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AN ADAPTIVE MONOPOLISTIC GENERAL EQUILIBRIUM MODEL WITH LOCAL KNOWLEDGE OF DEMAND

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Abstract: A general equilibrium model with monopoly is developed in which the monopolist has only local knowledge of the equilibrium manifold and behaves adaptively. We provide sufficient conditions ensuring that the iterative process activated by the monopolist converges to an equilibrium.

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1. INTRODUCTION

In this note we shall develop a general equilibrium model with a monopolistic producer who has local knowledge of the equilibrium manifold, and, by using an adaptive approach (see e.g. Day (1975)), the problem of convergence to a local monopolistic equilibrium is analysed. We do not provide a theory of how the local knowledge is formed, but we simply assume that the subset of the equilibrium manifold known by the monopolist is defined by a correspondence which is exogenously given. Obviously, the way in which this knowledge is formed deserves study, and our simplicistic approach is justified in that our main aim is to provide sufficient conditions on the above-mentioned correspondence which ensure that the iterative process activated by the monopolist in trying to maximize his/her profits converges to a local monopolistic equilibrium. Finally, our approach can be considered an intermediate approach between the "objective" and the "subjective" approaches usually employed by the literature on general equilibrium theory with imperfect competition (for a survey see Hart (1985)) because we assume that the monopolist has complete knowledge of only a subset of the equilibrium manifold. (General equilibrium models with monopoly within the "objective" and/or "subjective" demand approach are studied, among others, by Nikaido (1975), Cornwall (1977), Böhm (1990))

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2. THE MODEL

Consider the following ℓ-good economy: $\mathcal{E}((\geq_h)_{h\in H}, (X_h)_{h\in H}, (w_h)_{h\in H}, (\theta_h)_{h\in H}, (Y))$, where $H = 1, 2, \ldots, n$ is the set of indices of households, symbol $\geq_h$ (resp. X_h, resp. w_h, resp. θ_h) indicates preferences (resp. consumption set, resp. initial endowment, resp. the profit share) of household h. Symbol Y indicates the production set of the sole producer in the economy, which is interpreted as the monopolist. Symbol x_h (resp. y) indicates the generic element of set X_h (resp. of set Y). Finally, symbol p indicates the price vector and S is the price set $\{p \in R^{\ell}_{++} \mid p_1 = 1\}$.

ASSUMPTION 1. For every $h \in H$:

(i) $X_h = R^{\ell}$;

(ii) preferences are complete, transitive, reflexive, continuous, strictly convex and bounded from below;

(iii) preferences of agent h can be represented by a smooth utility function, u_h, which satisfies the conditions: (1) $\partial u_h/\partial x_j > 0$, for every $j \in G$; (2) the quadratic form defined by the hessian matrix of u_h, $X' \cdot Hu_h(x_h) \cdot X$, restricted to the space $\nabla u_h(x_h) \cdot X = 0$ is negative definite.

ASSUMPTION 2. Set Y is compact.

Assumption 1 is standard (see, e.g. Balasko (1988)), Assumption 2 can be replaced by any assumption ensuring boundedness of the feasible allocation set. Notice that as far as Propositions 1 and 2 below are concerned, Assumption 1 can be replaced by any standard assumption on consumers employed in non-differentiable models (see e.g. Debreu (1959)).

We shall consider two fictitious pure exchange economies: the first, denoted by $\mathcal{E}_w((\geq_h)_{h\in H}, (X_h)_{h\in H}, (w_h)_{h\in H})$, is obtained by suppressing the production sector in economy $\mathcal{E}$ and considering the initial allocation w a parameter in $R^{\ell n}$. Now, supposing that in economy $\mathcal{E}$ the monopolist chooses the production plan y, the second kind of pure exchange economy we shall consider is the pure exchange economy obtained from $\mathcal{E}$ by suppressing the production side and assuming that the "initial" endowment of agent h is $\omega_h(y) = w_h + \theta_h y$ (the "intermediate endowments"; see Gabszewicz and Vial (1972)). Denote this economy by $\mathcal{E}_y((\geq_h)_{h\in H}, (X_h)_{h\in H}, (\omega_h(y))_{h\in H})$. Note that in $\mathcal{E}_y$ the allocation w is the initial allocation of the original economy $\mathcal{E}$. The mapping $\omega : Y \to R^{\ell n}$, defined as follows: $\omega(y) = \omega_1(y) \times \omega_2(y) \times \cdots \times \omega_n(y)$ will be called the intermediate endowment function. Set $\Omega(Y) = \{w' \in R^{\ell n} \mid w'_h = \omega_h(y), h \in H, y \in Y\}$. Symbol $z(p, w)$ denotes the excess demand function in economy $\mathcal{E}_w$.

Define the sets: $E = \{(p, w) \in S \times R^{\ell n} \mid z(p, w) = 0\}$, $E_Y = \{(p, w) \in E \mid w \in \Omega(Y)\}$. Set E_Y is the equilibrium manifold.

The next Lemma provides an important property of this set; further interesting properties are provided in the Appendix.

LEMMA 1. *Set E_Y is compact.*

Proof. $E_Y = \pi^{-1}(\Omega(Y))$. By Assumption 2, set $\Omega(Y)$ is compact; thus the assertion is verified since the natural projection mapping is proper (see Balasko (1988, pp. 89–90)). ■

We assume that the monopolist behaves according to the following iterative process: given a correspondence $F : E_Y \twoheadrightarrow E_Y$ and an initial current strategy in E_Y, say (p°, y°), the monopolist chooses the next strategy in set $B(p^\circ, y^\circ) = \{(p, y) \in F(p^\circ, y^\circ) \mid p \cdot y \geq p' \cdot y', (p', y') \in F(p^\circ, y^\circ)\}$. Call this strategy (p^1, y^1). Then, the subset $F(p^1, y^1)$ is given and the monopolist chooses the next strategy in set $B(p^1, y^1)$, say (p^2, y^2), and so on. If there is a t ($t = 0, 1, \ldots$) such that (p^{t+1}, y^{t+1}) ($\in B(p^t, y^t)) = (p^t, y^t)$, then from t on the monopolist chooses strategy (p^t, y^t). This process can be intuitively interpreted as follows: time is discrete ($t = 0, 1, 2, \ldots$), and at the beginning of each period the monopolist has to choose the price/production plan configuration in E_Y. At the beginning of period 0, he/she chooses $(p^\circ, y^\circ) \in E_Y$, then during the period 0 the monopolist makes "experiments" in the set of price/production plans configurations; by means of these "experiments", at the end of period 0 the monopolist knows set $\Phi(p^\circ, y^\circ) \subset S \times Y$. Therefore, at the end of period 0 the subset of the equilibrium manifold known by the monopolist is $F(p^\circ, y^\circ) = \Phi(p^\circ, y^\circ) \cap E_Y$, and at the beginning of time 1 the monopolist chooses the profit maximizing price/production plan configuration in $F(p^\circ, y^\circ)$, say (p^1, y^1). And so on. If it happens that $(p^{t+1}, y^{t+1}) = (p^t, y^t)$, then (p^{t+1}, y^{t+1}) is the profit maximizing strategy in $F(p^t, y^t)$, hence the monopolist will stop the process. Therefore, the following definition is now obvious: A *local monopolistic equilibrium* (LME) is a strategy (p^*, y^*) which satisfies the following conditions: (i) $(p^*, y^*) \in E_Y$; (ii) $p^* \cdot y^* \geq p \cdot y$ for every $(p, y) \in F(p^*, y^*)$ (For an equilibrium concept similar to this, but within a game-theoretic context, see Bonanno (1988)).

ASSUMPTION 3. The correspondence $F : E_Y \twoheadrightarrow E_Y$ defined above is closed-valued, continuous and non-empty. Moreover, $(p, y) \in F(p, y)$ for every $(p, y) \in E_Y$.

Continuity means that the subset of E_Y which is known by the monopolist changes "smoothly" with respect to the current strategy. The condition $(p, y) \in F(p, y)$ can be justified on the ground that "experiments" are made "around" the status quo. Remark 1 below provides an example of a correspondence F satisying Assumption 3.

According to the monopolist's behaviour described above, a sequence of "temporary" optimal strategies is generated and it is natural to ask whether this sequence converges toward a LME.

PROPOSITION 1. *Under Assumptions* 1, 2 *and* 3 *and for whatever initial strategy* $(p^\circ, y^\circ) \in E_Y$, *the sequence of strategies defined by the above iterative process either converges to a LME or the limit of every convergent subsequence is a LME.*

Proof. By Assumption 3, by Lemma 1 and by Berge's Maximum Theorem (Berge (1963)) it follows that the correspondence $B : E_Y \twoheadrightarrow E_Y$ defined by set $B(p, y)$ is upper hemi-continuous; moreover, it is closed valued, hence it is closed (Border (1985, Proposition 11.9.(a))). The iterative behaviour of the monopolist as introduced above can be modelled as an algorithm *à la* Zangwill (1969), and it is possible to verify that all conditions for applying Convergence Theorem A in Zangwill (1969, p. 91) are satisfied; thus, by this theorem, the assertion holds true. ■

A weak aspect of the preceding result is that the iterative process may converge to two or more LMEs. A straightforward way to exclude this possibility is to assume that there is a unique LME (hence a unique monopolistic equilibrium) (see, for example, Bazaraa, Sherali and Shetty (1993, p. 250)). The following is an alternative weaker sufficient condition:

ASSUMPTION 4. There exists a family of disjoint compact neighbourhoods of the LMEs, say $\mathcal{D}$, such that each element of the family contains at most one LME and if (p, y) is a LME, then $F(p, y) \subset D(p, y)$, where $D(p, y) \in \mathcal{D}$.

For future reference, it is worthwhile to emphasize that Assumption 4 implies that: (i) if (p, y) and (p', y') are LMEs, then $(p', y') \notin F(p, y)$ and $(p, y) \notin F(p', y')$; (ii) LMEs are isolated; i.e. if (p, y) is a LME, then there exists an ε-ball around (p, y), $B_\varepsilon(p, y)$, such that in $B_\varepsilon(p, y) \cap E_Y$ no LMEs exist except (p, y).

PROPOSITION 2. *Under Assumptions* 1, 2, 3 *and* 4 *the whole sequence generated by the iterative process converges to a LME.*

Proof. Preliminarly we show that if $\{p^t, y^t\}$ is the sequence generated by the iterative process, then $d((p^t, y^t), (p^{t+1}, y^{t+1})) \to 0$ as $t \to \infty$, where $d(\cdot, \cdot)$ is the distance function. Suppose not. Then there exists a subsequence $\{p^{t_i}, y^{t_i}\}$ such that $d((p^{t_i}, y^{t_i}), (p^{t_i+1}, y^{t_i+1})) \to \beta > 0$ as $i \to \infty$. We may assume that $((p^{t_i}, y^{t_i})$ converges to (p°, y°) and that (p^{t_i+1}, y^{t_i+1}) converges to $(p^{\circ\circ}, y^{\circ\circ})$. Clearly, $d((p^\circ, y^\circ), (p^{\circ\circ}, y^{\circ\circ})) \geq \beta$. By Proposition 1, (p°, y°) and $(p^{\circ\circ}, y^{\circ\circ})$ are LMEs; moreover, $(p^{t_i+1}, y^{t_i+1})) \in F(p^{t_i}, y^{t_i})$, then by Assumption 3, $(p^{\circ\circ}, y^{\circ\circ}) \in F(p^\circ, y^\circ)$. But this contradicts Assumption 4.

Suppose now that the assertion of Proposition 2 is not true. Therefore, if $\{p^t, y^t\}$ is the sequence generated by the iterative process, there must exist (at least) two subsequences, say $(p^{t'}, y^{t'})$ and $(p^{t''}, y^{t''})$ converging to (p', y') and (p'', y''), respectively. By Proposition 1, every accumulation point of sequence $\{p^t, y^t\}$ is an LME; therefore, by Assumption 4, it is possible to take two positive numbers ε' and ε'' such that points (p', y') and (p'', y'') are the only accumulation points in $B_{\varepsilon'}(p', y')$ and $B_{\varepsilon''}(p'', y'')$, where $B_{\varepsilon'}(p', y') \subset D(p', y')$ and $B_{\varepsilon''}(p'', y'') \subset D(p'', y'')$.

Choose a positive number Z in such a way that $d((p^z, y^z), (p^{z+1}, y^{z+1})) < \varepsilon'/3$ for $z \geq Z$ (That such a number exists follows from the result at the beginning of this proof). However, (p', y') is an accumulation point of sequence $\{p^t, y^t\}$, therefore for infinitely many indices s one has that $d((p^s, y^s), (p', y')) < \varepsilon'/3$. On the other hand, (p'', y'') is another accumulation point of $\{p^t, y^t\}$, hence, by the fact that $(p'', y'') \notin B_{\varepsilon'}(p', y')$, there must exists infinitely many indices q such that $d((p^q, y^q), (p', y')) \geq 2\varepsilon'/3$. From the way in which Z has been defined, it follows that there exist infinitely many indices $r \geq Z$ such that $(\varepsilon'/3) \geq d((p^r, y^r), (p', y')) \geq (2\varepsilon'/3)$. This implies that there exists an accumulation point in the set $\{(p, y) \in E_Y \mid (\varepsilon'/3) \leq d((p, y), (p', y')) \leq (2\varepsilon'/3)\} \subset D(p', y')$. This result and Proposition 1 contradict Assumption 4. ■

It remains to show that Assumptions 3 and 4 are consistent. In order to do this, the following further assumption concerning the representation of the production set is adopted (see also Smale (1974)):

ASSUMPTION 5. $Y = \{y \in R^\ell \mid g(y) \leq 0\}$ where $g : R^\ell \to R$ is a smooth function such that $\nabla g(y) \neq 0$ if $g(y) = 0$.

LEMMA 2. *Under Assumptions* 1, 2 *and* 5 *set* E_Y *is a* ℓ*-dimensional differentiable manifold with boundary.*

Proof. Consider the function $\varphi_E : E \to R^\ell$ defined as follows: $\varphi_E(p, w') = (w'_h - w_h)/\theta_h$ where h is chosen in such a way that $\theta_h > 0$. Obviously, φ_E is smooth. This and Assumption 5 yields that also function $G : E \to R$ defined as follows: $G = g \circ \varphi_E$ is smooth. Obviously, $G(p, w) \leq 0$ if $(p, w) \in E_Y$ and that $G(p, w) = 0$ if and only if $g(\varphi_E(p, w)) = 0$; i.e., the zero values of G are the zero values of g. If we show that G has zero as regular value, then E_Y is a manifold with boundary (see e.g. Milnor (1965), Lemma 3, p. 12); i.e. we have to show that $\nabla G(p, w) \neq 0$ if $G(p, w) = 0$. Take (p, w) such that $G(p, w) = g(\varphi_E(p, w)) = 0$. But since $\nabla G(p, w) = [\mathbf{0}' \mid \nabla g(\varphi_E(p, w))]/\theta_h$, where $\mathbf{0}'$ is a $1 \times \ell$ zero vector, by Assumption 5 it follows that $\nabla G(p, w) \neq 0$. As for dimension, notice that E_Y is diffeomorphic to, hence has the same dimension of, set $\{(p, y) \in S \times Y \mid z(p, \omega(y)) = 0\}$. By employing the same technique in Balasko (1988, p. 68–69)) it can be seen that the rank of the Jacobian matrix of $z^{(-\ell)}(p, \omega(y))$ (i.e. $z(p, \omega(y))$ without the ℓ-th element) is equal to $\ell - 1$. ■

REMARK 1. Under Assumptions 1, 2 and 5:

(i) There exists a correspondence $F : E_Y \to E_Y$ which satisfies Assumption 3.

(ii) If LMEs are isolated, then there exists a correspondence F satisfying Assumption 4 as well.

Proof. (i) Set: $V = \mu^{-1}(E_Y)$, where μ denotes generically a parametrization function (they exist from Lemma 2). Because of Lemma 1, set V is a compact subset of R^ℓ. Choose $\delta > 0$ and denote by $B_\delta\mu^{-1}(p, y)$ the closed ball of radius δ in R^ℓ centered at $\mu^{-1}(p, y)$. Define the correspondence $F : E_Y \to E_Y$ as follows: $F(p, y) = \{(p', y') \in E_Y \mid (p', y') \in \mu(V \cap B_\delta\mu^{-1}(p, y))\}$. Obviously, for every $(p, y) \to E_Y$, $F(p, y)$ is non-empty, closed and, moreover, $(p, y) \in F(p, y)$. Since continuity is invariant under homeomorphism, it is enough to prove that $V \cap B_\delta\mu^{-1}(p, y)$ is continuous. That $B_\delta\mu^{-1}(p, y)$ is upper hemi-continuous (uhc) is immediate. This and compactness of V allow to prove by standard arguments that $V \cap B_\delta\mu^{-1}(p, y)$ is uhc as well. Suppose that $V \cap B_\delta\mu^{-1}(p, y)$ is not lower hemi-continuous. Thus, given an open set T such that $T \cap (V \cap B_\delta\mu^{-1}(p, y)) \neq \emptyset$, one obtains that for every neighbourhood of (p, y) in $R^{2\ell}$, say U_α, there exists at least one $(p^\alpha, y^\alpha) \in U_\alpha \cap E_Y$ such that $T \cap (V \cap B_\delta\mu^{-1}(p^\alpha, y^\alpha)) = \emptyset$. This generates a net (p^α, y^α) in E_Y with limit (p, y), and which satisfies the condition that $T \cap (V \cap B_\delta\mu^{-1}(p^\alpha, y^\alpha) = \emptyset$ for every α. Since

$T \cap (V \cap B_\delta \mu^{-1}(p, y) \neq \emptyset$, there exists $q \in T$ with $d(\mu^{-1}(p, y), q) < \delta$. Since $d(\cdot)$ is continuous and since (p^α, y^α) has limit (p, y), it follows that there exists α^* such that for every $\alpha > \alpha^*$ one obtains: $d(\mu^{-1}(p^\alpha, y^\alpha), q) < \delta$, thus $T \cap (V \cap B_\delta \mu(p^\alpha, y^\alpha)) \neq \emptyset$, a contradiction.

(ii) Suppose that there are h isolated LMEs, $\{(p^{(i)}, y^{(i)}), i = 1, 2, \cdots, h\}$. Consider the collection $\mathcal{D} = \{D(p^{(i)}, y^{(i)})\}$ of disjoint neighbourhoods of the LMEs in $R^{2\ell}$. For each i take a closed ball BV_i in $\mu^{-1}(D(p^{(i)}, y^{(i)}) \cap R^{2\ell})$, and denote by ε_i the radius of ball BV_i. Take $\beta < \min\{\delta, \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_h\}$, where δ has been defined in point (i), and define the correspondence F as follows $F(p, y) = \{(p', y') \in E_Y \mid (p', y') \in \mu B_\beta \mu^{-1}(p, y)\}$. It clearly satisfies all the conditions required. ■

APPENDIX

This Appendix is devoted to analyse some further topological properties of set E_Y and establishing sufficient conditions ensuring that this set is arc-connected, simple connected or contractible (For justification for such an analysis see Balasko (1988, pp. 69–72).) Notice first that set E is a differentiable manifold diffeomorphic to $R^{\ell n}$ (Balasko (1988, p. 73)). By contrast, the following example shows that set E_Y can be not connected. However, Proposition A below provides sufficient conditions ensuring the desired topological properties for set E_Y.

Denote by $\mathcal{R}$ the set of regular initial allocations for the pure exchange economy $\mathcal{E}_w$, by $\mathcal{U}$ the set of initial allocations yielding a unique walrasian equilibrium, and by $\mathcal{C}$ the connected component of the set of regular allocations which contains the set of Pareto-optima (Balasko (1988, Theorem (4.5.3.)).

EXAMPLE. Suppose that $w \in \mathcal{R}$, and that $w \in \mathcal{U}$. There exists a neighbourhood of w, say $N(w)$, such that $\pi^{-1}(N(w))$ is the disjoint union of a family of open subset of $S \times R^{ln}$ (see Balasko (1988, p. 91)). Consider now economy $\mathcal{E}$ where Y satisfies the condition $\Omega(Y) \subset N(w)$. It follows that E_Y is the disjoint union of a family of closed sets; i.e. it is not connected.

PROPOSITION A. *Under Assumption* 1, *if* $\Omega(Y) \subset \mathcal{U} \cap \mathcal{R}$, *then set* E_Y *is arc-connected, simple connected or contractible if set* Y *is arc-connected, simple connected or contractible.*

Proof. Since $\Omega(Y) \subset \mathcal{U} \cap \mathcal{R}$, there exists one diffeomorphism $p : \Omega(Y) \to S$ defining the walrasian price vector as a function of the initial allocation (see Balasko (1988, p. 93)). Consider the function $\eta : Y \to E_Y$ defined as follows: $\eta = (p \circ \omega, \omega)$ where function p has been just defined and ω is the "indirect" endowment function. Function η is differentiable because functions ω and p are differentiable. Consider now the function $\varphi : E_Y \to Y$ defined as follows $\varphi(p, w') = \{y \in Y \mid w' = \omega(y)\}$. Obviously, $\varphi \circ \eta = i$; moreover, φ is a differentiable function since $\varphi(p, w') = (w'_h - w_h)/\theta_h$ where w_h is the initial allocation of household h, and index h satisfies the condition $\theta_h > 0$.Thus, if $\Omega(Y) \subset \mathcal{U} \cap \mathcal{R}$, set E_Y is diffeomorphic to set Y. ■

By recalling that there is uniqueness of walrasian equilibrium for every economy whose initial allocation belongs to set $\mathcal{C}$ (Balasko (1988, Corollary (4.5.4.)) one obtains the result:

COROLLARY. *If* $\Omega(Y) \subset \mathcal{C}$, then E_Y is arc-connected, simple connected or contractible if Y is, respectively, arc-connected, simple connected or contractible.

Corollary has a very clear economic meaning: it shows that E_Y is diffeomorphic to the production set Y if the initial allocation is "close enough" to the set of Pareto-optimal allocations (i.e. $w \in \mathcal{C}$) and the monopolist is "small enough" with respect to the economy (i.e. if $\Omega(Y) \subset \mathcal{C}$). Actually, by means of a replication argument, it is possible to show that if the initial allocation of the economy $\mathcal{E}$ is in $\mathcal{C}$, then there exists a number of replication of the consumption sector beyond which the condition $\Omega(Y) \subset \mathcal{C}$ is satisfied.

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