_{11}}{\partial e_{11}} = \lambda'(e_{11}+e_{21})\left(\frac{R}{2} - P_{12}\right) - h_1 c(e_{11}) = 0, \tag{5}$$

$$\frac{\partial P_{21}}{\partial e_{21}} = \lambda'(e_{11}+e_{21})\left(\frac{R}{2} - P_{22}\right) - h_2 c(e_{21}) = 0. \tag{6}$$

Here P_{i2} ($i = 1, 2$) denotes the second period payoff of firm i, evaluated at the Nash equilibrium effort levels of the second period game. We can argue as before that the period 1 reaction functions, R_{11} (derived from equation (5)) and R_{21} (derived from equation (6)), are negatively sloped and that a unique, interior solution exists.

Consider the case where, for a mean preserving spread of the technology levels (h_1, h_2), P_{12} and P_{22} increase. From equations (5) and (6) it is obvious that this will have a negative effect on the first period efforts, since the consequences of a failure in the first period is reduced.

Even for the especially simple example of a linear cost function, where $C_i(e_{ij}) = h_i e_{ij}$, we find that the effect is ambiguous and the sign of $\frac{de_{11}+de_{21}}{dh_1}$ could go either way.

The period 2 reaction functions are given by

$$\lambda'(e_{12}+e_{22})\frac{R}{2} = h_1, \tag{7}$$

$$\lambda'(e_{12}+e_{22})\frac{R}{2} = h_2. \tag{8}$$

Clearly for $h_1 = h_2$, the reaction functions coincide and all points can be a Nash equilibrium. For $h_1 < h_2$, it is clear that R_{12} lies above R_{22}. Consequently, the equilibrium involves $e_{22} = 0$ and $e_{12} = \overline{e_{12}}$, where $\lambda'(\overline{e_{12}})\frac{R}{2} = h_1$. Thus, for a mean preserving spread of h_1 and h_2, e_{12} would increase and e_{22} would still be zero. Besides, both P_{12} and P_{22} would increase. In fact $dP_{12} = -e_{12}dh_1 > 0$.

The period 1 reaction functions are given by

$$\lambda'(e_{11}+e_{21})\left[\frac{R}{2} - P_{12}\right] = h_1, \tag{9}$$

$$\lambda'(e_{11}+e_{21})\left[\frac{R}{2} - P_{22}\right] = h_2. \tag{10}$$

Since $P_{12} < P_{22}$, R_{21} lies below R_{11} and the equilibrium involves $e_{21} = 0$ and $e_{11} = \overline{e_{11}}$, where $\lambda'(\overline{e_{11}})\left(\frac{R}{2} - P_{12}\right) = h_1$.

Differentiating the above and taking into account the change in P_{12}, we obtain

$$\frac{de_{11}}{dh_1} = \frac{1 - \lambda'(\overline{e_{11}})e_{12}}{\lambda''(\overline{e_{11}})\left(\frac{R}{2} - P_{12}\right)}. \tag{11}$$

Clearly the sign of this is ambiguous. In fact if we assume that $\lambda'(E) < 1$ for all E and that the costs h_1 is not very high (specifically $\frac{2h_1}{R} < \lambda'(1)$) then it is obvious that $\frac{de_{11}}{dh_1} > 0$. In this example it is clearly seen that the problem stems from the fact that P_{12} increases.

4. THE INFINITE HORIZON GAME

We next investigate whether the indeterminacy problem in the two period game could be due to the finite horizon formulation. The dissimilarity between the first and the second period is artificial. If second period failure is followed by the possibility of going into another period then second period profits may not increase and consequently the first period effort may not decrease. In order to examine this question we set up an infinite horizon version of the above game where at each $t = 0, 1, 2, \cdots, \infty$ the firms simultaneously decide on their effort levels. We show that if the costs are linear then the effort levels increase for a mean-preserving spread of h_1 and h_2. In this case we also find that an increase in technological dissimilarity widens the gap between the efficient and the equilibrium effort stream.

For the sake of simplicity we assume that the discount factor is one (i.e. there is no discounting of the future) and that there is no memory. The assumption of no memory implies that the game at any period is identical, except for the date, to the game at any other period.[3]

In this section we introduce establishment costs, K, which have to be met even when effort levels are zero. Of course once the firms succeed in developing the product, the R&D establishment can be dissolved and there is no need to incur the establishment costs any more. We assume that dissolving the establishment means that the firms opt out of the R&D venture. So we can rewrite the cost function as, $C_i(e_i) = K + h_i \int_0^{e_i} c(\tilde{e}_i)d\tilde{e}_i$, where $K > 0$. (Since there is no discounting and since assumption F (yet to be introduced) ensures that even with zero effort the venture will ultimately succeed, in the absense of the establishment costs there is no incentive for the firms to put in a positive level of effort.)

We restrict attention to stationary Markov equilibria. In our setup this implies that the prescribed strategies must be identical at every period.

[3] With positive memory we would have a stochastic game with an uncountable state and action space, which raises a number of technical difficulties. In view of the problems in even guaranteeing existence of equilibria in such games, we do not try to examine the game with positive memory.

The technique followed is to approximate this game by a sequence of truncated games where the freedom of action of the firms is taken away after a certain period. Define G_T to be the game truncated at T, if for $t \geq T+1$, it is the case that $e_{1t} = e_{1T}$ and $e_{2t} = e_{2T}$, where e_{ij} is the amount of effort put in by the ith firm in period j. Effectively after the Tth period the firms' freedom of action is taken away and they are forced to choose the same effort levels as in the Tth period.

Fortunately there exists a unique subgame perfect equilibrium for any truncation period. Besides, the same strategies are prescribed for all truncation periods. Next we let these games approximate the real game by indefinitely postponing the truncation date to later periods. We find that the equilibrium of these truncated games is an equilibrium of the untruncated game as well. (This technique draws heavily on Harris (1985).)

Consider the game G_T at period T. Clearly, if P_i denotes the profit of the ith firm, then, under the above truncation rules we obtain

$$P_1 = \lambda(e_1+e_2)\frac{R}{2} - K - h_1\int_0^{e_1} c(\tilde{e}_1)d\tilde{e}_1 + (1-\lambda(e_1+e_2))P_1\,, \tag{12}$$

$$P_2 = \lambda(e_1+e_2)\frac{R}{2} - K - h_2\int_0^{e_2} c(\tilde{e}_2)d\tilde{e}_2 + (1-\lambda(e_1+e_2))P_2\,. \tag{13}$$

Taking the Nash equilibrium of the above game we obtain the following first order conditions

$$\frac{\lambda(e_1+e_2)}{\lambda'(e_1+e_2)} = \frac{K + h_1\int_0^{e_1} c(\tilde{e}_1)d\tilde{e}_1}{h_1 c(e_1)}\,, \tag{14}$$

$$\frac{\lambda(e_1+e_2)}{\lambda'(e_1+e_2)} = \frac{K + h_2\int_0^{e_2} c(\tilde{e}_2)d\tilde{e}_2}{h_2 c(e_2)}\,. \tag{15}$$

In Proposition 4(ii) we show that the above equations have a unique solution. Denote the solution vector by (e_1^*, e_2^*).

From equations (12) and (13) we find

$$P_1(e_1^*, e_2^*) = \frac{R}{2} - \frac{K + h_1\int_0^{e_1^*} c(\tilde{e}_1)d\tilde{e}_1}{\lambda(e_1^*+e_2^*)}\,, \tag{16}$$

$$P_2(e_1^*, e_2^*) = \frac{R}{2} - \frac{K + h_2\int_0^{e_2^*} c(\tilde{e}_2)d\tilde{e}_2}{\lambda(e_1^*+e_2^*)}\,. \tag{17}$$

We next look at the game starting from period $T-1$. Letting $\overline{P_i}$ denote the profit of the ith firm

$$\overline{P_1} = \lambda(e_1+e_2)\frac{R}{2} - h_1\int_0^{e_1} c(\tilde{e}_1)d\tilde{e}_1 + (1-\lambda(e_1+e_2))P_1(e_1^*, e_2^*)\,, \tag{18}$$

$$\overline{P_2} = \lambda(e_1+e_2)\frac{R}{2} - h_2\int_0^{e_2} c(\tilde{e}_2)d\tilde{e}_2 + (1-\lambda(e_1+e_2))P_2(e_1^*, e_2^*)\,. \tag{19}$$

The subgame perfect solution in this case is therefore given by

$$\lambda'(e_1 + e_2)\left[\frac{R}{2} - P_1(e_1^*, e_2^*)\right] - h_1 c(e_1) = 0\,, \tag{20}$$

$$\lambda'(e_1 + e_2)\left[\frac{R}{2} - P_2(e_1^*, e_2^*)\right] - h_2 c(e_2) = 0\,. \tag{21}$$

From equations (16) and (17) it is easy to see that, (e_1^*, e_2^*) solves the above equation pair.

Since equations (20) and (21) have a unique solution[4] it must be (e_1^*, e_2^*). Therefore at $T-1$ also the equilibrium levels of effort will be (e_1^*, e_2^*). Hence F_T, the equilibrium of the game truncated at T, involves playing (e_1^*, e_2^*) at each period. Clearly this strategy is independent of the period of truncation and for all G_T the same unique strategy $F = F_T$ obtains.

Next we identify the unique stationary equilibrium of the game. We begin by introducing some notations and assumptions.

If k is some strategy pair of the infinite horizon game, we let (k, x) denote the continuation strategies prescribed by k following some subgame x. The strategy pair where firm i plays h_i instead of the action sequence prescribed by k, is denoted by k/h_i. Finally $\pi_s h_i$ denotes a truncation of firm i's strategy (h_i) at period s i.e. it denotes the strategy where from period $s+1$ onwards firm i will choose the same action as in period s.

ASSUMPTION E. $\lambda(0) > 0$.

Note that in this model the discount factor is 1. Instead $(1 - \lambda(E))$ plays the role of a discount factor in this model. Though this is endogenous rather than exogenous, the above assumption ensures that it is always strictly less than 1.

ASSUMPTION F. $e_i \in [0, \overline{e_i}]$.

This assumption can be justified on the following grounds. Take $\overline{e_i}$ such that $\frac{R}{2} = h_i \int_0^{\overline{e_i}} c(\tilde{e}_i)d\tilde{e}_i$. Clearly firm i is never going to put in an effort level greater than $\overline{e_i}$ because for any higher effort level his payoff is going to be negative. Technically we need this assumption to put a bound on the one-shot payoffs. Clearly this implies that the deviations in one-shot payoffs are bounded.

Note that assumption E and the fact that the deviations in one-shot payoffs are bounded implies that $P_j(k/\pi_s h_i) - P_j(k/h_i)$ goes towards zero as s goes towards infinity.

Define $g(e_1 + e_2) = \frac{\lambda(e_1+e_2)}{\lambda'(e_1+e_2)}$ and $y(e_i) = \frac{K + h_i \int_0^{e_i} c(\tilde{e}_i)d\tilde{e}_i}{h_i c(e_i)}$.

We next introduce the following assumption which is required in Proposition 4(ii) to prove that the strategy pair F is unique in the class of stationary equilibrium.

[4] This follows because in equation (20), E decreases for an increase in e_1, (i.e. $\frac{de_2}{de_1} < -1$), and symmetrically in equation (21), E decreases for an increase in e_2, (i.e. $\frac{de_2}{de_1} > -1$).

ASSUMPTION G. Either (i) $y(e_i)$ is decreasing in e_i or (ii) $y(e_i)$ is increasing in e_i[5] and $\frac{\partial g(e_i+e_j)}{\partial e_i} > \frac{\partial y(e_i)}{\partial e_i}, \forall e_i, e_j$.

Proposition 4 shows that the unique equilibrium of the truncated games is a stationary equilibrium of the untruncated infinite horizon game as well. We also demonstrate that for simple linear cost functions the probability of success increases for a mean preserving spread of the technology levels.

PROPOSITION 4. (i) *F is a stationary equilibrium of the infinite horizon game.*

(ii) *F is unique in the class of stationary Markov equilibria.*

(iii) *For a linear cost function of the form $C_i(e_i) = K + h_i e_i$, the joint effort stream increases for a mean preserving spread of h_1 and h_2.*[6]

Proof. (i) We have to show that for any subgame x, (F, x) is a Nash equilibrium for the subgame x.

$$\begin{aligned} &P_i(F/h_i, x) - P_i(F, x) \\ &\quad = P_i(F/h_i, x) - P_i(F/\pi_s h_i, x) + P_i(F_T/\pi_s h_i, x) - P_i(F_T, x) \qquad (22) \end{aligned}$$

For $T \geq s$, the second difference is less than equal to zero, as $\pi_s h_i$ can be taken to be any strategy of the truncated game and F_T is optimal in the truncated game. Now keeping $T \geq s$ take (T, s) towards infinity. Clearly $\pi_s h_i \to h_i$ so the first difference goes towards zero. This follows from assumption E and the fact that the deviations in one shot payoff are bounded. The second difference is less than equal to zero, therefore

$$P_i(F/h_i, x) - P_i(F, x) \leq 0 .$$

(ii) First observe that assumption G implies that equations (14) and (15) are negatively sloped.

If assumption G(i) holds then it is clear that for equation (14), $\frac{de_2}{de_1} < -1$ and for equation (15), $\frac{de_2}{de_1} > -1$. Therefore equations (14) and (15) have a unique intersection. If, however, assumption G(ii) holds a similar argument applies, with the sign of the slopes reversed.[7]

(iii) Consider the case where $h_1 < h_2$. Totally differentiating equations (14) and (15) and manipulating we obtain,

$$\frac{de_1 + de_2}{dh_2} = \left(\frac{K}{h_1^2} - \frac{K}{h_2^2}\right) \frac{1}{(2g'(E) - 1)}$$

[5] Note that this implies that assumption D can no longer hold.

[6] In this case clearly $y'(e_i) = 1$. Assumption G is however satisfied since, for any return function satisfying $\lambda''(E) < 0$, it follows that $g'(E) > 1$.

[7] Consider the case where assumption G(i) holds. Define e_1' by $\frac{\lambda(e_1')}{\lambda'(e_1')} = \frac{K + h_1 \int_0^{e_1'} c(e_1) de_1}{h_1 c(e_1')}$ and e_1'' by $\lambda'(e_1'') = 0$ (using assumption D). Therefore, $e_1' < e_1''$. We can similarly define e_2' and e_2'' and show that $e_2' < e_2''$. Therefore it follows that the intersection is interior. If, however, assumption G(ii) holds, we cannot use the previous argument. In this case we must assume that the technology levels are not too far apart.

for $dh_2 = -dh_1 > 0$. For all $\lambda(E)$ such that $\lambda''(E) \leq 0$ it follows that $g'(E) = \frac{\lambda'(E)^2 - \lambda(E)\lambda''(E)}{\lambda'(E)^2} \geq 1$ and so the effort stream increases. ■

Note that the equilibrium first order conditions are independent of the sharing rule, so that the analysis in this section goes through even when the sharing rule is endogenously determined. In particular this shows that the result in the previous section, that for some endogenous sharing rules an increase in technological diversity may lead to a decrease in the divergence between the two effort streams,[8] is, to some extent, an artifact of the finite horizon nature of the model.

Therefore in the infinite horizon game also we find that while an increase in technological diversity increases the success probability in absolute terms, it also leads to an increase in the divergence between the efficient and the equilibrium effort levels.

We then compare the profit levels of the two firms under joint product development and competitive R&D.[9] Given the complexity of the question, however, we restrict attention to the case where both the cost and the return functions are linear, i.e. the cost function is $K + h_i e_i$ and the return function is $\min\{e, 1\}$.

We first consider the profit level of the two firms under joint product development. Note that from equations (14) and (15) we can write that $h_i = \frac{K + h_i e_i^*}{(e_1^* + e_2^*)}$. Next from equations (16) and (17) we find that the profit level of the i-th firm is $\frac{R}{2} - \frac{K + h_i e_i^*}{e_1^* + e_2}$. Thus in equilibrium the profit level of the i-th firms is

$$\frac{R}{2} - h_i \,.$$

We then consider the case of competitive R&D. Clearly, the competitive profit level D_i for a constant effort stream (e_1, e_2) is given by

$$D_i = e_i(1 - e_j)R - K - h_i e_i + (1 - e_1)(1 - e_2)D_i \,, \quad i \neq j \,. \tag{23}$$

Re-arranging the above equation we can write

$$D_i = \frac{e_i(1 - e_j)R}{e_1 + e_2 - e_1 e_2} - \frac{K + h_i e_i}{e_1 + e_2 - e_1 e_2} \,, \quad i \neq j \,. \tag{24}$$

We can then mimic the argument for the case of joint product development to argue that the first order condition yields

$$-\frac{K + h_i e_i}{e_1 + e_2 - e_1 e_2} = \frac{e_j R}{e_1 + e_2 - e_1 e_2} - \frac{h_i}{1 - e_j} \,, \quad i \neq j \,. \tag{25}$$

Note that we can use equations (24) and (25) to write that $D_i = R - \frac{h_i}{1 - e_j}$. Moreover, from equation (25) we find that $\frac{h_i}{1 - e_j} = R + \frac{K}{e_j}$. Combining the two together we find that

$$D_i = -\frac{K}{e_j} > 0 \,. \tag{26}$$

[8] For sharing rules that are responsive enough to a change in efficiency.

[9] We are indebted to an anonymous referee for pointing out the importance of this issue and the need for analysis.

Thus whenever $\frac{R}{2} - h_i > 0$, for $i = 1, 2$, both firms prefer to opt for joint product development rather than pursue competitive R&D. Summarising the above discussion we obtain our last proposition.

PROPOSITION 5. *Assume that the cost function is* $K + h_i e_i$, *the return function is* $\min\{e, 1\}$ *and* $\frac{R}{2} - h_i > 0$, *for* $i = 1, 2$. *Then both the firms strictly prefer joint product development to competitive R&D.*

Note that the condition that $\frac{R}{2} - h_i > 0$, for $i = 1, 2$, is more likely to be satisfied if the firms are symmetric. Whereas if the firms are asymmetric then it is possible that $\frac{R}{2} - h_1 > 0 > \frac{R}{2} - h_2$. In that case profit from joint product development is also negative and firm 2 might prefer to opt for competitive R&D. Thus even in the infinite horizon case the results are qualitatively similar to that in Section 2, that joint product development is more likely if the firms are similar, whereas with asymmetric firms we might have competitive R&D.

5. CONCLUSION

In this paper we examine the impact of technological diversity on the incentive for joint product development.

In a static framework our model predicts that if technological diversity increases, then the chance that a joint venture forms at all is reduced. Given that a joint venture forms however, an increase in technological diversity increases the probability of success. Extending our analysis to a dynamic two period model we find that the earlier sufficient conditions no longer ensure that the effort stream increases as the firms become more technologically diverse. In the infinite horizon game, however, we find that under some restrictions on the cost and the return functions all the earlier results still hold qualitatively. Moreover, the above results are independent of the nature of the sharing rule and are therefore consistent with any endogenous sharing rule.

Thus the results are critically dependent on the perceptions of the partner firms regarding the longevity of the joint venture, i.e. whether they expect the joint venture to break up soon, or whether they expect it to last for a long time.

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