Title	STOCHASTICALLY STABLE STATES IN A DUOPOLY WITH DIFFERENTIATED GOODS
Sub Title	
Author	TANAKA, Yasuhito
Publisher	Keio Economic Society, Keio University
Publication year	2000
Jtitle	Keio economic studies Vol.37, No.2 (2000.) ,p.25- 36
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-20000002-0025

KEIO ECONOMIC STUDIES **37**(2), 25–36 (2000)

STOCHASTICALLY STABLE STATES IN A DUOPOLY WITH DIFFERENTIATED GOODS

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First version received November 1999; final version accepted June 2000

Abstract: We present results on finite population evolutionarily stable strategies (ESSs) and stochastically stable states for a model of evolution with an imitative rule of strategy choice in a symmetric duopoly with differentiated goods. Two firms play price setting and quantity setting duopoly games under general demand functions. We will show that the stochastically stable state in a price setting duopoly and that in a quantity setting duopoly coincide.

JEL Classification Number: C72, L13

1. INTRODUCTION

Duopolistic or oligopolistic markets are typically analyzed under two alternative assumptions about firms' behavior: a quantity setting or Cournot approach and a price setting or Bertrand approach. It is well known that, when goods are substitutes, the Bertrand equilibrium is more efficient than the Cournot equilibrium (see Singh and Vives (1984), Cheng (1985) and Vives (1985)). These analyses are based on the Nash equilibrium concept. In this paper we present an evolutionary game theoretic analysis of duopoly. We consider a duopoly with differentiated goods, and study finite population evolutionarily stable strategies (ESSs) defined by Schaffer (1988) and stochastically stable states (or long run equilibria in terms of Kandori et al. (1993)) for a model of evolution with an imitative rule of strategy choice with mutations. A stochastically stable state is a state where most of the time is spent in the long run when the probability of mutation becomes very small. Our formulation of a model of evolution with an imitative rule of strategy choice follows Robson and Vega-Redondo (1996) and Vega-Redondo (1997).

Vega-Redondo (1997) studied imitative behavior in a symmetric oligopoly with a homogeneous good, and showed that Walrasian behavior (profit maximization given the market clearing price) is a stochastically stable strategy. Tanaka (1999) extended the result of Vega-Redondo (1997) to a case of *asymmetric* homogeneous oligopoly with low cost and high cost firms, and showed that under the assumption that marginal cost is increasing a stochastically stable outcome is the competitive (Walrasian) output

Acknowledgement. I wish to thank an anonymous referee for very helpful comments.

for each group of firms. Rhode and Stegeman (2001) analyzed Darwinian dynamics of a symmetric, differentiated duopoly with linear demand functions. They showed that firms' strategy choices cluster around a strategy profile that is not a one-shot Nash equilibrium, and this profile is invariant under a class of transformations of the strategy spaces (Bertrand vs. Cournot). They considered a stationary distribution of a Markov chain with large and frequent mutations. By contrast, we consider a limit of a stationary distribution of a Markov chain as mutations vanish according to the formulation by Robson and Vega-Redondo (1996) and Vega-Redondo (1997).

Schaffer (1988) proposed a concept of an evolutionarily stable strategy (ESS) for a finite (or small) population as a generalization of the standard ESS concept for an infinite (or large) population by Maynard-Smith(1982). We call it a *finite population ESS*. He showed that a finite population ESS is not generally a Nash equilibrium strategy. In Schaffer (1989) he applied this concept to an economic game, and showed that the strategy which survives in economic natural selection is a relative, not absolute, payoff maximizing strategy. He considered the following survival rule. Firms are born with strategies and cannot change their strategies in response to changing circumstances. At the end of each period, if the payoff of Firm i is larger than the payoff of Firm j, the probability that Firm i survives in the next period is larger than the probability that Firm j survives in the next period. Alternatively we consider that the survival rule operates on strategies, not firms, and the proportion of successful strategies in the population grows by firms' imitation of strategies.[1]

In this paper we consider the following model of a duopoly. Every firm can observe decisions each other, but does not know the exact form of demand functions, and can not compute its best response to the other firm's strategy. On the other hand, the firms know that the demand functions for them are symmetric, and their cost functions are the same. When two firms choose the same strategy (output or price), denoting it by s_1, in a symmetric duopoly their profits are equal, and they do not have incentive to change their strategies. Now, suppose that one firm (a mutant firm) experiments a different strategy, s_2. If this firm makes higher profit than the other firm, it will wish to imitate the mutant firm's success. On the other hand, if the mutant firm makes lower profit than the other firm, it will not wish to imitate the mutant firm's failure, and in fact the mutant firm will wish to switch from s_2 to s_1. If, starting from s_1, experimenting always leads to lower profit for the mutant firm than for the other firm, then s_1 is a finite population ESS.

The mechanism of an imitative strategy choice will be explained in Section 4. Some recent papers such as Robson and Vega-Redondo (1996) and Vega-Redondo (1997)

[1] Hansen and Samuelson (1988) also presented analyses about evolution in economic games. They showed that with small number of firms a surviving strategy in economic natural selection, they called such a strategy a *universal survival strategy*, is not a Nash equilibrium strategy. Their universal survival strategy is essentially equivalent to Schaffer's finite population evolutionarily stable strategy. They said, "In real-world competition, firms will be uncertain about the profit outcomes of alternative strategies. This presents an obvious obstacle to instantaneous optimization. Instead, firms must search for and learn about more profitable strategies. As Alchian (1965) emphasizes, an important mechanism for such a search depends on a comparison of observed profitability across the strategies used by market participants. That is, search for better strategies is based on *relative* profit comparisons." For more recent analyses of imitation behavior, see Schlag (1998) and (1999).

considered a model of evolution with an imitative strategy choice. On the other hand, some other papers such as Kandori and Rob (1995) and (1998), and Galesloot and Goyal (1997) considered a model of evolution based on best response dynamics. In best response dynamics each player chooses a strategy in period $t+1$ which is a best response to other players' strategies in period t. Thus players must know the whole payoff structure of the game, and be able to compute their best responses. While in imitation dynamics, players simply mimic successful strategies of other players. We think that imitation dynamics is more appropriate than best response dynamics for an economic game with boundedly rational players.

We are concerned with showing the following results. In a quantity setting duopoly the finite population ESS output is a stochastically stable strategy, and when the goods are substitutes, it is between the Nash-Cournot equilibrium output and the competitive output. In a price setting duopoly the finite population ESS price is a stochastically stable strategy, and when the goods are substitutes, it is between the Nash-Bertrand equilibrium price and the competitive price. The ESS output in the quantity setting case and the ESS price in the price setting case yield equivalent outcomes. Therefore the stochastically stable state in a quantity setting duopoly and that in a price setting duopoly coincide.

In the next section we consider finite population ESSs. In Section 3 we will show the equivalence of dual ESSs. In Section 4 we will show that the finite population ESSs are stochastically stable strategies in both quantity setting and price setting cases. From the equivalence of the finite population ESSs we obtain the conclusion that the stochastically stable state in a quantity setting duopoly and that in a price setting duopoly coincide. The last section contains concluding remarks.

2. FINITE POPULATION EVOLUTIONARILY STABLE STRATEGIES

There is a duopoly with two firms producing differentiated goods. We call two firms Firm 1 and Firm 2. Let x_1 and x_2 be the outputs of Firm 1 and Firm 2, and let p_1 and p_2 be the prices of the goods of Firm 1 and Firm 2. Then, the direct demand functions for the goods are given by

$$x_1 = x_1(p_1, p_2), \tag{1}$$

and

$$x_2 = x_2(p_1, p_2). \tag{2}$$

We assume that the demand functions are symmetric for two firm, that is,

$$x_1(p_1, p_2) \equiv x_2(p_2, p_1).$$

And we assume that $x_1(p_1, p_2)$ and $x_2(p_1, p_2)$ are twice differentiable, and

$$\frac{\partial x_1}{\partial p_1} < 0, \quad \frac{\partial x_2}{\partial p_2} < 0, \quad \left|\frac{\partial x_1}{\partial p_1}\right| > \left|\frac{\partial x_1}{\partial p_2}\right| \quad \text{and} \quad \left|\frac{\partial x_2}{\partial p_2}\right| > \left|\frac{\partial x_2}{\partial p_1}\right| .$$

The latter two inequalities mean that own effects are larger than cross effects. From the above demand functions, we obtain the following inverse demand functions

$$p_1 = p_1(x_1, x_2), \tag{3}$$

and

$$p_2 = p_2(x_1, x_2). \tag{4}$$

$p_1(x_1, x_2)$ and $p_2(x_1, x_2)$ are also symmetric and twice differentiable, and

$$\frac{\partial p_1}{\partial x_1} < 0, \quad \frac{\partial p_2}{\partial x_2} < 0, \quad \left|\frac{\partial p_1}{\partial x_1}\right| > \left|\frac{\partial p_1}{\partial x_2}\right| \quad \text{and} \quad \left|\frac{\partial p_2}{\partial x_2}\right| > \left|\frac{\partial p_2}{\partial x_1}\right| .$$

The cost function of Firm i is denoted by $c(x_i)$, which is twice differentiable. Two firms have the same cost function. The marginal cost of Firm i, $c'(x_i)$, is positive and increasing.

Further we assume that the following relations hold.

$$2\frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial x_j} + \left(\frac{\partial^2 p_i}{\partial x_i^2} + \frac{\partial^2 p_i}{\partial x_i x_j}\right) x_i - c''(x_i) < 0, \quad j \neq i, \tag{5}$$

and

$$2\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial p_j} + (p_i - c'(x_i))\left(\frac{\partial^2 x_i}{\partial p_i^2} + \frac{\partial^2 x_i}{\partial p_i p_j}\right) - \frac{\partial x_i}{\partial p_i}\left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial p_j}\right) c''(x_i) < 0, \quad j \neq i. \tag{6}$$

$c''(x)$ is the second order derivative of $c(x)$. Eq. (5) is derived from the stability condition with the second order condition for the Nash-Cournot equilibrium, and Eq. (6) is derived from the stability condition with the second order condition for the Nash-Bertrand equilibrium (see Appendix).

In a quantity setting duopoly the profit of Firm 1 is

$$\pi_1(x_1, x_2) = p_1(x_1, x_2)x_1 - c(x_1),$$

and the profit of Firm 2 is

$$\pi_2(x_1, x_2) = p_2(x_1, x_2)x_2 - c(x_2).$$

We consider an evolutionary game in which two firms repeatedly play a duopoly stage game. In this game the population is two, and the stage game is also a two players game. Thus it is a so called *playing the fields model.* Strategies for the firms are their outputs. The firms repeatedly play the stage game in each period, and may change their strategies between one period and the next period. Such a dynamic problem is treated in Section 4. In this section we consider finite population evolutionarily stable strategies of the stage game.

Consider a state in which both firms choose x^*. If, when one firm (a mutant firm) chooses a different strategy x', the profit of the firm who chooses x^* is larger than the profit of the mutant firm, and this relation holds for all $x' \neq x^*$, then x^* is a finite population evolutionarily stable strategy (ESS).[2] Without loss of generality, assuming that the mutant player is Firm 1, x^* is a finite population ESS if

$$\pi_2(x_1, x^*) > \pi_1(x_1, x^*), \quad \forall x_1 \neq x^*. \tag{7}$$

We define

$$\bar{x}_1(x_2) = \arg\max_{x_1} \varphi_1(x_1, x_2), \tag{8}$$

where

$$\begin{aligned}\varphi_1(x_1, x_2) &= \pi_1(x_1, x_2) - \pi_2(x_1, x_2) \\ &= p_1(x_1, x_2)x_1 - c(x_1) - p_2(x_1, x_2)x_2 + c(x_2).\end{aligned} \tag{9}$$

If there is a unique maximizer x^* in Eq. (8) such that $x^* = \bar{x}_1(x^*)$, then x^* satisfies Eq. (7) since $\varphi_1(x_1, x^*)$ has the maximum value, which is zero, only when $x_1 = x^*$.

Differentiating Eq. (9) with respect to x_1 yields

$$p_1 + \frac{\partial p_1}{\partial x_1}x_1 - c'(x_1) - \frac{\partial p_2}{\partial x_1}x_2 = 0.$$

Substituting $x_1 = x_2 = x^*$ into this, we obtain the following condition for a finite population ESS,

$$p_i(x^*, x^*) + \left(\frac{\partial p_i}{\partial x_i} - \frac{\partial p_j}{\partial x_i}\right)x^* - c'(x^*) = 0, \quad j \neq i. \tag{10}$$

We assume that there is a unique ESS output x^*.

Now suppose that the goods of the firms are substitutes. Then we have $\frac{\partial p_j}{\partial x_i} < 0$, $j \neq i$. The profit maximizing condition for Firm i, $i = 1, 2$, in a Cournot game (a quantity game) is

$$p_i + \frac{\partial p_i}{\partial x_i}x_i - c'(x_i) = 0. \tag{11}$$

Let x_c be the Nash-Cournot equilibrium output. We assume $x_c > 0$. Since the demand functions are symmetric, and the firms have the same cost function, the Nash-Cournot equilibrium is symmetric. Then, when $x_1 = x_2 = x_c$, the left hand side of (11) is zero. From Eq. (10) and $\frac{\partial p_j}{\partial x_i} < 0$ we find that when $x_1 = x_2 = x^*$ the left hand side of Eq. (11) is equal to $\frac{\partial p_j}{\partial x_i}x^*$, and it is negative. Eq. (5) implies that the left hand side of Eq. (11) is decreasing with respect to the outputs of the firms provided $x_1 = x_2$. Thus we obtain $x^* > x_c$.

In a competitive industry the profit maximizing condition for Firm i is

$$p_i - c'(x_i) = 0. \tag{12}$$

Let x_w be the competitive (or Walrasian) equilibrium output. When $x_1 = x_2 = x_w$, the left hand side of (12) is zero. In symmetric situations we have $\frac{\partial p_j}{\partial x_i} = \frac{\partial p_i}{\partial x_j}$. From Eq.

[2] Schaffer's original definition is weaker. He defines x^* as a finite population ESS if Eq. (7) is satisfied with weak inequality. We adopt the definition with strong inequality. About the definition of a finite population ESS, see Crawford (1991).

(10) we obtain

$$p_i(x^*, x^*) - c'(x^*) = -\left(\frac{\partial p_i}{\partial x_i} - \frac{\partial p_i}{\partial x_j}\right)x^* > 0\,, \quad j \neq i\,.$$

That is, when $x_1 = x_2 = x^*$, the left hand side of (12) is positive. Since $\frac{\partial p_i}{\partial x_i} + \frac{\partial p_i}{\partial x_j} - c'' < 0$, the left hand side of Eq. (12) is decreasing with respect to the outputs of the firms provided $x_1 = x_2$. Thus we obtain $x^* < x_w$.

Next, in a price setting duopoly the profits of Firm 1 and 2 are represented as

$$\pi_1(p_1, p_2) = p_1 x_1(p_1, p_2) - c(x_1(p_1, p_2))\,,$$

and

$$\pi_2(p_1, p_2) = p_2 x_2(p_1, p_2) - c(x_2(p_1, p_2))\,.$$

Similarly to the quantity setting case we consider an evolutionary game in which two firms repeatedly play the duopoly stage game. Denote the finite population ESS price by p^*. The condition for the finite population ESS price is written as follows,

$$x_i(p^*, p^*) + \left(\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i}\right)(p^* - c'(x_i(p^*, p^*))) = 0\,, \quad j \neq i\,. \tag{13}$$

We assume that there is a unique ESS price p^*.

Now suppose that the goods of the firms are substitutes. Then we have $\frac{\partial x_j}{\partial p_i} > 0$, $j \neq i$. The profit maximizing condition for Firm i, $i = 1, 2$, in a Bertrand game (a price game) is

$$x_i + \frac{\partial x_i}{\partial p_i}(p_i - c'(x_i)) = 0\,. \tag{14}$$

Let p_b be the Nash-Bertrand equilibrium price.[3] Then, when $p_1 = p_2 = p_b$, the left hand side of (14) is zero. From Eq. (13) and $\frac{\partial x_j}{\partial p_i} > 0$ we find that when $p_1 = p_2 = p^*$ the left hand side of Eq. (14) is equal to $\frac{\partial x_j}{\partial p_i}(p^* - c'(x_i))$, and it is positive. Eq. (6) implies that the left hand side of Eq. (14) is decreasing with respect to the prices of the goods provided $p_1 = p_2$. Thus we have $p^* < p_b$.

In a competitive industry the profit maximizing condition for Firm i is the same as (12). Let p_w be the competitive (or Walrasian) equilibrium price. Since $c'(x_i) > 0$ we have $p_w > 0$. When $p_1 = p_2 = p_w$, the left hand side of (12) is zero. From Eq. (13) we have

$$p^* - c'(x_i(p^*, p^*)) = -\frac{x_i(p^*, p^*)}{\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i}} > 0\,, \quad j \neq i\,. \tag{15}$$

That is, when $p_1 = p_2 = p^*$, the left hand side of (12) is positive. Since $1 - c''(x_i)\left(\frac{\partial x_i}{\partial p_i} + \frac{\partial x_i}{\partial p_j}\right) > 0$, the left hand side of Eq. (12) is increasing with respect to the prices of the goods provided $p_1 = p_2$. Thus we have $p^* > p_w$.

[3] Since the demand functions are symmetric, and the firms have the same cost function, the Nash-Bertrand equilibrium is symmetric.

Therefore we have shown the following proposition.[4]

PROPOSITION 1. *When the goods of the firms are substitutes,*

1. *The finite population ESS output in a quantity setting duopoly is between the Nash-Cournot equilibrium output and the competitive output; and*
2. *The finite population ESS price in a price setting duopoly is between the Nash-Bertrand equilibrium price and the competitive price.*

3. THE EQUIVALENCE OF DUAL FINITE POPULATION ESSs

The inverse demand functions in the quantity setting case, Eq. (3) and Eq. (4), are obtained by inverting the demand functions in the price setting case, Eq. (1) and Eq. (2). These two systems represent the same demand structure.

Totally differentiating Eq. (3) and Eq. (4) yields the following expressions,

$$dp_1 = \frac{\partial p_1}{\partial x_1}dx_1 + \frac{\partial p_1}{\partial x_2}dx_2 ,$$

and

$$dp_2 = \frac{\partial p_2}{\partial x_1}dx_1 + \frac{\partial p_2}{\partial x_2}dx_2 .$$

Solving these equations for dx_1 and dx_2, we obtain

$$dx_1 = \frac{1}{D}\left(\frac{\partial p_2}{\partial x_2}dp_1 - \frac{\partial p_1}{\partial x_2}dp_2\right) , \tag{16}$$

and

$$dx_2 = \frac{1}{D}\left(-\frac{\partial p_2}{\partial x_1}dp_1 + \frac{\partial p_1}{\partial x_1}dp_2\right) , \tag{17}$$

where

$$D = \frac{\partial p_1}{\partial x_1}\frac{\partial p_2}{\partial x_2} - \frac{\partial p_1}{\partial x_2}\frac{\partial p_2}{\partial x_1} .$$

In symmetric situations in which $x_1 = x_2$, we have

$$\frac{\partial p_2}{\partial x_2} = \frac{\partial p_1}{\partial x_1} \quad \text{and} \quad \frac{\partial p_1}{\partial x_2} = \frac{\partial p_2}{\partial x_1} . \tag{18}$$

Then D is rewritten as follows,

$$D = \left(\frac{\partial p_1}{\partial x_1}\right)^2 - \left(\frac{\partial p_2}{\partial x_1}\right)^2 = \left(\frac{\partial p_1}{\partial x_1} - \frac{\partial p_2}{\partial x_1}\right)\left(\frac{\partial p_1}{\partial x_1} + \frac{\partial p_2}{\partial x_1}\right) .$$

From Eq. (16), Eq. (17) and Eq. (18) we obtain

$$\frac{\partial x_i}{\partial p_i} = \frac{1}{D}\frac{\partial p_i}{\partial x_i} ,$$

and

$$\frac{\partial x_j}{\partial p_i} = -\frac{1}{D}\frac{\partial p_j}{\partial x_i} , \quad j \neq i .$$

[4] On the other hand, when the goods are complements, we obtain $x^* < x_c < x_w$ and $p^* > p_b > p_w$.

Then we find

$$\frac{\partial x_i}{\partial p_i} - \frac{\partial x_j}{\partial p_i} = \frac{1}{D}\left[\frac{\partial p_i}{\partial x_i} + \frac{\partial p_j}{\partial x_i}\right] = \frac{1}{\dfrac{\partial p_i}{\partial x_i} - \dfrac{\partial p_j}{\partial x_i}}, \quad j \neq i. \tag{19}$$

Substituting Eq. (19) into Eq. (13), which is the condition for the finite population ESS in a price setting duopoly, yields

$$p^* + \left(\frac{\partial p_i}{\partial x_i} - \frac{\partial p_j}{\partial x_i}\right) x_i(p^*, p^*) - c'(x_i(p^*, p^*)) = 0, \quad j \neq i.$$

This is equivalent to Eq. (10), which is the condition for the finite population ESS in a quantity setting duopoly. Thus we obtain the following proposition.

PROPOSITION 2. *The finite population ESS in a quantity setting duopoly and that in a price setting duopoly are equivalent.*

Note that this result holds regardless of whether the goods are substitutes or complements. It means that the ESS outputs in a quantity setting duopoly and the outputs with the ESS prices in a price setting duopoly are equal, and the ESS prices in a price setting duopoly and the prices with the ESS outputs in a quantity setting duopoly are equal.

4. STOCHASTICALLY STABLE STATES

In this section we will show that the finite population ESS output and the finite population ESS price obtained in Section 2 are stochastically stable strategies for a model of evolution with an imitative rule of strategy choice with mutations. Kandori et al. (1993), Kandori and Rob (1995), Robson and Vega-Redondo (1996) and Vega-Redondo (1997) presented analyses of stochastically stable states in evolutionary games. In our model, two players (firms) play a symmetric duopoly game in each period. According to Robson and Vega-Redondo (1996) and Vega-Redondo (1997) we consider the following imitation dynamics of the firms' strategies. In period $t+1$ every firm has a chance with positive probability less than one to change its strategy to the strategy which achieved the highest profit in period t among the strategies chosen by the firms in period t. If the strategy of a firm in period t achieved the strictly highest profit, this firm does not change its strategy. If the profits of two firms were equal even when they chose different strategies in period t, each firm may choose either strategy in period $t+1$ among the strategies chosen by some firms in period t.

First consider the quantity setting case. As in Vega-Redondo (1997) we assume that the firms must choose their outputs from a finite grid $\Gamma = \{0, \delta, 2\delta, \cdots, v\delta\}$ where $\delta > 0$ and $v \in \mathbb{N}$ are arbitrary. It is required that the finite population ESS output belongs to this grid. A state of the imitation dynamics is represented by the number of firms choosing each output. The state space is denoted by Ω which is equal to Γ^2. Denote the transition matrix of this dynamics by $T(\omega, \omega')$, and by $T^{(m)}(\omega, \omega')$ the corresponding m-step transition matrix, where $\omega, \omega' \in \Omega$.

In addition to this dynamic adjustment, there is a random mutation. In each period, each firm switches (mutates) its strategy with probability ε. Mutation may be interpreted as experimentation of a new strategy by the firms. All strategies may be chosen with positive probability. Thus the complete dynamic is an ergodic Markov chain, and it has a unique stationary distribution. Consider the limit of the stationary distribution of the Markov chain as $\varepsilon \to 0$. Stochastically stable states are states which are assigned positive probability in the limit.[5]

We define a *limit set* of the dynamics without mutation. A set A is a limit set of T if this set is closed in a finite chain of positive probability transitions. That is,

$$(1) \quad \forall \omega \in A, \ \forall \omega' \notin A, \ T(\omega, \omega') = 0\,.$$

$$(2) \quad \forall \omega \in A, \ \omega' \in A, \ \exists m \in \mathbb{N} \text{ such that } T^{(m)}(\omega, \omega') > 0\,.$$

If in period t two firms choose different strategies, at least one firm has a chance to change its strategy with positive probability without mutation. Thus, such a state can not be included in a limit set, and in any state included in some limit set two firms must choose the same strategy.[6] On the other hand, in any state in which two firms choose the same strategy, no firm has incentive to change its strategy except for mutation. Accordingly, a limit set is identified as a set which includes a single state in which two firms choose the same strategy. We need no mutation to move from any state which is not included in a limit set to a state in some limit set. Thus a stochastically stable state must be in some limit set.

Denote the state in which two firms choose the output x by $\omega(x)$. The number of the states (including the state where $x = 0$) is $v + 1$. Denote the subset of Ω consisting of limit sets of T by Ω_l. Define an $\omega(x)$-tree as follows. An $\omega(x)$-tree is a function $t : \Omega_l \to \Omega_l$ such that $t(\omega(x)) = \omega(x)$ and such that for all $\omega \neq \omega(x)$, there exists m with $t^m(\omega) = \omega(x)$. We may think of an $\omega(x)$-tree as a set of arrows connecting elements of Ω_l in which every element has a unique successor $t(\omega)$, and all paths eventually lead to $\omega(x)$.[7] Define the cost of a move from ω to $t(\omega)$, $c(\omega, t(\omega))$, to be the minimum number of mutations needed to transit from ω to $t(\omega)$ under T_ε, where T_ε is the transition matrix on Ω_l when the mutation probability is ε. Then the cost of an ω-tree is the total cost of all moves in the tree,

$$\sum_{\omega \in \Omega_l} c(\omega, t(\omega))\,.$$

And finally define $C(x)$ to be the minimum cost of all possible $\omega(x)$-trees. This is the minimum number of mutations needed to reach $\omega(x)$ from all the other limit sets. Based on the results in Freidlin and Wentzel (1984), in their Proposition 4 Kandori and Rob

[5] This adjustment process is the same as that in Robson and Vega-Redondo (1996) and Vega-Redondo (1997). It has a stochastic nature even without mutation since each firm has a chance to change its strategy independently with some positive probability, and the number of firms who change their strategies in period $t + 1$ to the most profitable strategy in period t is a stochastic variable without mutation. In period $t + 1$ both firms may choose the most profitable strategy in period t with strictly positive probability.

[6] This result is similar to Proposition 1 in Vega-Redondo (1997).

[7] For more details about a tree see Kandori et al. (1993), Vega-Redondo (1996), Vega-Redondo (1997) and Young (1998).

(1995) showed that the stochastically stable states comprise the states having minimum $C(x)$.

From the arguments in the previous section we see that, since x^* is the finite population ESS and any other output is not ESS, and we have $\pi_1(x^*, x_2) > \pi_2(x^*, x_2)$ for $x_2 \neq x^*$, one mutation is sufficient to reach the state $\omega(x^*)$ from any state $\omega(x), x \neq x^*$. Therefore $C(x^*) = v$. On the other hand, one mutation is not sufficient and we need two mutations to move from the state $\omega(x^*)$ to some other state. Thus $C(x) \geqq v + 1$ for $x \neq x^*$. Hence x^* is the stochastically stable output. The transition to $\omega(x^*)$ from any other state occurs with one mutation. On the other hand the transition from $\omega(x^*)$ to any other state occurs with two mutations. Thus the former transition is more probable than the latter. This is the reason why $\omega(x^*)$ is the stochastically stable state. Therefore we have shown the following result.

PROPOSITION 3. *In a quantity setting duopoly the finite population ESS output, which is obtained from Eq.* (10), *is a stochastically stable output.*

In the price setting case, by essentially the same procedures as in the quantity setting case we can show the following result.

PROPOSITION 4. *In a price setting duopoly the finite population ESS price, which is obtained from Eq.* (13), *is a stochastically stable price.*

In Proposition 2 we have shown that the finite population ESS in a quantity setting duopoly and the finite population ESS in a price setting duopoly are equivalent. Therefore we obtain the following conclusion.

PROPOSITION 5. *The stochastically stable state in a quantity setting duopoly and that in a price setting duopoly coincide.*

5. CONCLUDING REMARKS

Let us consider the difference between stochastically stable strategies and Nash equilibrium strategies. In a quantity setting duopoly, when the goods are substitutes, each firm determines its output with a conjecture that even if it increases its output, the output of the other firm keeps constant. On the other hand, in a price setting duopoly, each firm determines the price of its good with a conjecture that if it reduces the price of its good, the output of the other firm will decrease (the price of the other firm's good keeps constant). Then the firms in a price setting duopoly should be more aggressive than the firms in a quantity setting duopoly. These are why the Nash equilibrium in a quantity setting duopoly and that in a price setting duopoly are different.[8]

With imitation dynamics what matters is that a quantity increase in a quantity setting duopoly or a price increase in a price setting duopoly, as long as it raises a firm's profit

[8] When the goods are complements, in a price setting duopoly, each firm determines the price of its good with a conjecture that if it reduces the price of its good, the output of the other firm will *increase*. Then the firms in a price setting duopoly should be less aggressive than the firms in a quantity setting duopoly. The Nash equilibrium in a quantity setting duopoly and that in a price setting duopoly are different in this case, too.

relatively to that of the other firm, it will be imitated. This suggests why both games lead to the same outcome.

APPENDIX: DERIVATIONS OF EQ. (5) AND (6)

The stability condition for the Nash-Cournot equilibrium is that the slopes of the reaction curves of the firms are smaller than one. This condition for Firm 1 is

$$\left| -\frac{\dfrac{\partial p_1}{\partial x_2} + \dfrac{\partial^2 p_1}{\partial x_1 x_2} x_1}{2\dfrac{\partial p_1}{\partial x_1} + \dfrac{\partial^2 p_1}{\partial x_1^2} x_1 - c''(x_1)} \right| < 1 .$$

The denominator is negative from the second order condition. From this expression we obtain

$$2\frac{\partial p_1}{\partial x_1} + \frac{\partial p_1}{\partial x_2} + \left(\frac{\partial^2 p_1}{\partial x_1^2} + \frac{\partial^2 p_1}{\partial x_1 x_2} \right) x_1 - c''(x_1) < 0 .$$

The stability condition for the Nash-Bertrand equilibrium is that the slopes of the reaction curves of the firms are smaller than one. This condition for Firm 1 is

$$\left| -\frac{\dfrac{\partial x_1}{\partial p_2} + (p_1 - c'(x_1))\dfrac{\partial^2 x_1}{\partial p_1 p_2} - \dfrac{\partial x_1}{\partial p_1}\dfrac{\partial x_1}{\partial p_2} c''(x_1)}{2\dfrac{\partial x_1}{\partial p_1} + \dfrac{\partial^2 x_1}{\partial p_1^2}(p_1 - c'(x_1)) - \left(\dfrac{\partial x_1}{\partial p_1}\right)^2 c''(x_1)} \right| < 1 .$$

The denominator is negative from the second order condition. From this expression we obtain

$$2\frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2} + (p_1 - c'(x_1)) \left(\frac{\partial^2 x_1}{\partial p_1^2} + \frac{\partial^2 x_1}{\partial p_1 p_2} \right) - \frac{\partial x_1}{\partial p_1} \left(\frac{\partial x_1}{\partial p_1} + \frac{\partial x_1}{\partial p_2} \right) c''(x_1) < 0 .$$

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