Title	ECONOMIC EXPANSION AND EQUILIBRIUM UNEMPLOYMENT
Sub Title	
Author	BELADI, Hamid FRASCA, Ralph
Publisher	Keio Economic Society, Keio University
Publication year	1999
Jtitle	Keio economic studies Vol.36, No.2 (1999.) ,p.81- 90
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19990002-0081

KEIO ECONOMIC STUDIES **36**(2), 81–90 (1999)

ECONOMIC EXPANSION AND EQUILIBRIUM UNEMPLOYMENT

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First version received June 1999; final version accepted September 1999

Abstract: The authors examine economic expansion within a Heckscher-Ohlin model with two factors and two goods under equilibrium unemployment. The labor supply function is obtained from an optimization process wherein consumers choose between real income and leisure. Technological change is exogenous, but can be influenced by government tax incentives. Within this model, programs that stimulate technological change in capital intensive industries may reduce employment. Alternatively, technological change in labor intensive industries may increase labor force participation through reducing equilibrium unemployment.

1. INTRODUCTION

In recent years there has been a remarkable growth in the literature on the implications of economic expansion for unemployment and national income, among other things, in developed as well as developing countries.[1] The two types of models which are generally used to analyze economic expansion in the presence of unemployment are the general wage rigidity model and the Harris–Todaro model. The general wage rigidity framework generates a linear transformation curve or constant average cost in each industry and it leads a trading country to complete specialization. The Harris–Todaro model, on the other hand, avoids the constancy of costs, however in the Harris–Todaro model, a rise in the labor supply of a small economy causes a fall in unemployment whereas a rise in the capital stock leads to an increase in unemployment.

The purpose of this paper is to study the implications of economic expansion on the results of the Heckscher–Ohlin model with two factors and two goods under equilibrium unemployment.[2] Specifically, in the present paper we draw upon Batra and Beladi (1997) and introduce a labor supply function with the quantity of labor supplied dependent upon both the market wage and unearned income. In a two-sector general equilibrium model this allows us to examine the impact of technological change in the presence of a variable labor supply that responds to changes in both the market wage and the market return to capital. The present model is an expansion of both the traditional neoclassical model with an inelastic labor supply and the keynesian model with rigid wages. In the traditional neoclassical model, the labor force is fully employed and

[1] See, for instance, Chao and Yu (1992, 1993), and Yu (1978).

[2] For different modeling of equilibrium unemployment see Bruenllo (1996), Moomaw (1995), and Groenewold (1994). Also see Kemp and Jones (1962) on variable labor supply literature.

there is zero equilibrium unemployment. The wage is determined by the demand for labor and at this wage all potential workers are labor market participants. Alternatively, in models with rigid wages the concept of equilibrium unemployment is unimportant because it does not impact on either wage or output. The controlling factor in these models is the presence of involuntary unemployment.

With a variable labor supply function we can examine the existence of equilibrium unemployment. By equilibrium unemployment we mean the number of non-participants in the labor force. These are individuals who have the skills demanded in the labor market, but who voluntarily choose not to offer their services given the equilibrium wage and their current unearned income. Policies designed to stimulate technological change in either relatively capital-intensive industries or relatively labor-intensive industries, through their impact on wages and unearned income, can influence the labor market decision. Consequently, these policies may have unintended consequences on industrial production, national income and employment. These impacts may be analyzed within the context of the current model.

We examine the effects of labor-saving, capital saving and Hicks-neutral technical progress and of factor accumulation upon the equilibrium rate of unemployment, sectoral outputs, national income as well as resource allocation in a two-sector general equilibrium model with equilibrium unemployment. Our analysis has definite policy implications for the rising tide of technological change in specific targeted industries.

The paper is organized as follows: Section 2, sets out the basic model and section 3 analyzes the comparative static results for some key variables of interest. We draw some concluding remarks and suggestions for possible extensions of the model in Section 4.

2. THE BASIC MODEL

The main features of the model maybe described as follows: we assume an economy with two aggregate competitive sectors, A (advancing sector) and M (manufacturing sector). The advancing sector embodies the new technology. For analytical convenience, the manufacturing sector encompasses all sectors other than the advancing sector. The output from each of the sectors is respectively, X_a and X_m. Each sector utilizes two factors of production, capital (K) and labor (L), in a linearly homogenous and concave production function with the standard properties, i.e. positive and decreasing marginal products. Perfect competition in the product market and full employment of inelastically supplied capital are also assumed. Labor is, however, not fully employed and the economy suffers from equilibrium unemployment. Technical advance is limited to the sector A, the sector experiencing the technological stimulus.

The two aggregate production functions are given by:

$$X_A = A(\beta K_A, \theta L_A) = \theta L_A f_A(k_A \beta/\theta) \tag{1}$$

$$X_M = M(K_M, L_M) = L_M f_M(k_M) \tag{2}$$

Where K_i and L_i are for ($i = A, M$) the employment of capital and labor, and $k_i = (K_i/L_i)$ is the capital-labor ratio in the ith sector. The shift parameters (θ, β)

incorporate technical progress in the advancing sector. It is assumed that θ, β are initially equal to unity. The impact from technical progress in the manufacturing sector, M, will be just the opposite from that in the advancing sector, A. An increase in β above unity indicates capital saving (labor using) technical advance, as the same level of output now can be produced by a smaller amount of capital so that at the original equilibrium factor ratio the rise in the marginal product of labor is more than the concurrent increase in the marginal product of capital. Similarly, labor saving (capital using) technical progress is represented by an increase in θ; whereas, Hicks-neutral technical progress is defined as an identical proportionate increase in the marginal product of both inputs at the original equilibrium factor price ratio (an equal rise in θ and β).

Let us assume that consumer's utility is a function of real income and leisure, then,

$$U = U(Y, Z) \tag{3}$$

where U is utility, Y is real income, Z is leisure and,

$$Z = (\bar{H} - L) \tag{4}$$

and,

$$Y = [WL + r\bar{K}] \tag{5}$$

where $\bar{H}$ is the given number of hours per day, W is the real wage rate, L is the supply of labor, $\bar{K}$ is the given stock of capital, and r is the real return to capital. Note that $r\bar{K}$ is referred to as unearned income in the literature.

Now, given (4) and (5), (3) can be written as,

$$U = U[(WL + r\bar{K}), (\bar{H} - L)] \tag*{(3)$'$}$$

Maximizing the utility function given by (3)$'$ we obtain

$$(dU/dL) = [WU_y - U_z] = 0 \tag{6}$$

The second order condition for maximization is $[W^2U_{yy} - 2W_{yz} + U_{zz}] < 0$. In (6), U_y stands for marginal utility of income; whereas, U_z denotes the marginal utility of leisure.

From (4), we define the labor supply as a function of the real wage and unearned income, I, which is equal to $r\bar{K}$. Thus,

$$L = L(W, I) \tag{7}$$

Differentiating (7) with respect to W, we can show that,

$$L_W = (\partial L/\partial W) = [S + L_I L] \tag{8}$$

It is assumed that labor positively responds to an increase in the real wage so that $L_W = (\partial L/\partial W) > 0$. L_I is the income effect and S is the corresponding positive substitution effect of a change in the wage rate on labor supply. Assuming that leisure consumption is non-inferior, then $L_I \leq 0$.

Totally differentiating (7) and utilizing (8) yields,

$$dL = (S + L_I L)dW + L_I dI \tag{9}$$

At this point, a few remarks regarding equilibrium unemployment are in order. As stated earlier, equation (7) determines the equilibrium level of employment as well as equilibrium rate of unemployment. A geometrical exposition of these two concepts is provided by Fig. 1, where S_L is the classical labor supply curve and D_L is the negatively sloped aggregate demand for labor. If the labor supply were inelastic at $O\bar{L}$, as assumed in traditional models, then the equilibrium real wage is W_o. Alternatively, in the present model the labor supply function is given by equation (7) and illustrated by $S_L N W_I$ in Fig. 1. Along this supply curve W_1 is the subsistence wage and the equilibrium wage is W_2. At equilibrium the amount of labor employed is L_2. Consequently, the traditional model with an inelastic labor supply understates the level of the real wage rate and overstates the level of employment and hence of national income. In Fig. 1, $L_2\bar{L}$ is the equilibrium level of unemployment and $\lambda = (L_2\bar{L}/O\bar{L})$ is the equilibrium rate of unemployment.

At equilibrium factor prices are the same in both sectors. Therefore, letting P equal the relative price of X_M and expressing all values in terms of X_A, it must hold that,

$$\beta f'_A(k_A\beta/\theta) P f'_A(k_M) = r \tag{10}$$

Where $\beta f'_A(k_A\beta/\theta) \equiv A_K$ is the marginal product of capital in sector A, $f'_M(k_M) = M_K$ is marginal product of capital in sector M, and r is the real rental rate of capital. The hiring of labor in the two sectors is determined by,

$$W = A_L \equiv (\theta f_A - \beta k_A f'_A) \tag{11}$$

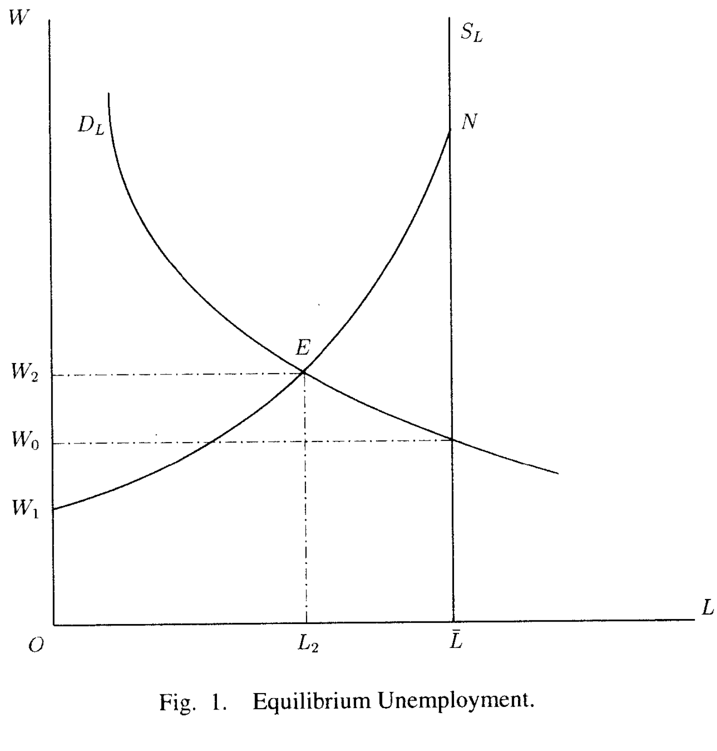

Fig. 1. Equilibrium Unemployment.

and

$$W = PM_L \equiv P(f_M - k_M f'_M) \tag{12}$$

With perfect competition in all markets, factor rewards are the same in both sectors. Consequently, (11) and (12) yield the following factor market equilibrium condition,

$$W = P(f_M - k_M f'_M) = (\theta f_A - \beta k_A f'_A) \tag{13}$$

It is assumed that the aggregate supply of capital ($\bar{K}$) is fixed, and that the aggregate quantity of labor supplied is variable. Therefore, by definition the following factor constraints must hold:

$$K_A + K_M \equiv k_A L_A + k_M L_M = \bar{K} \tag{14}$$

and,

$$L_A + L_M = L(W, I) \tag{15}$$

Finally, let the real national income, Y, be rewritten as,

$$Y = PX_M + X_A = WL + r\bar{K} \tag*{(5)$'$}$$

With this last equation the production structure of the model is complete. We assume that the economy under study is small and, therefore, experiences fixed terms of trade. Accordingly, P is determined exogenously.

3. EQUILIBRIUM SOLUTION AND TECHNICAL PROGRESS

The basic model developed above can now be used to explore the impact of technical progress on equilibrium unemployment as well as some other important variables of interest. From the market equilibrium conditions in (10) through (12), it follows that,

$$\beta f'_A(k_A \beta/\theta) = P f'_M(k_M) \tag{16}$$

and,

$$(\theta f_A - \beta k_A f'_A) = P(f_M - k_M f'_M) \tag{17}$$

Without loss of generality, we assume that $P = 1$. By definition, P is constant and θ and β initially equal to one. Consequently total differentiation of (16) and (17) yields the following matrix system,

$$\begin{bmatrix} f''_A & -f''_M \\ -k_A f''_A & k_M f''_M \end{bmatrix} \begin{bmatrix} dk_A \\ dk_M \end{bmatrix} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \tag{18}$$

where,

$$G_1 = [f''_A k_A d\theta - (r - f''_A k_A) d\beta]$$

and,

$$G_2 = [f''_A k_A^2 d\beta - (W + k_A^2 f''_A) d\theta]$$

Differentiating the conditions for market equilibrium in (10) and (13) along with the solution to the matrix system in (18), and using the labor supply function in (8) yields the following results:

$$(dL/d\beta) = [rk_A/(k_M - k_A)][L_W k_M - L_I \bar{K}] \tag{19}$$

$$(dL/d\theta) = [W/(k_M - k_A)][L_W k_M - L_I \bar{K}] \tag{20}$$

and,

$$(dL/d\beta)_{\beta=\theta} = [f_A/(k_M - k_A)][L_W k_M - L_I \bar{K}] \tag{21}$$

Given these results, the following theorem is now in order.

THEOREM 1: *All types of technical progress, including the purely labor-using variety may cause a rise in the equilibrium unemployment rate.*

Equations (19) through (21) illuminate the effect of technical progress on equilibrium employment. It is clear that irrespective of the type of technical improvement, the effect of technical progress on equilibrium unemployment depends on the substitution effect, factor intensities, and the income effect. Give that $L_W > 0$ and $L_I \leq 0$, the nature of the impact on unemployment from all types of technical progress will be determined by factor intensities. If the advancing sector enjoying technical improvement is relatively labor-intensive, $k_A < k_M$, then all types of technical progress, including the purely labor-saving variety, cause a fall in the equilibrium rate of unemployment.

Given these relative factor intensities, the rental rate of capital must fall and cause a reduction in unearned income ($dI < 0$). Consequently, the fall in unearned income will stimulate an increase in the supply of labor. The point to remember is that the real wage increases by less than it does when the labor supply is inelastic (as is assumed in the traditional two-sector model). It is interesting to note that if the first sector (the advancing sector) is capital intensive relative to the second sector (the manufacturing sector), then all types of technical progress including the purely capital-saving variety, can cause a rise in equilibrium unemployment. The decline in the quantity of labor supplied is a result of both a fall in the real wage and a rise in unearned income.

We are now in a position to examine the impact of the technical advance on the sectoral outputs. Let us consider the case of neutral technical progress, so that $\beta = \theta$. Totally differentiating (1), and remembering that θ and β are initially equal to one, we obtain,

$$dX_A = f_A(dL_A + L_A d\theta) + L_A f'_A[dk_A + k_A(d\beta - d\theta)] \tag{22}$$

Now, differentiating (14) and (15) and using (7) we get,

$$dL_A = [1/(k_M - k_A)]\{L_A dk_A + L_M dk_M + k_M[SdW + L_I(dI + LdW)]\} \tag{23}$$

and,

$$dL_M = [-1/(k_M - k_A)]\{L_A dk_A + L_M dk_M + k_A[SdW + L_I(dI + LdW)]\} \tag{24}$$

where,

$$dk_M = [(G_1 k_M + G_2)/f''_A(k_M - k_A)] \quad \text{and} \quad dk_A = [(G_1 k_A + G_2)/f''_M(k_M - k_A)].$$

Introducing (10) through (12) into (22), and after a little manipulation we can derive the following,

$$\begin{aligned}(dX_A/d\beta)_{\beta=\theta} = {} & X_A + [rk_A/(k_M - k_A)^2] \\ & \times \{k_M f_A(L_W k_M - L_I) - (L_A f_M/f''_A) - (L_M f_A/f''_M)\}\end{aligned}$$

(25)

and similarly,

$$(dX_M/d\beta)_{\beta=\theta} = [rk_A/(k_M - k_A)^2] \times \{-k_A f_M(k_M L_W - L_I) + (L_M f_A/f''_M) + (L_A f_M/f''_A)\} \quad (26)$$

It is fairly obvious from (25) and (26) that $(dX_A/d\beta)_{\beta=\theta} > 0$ and $(dX_M/d\beta)_{\beta=\theta} < 0$, when $L_W > 0$ and $L_I \leq 0$. On this basis, the next theorem immediately follows:

THEOREM 2: *In an economy characterized by equilibrium unemployment, if technical progress is neutral or intensive-factor saving, the output of the advancing sector rises and that of the other sector falls.*

This may be explained by examining the impact of technical progress on cost and employment. The cost effect is the traditional effect that tends to raise X_A and lower X_M, i.e. at constant terms of trade, neutral or intensive-factor saving technical change raises the output of the advancing sector and lowers the output of the other sector. However if the technical advance is intensive factor using then output effects are indeterminate. The employment effect corresponds to the well known Rybczynski Theorem; whereby, a rise in the supply of labor raises the output of the labor-intensive good and lowers that of the capital-intensive product at constant relative prices. In our model, the employment effect reinforces the cost effect.

Therefore, with neutral technical advance in X_A, both the real wage and employment rise if X_A is labor intensive. Concurrently, as employment rises, the output of X_A also rises from the Rybczynski effect. Conversely, in the second sector the output of X_M falls from both effects.

This result is also clearly valid for intensive-factor saving technical improvement. If X_A is capital-intensive, the real wage and employment fall. However, output in sector A receives an additional stimulus from the employment effect. Thus, both the cost effect and the employment effect tend to raise X_A. Similarly for X_M, both effects are negative regardless of its factor-intensity.

Given a neutral technical improvement the effects are the same as in the traditional model. When the improvement is intensive-factor using, however, the traditional results themselves are indeterminate; and our model does not change that.[3]

With these results at hand, we are now in a position to examine the effects of technical progress on national income. Differentiating (5)′, (1) and (2) totally, we obtain,

$$\begin{aligned} dY &= dX_A + PdX_M \\ &= A_L(dL_A + L_A d\theta) + A_K(dK_A + K_A d\beta) + P(M_L dL_M + M_K dK_M) \end{aligned}$$

Now from (14) and (15) we have $dK_A + dK_M = 0$ and $dL_A + dL_M = dL$. Moreover, given factor market equilibrium in (10) and (13) we get,

$$dY = W(dL + L_A d\theta) + rK_A d\beta \quad (27)$$

[3] See Murray Kemp (1969) for an eloquent presentation of this result.

It may be observed from (27) that all types of technical change in sector A cause an increase in national income provided that $k_M > k_A$, $L_W > 0$ and $L_I \leq 0$. The results are ambiguous when $k_A > k_M$. In that case, the relative impact of changes in wages and unearned income on the supply of labor would have to be considered.

Similarly for the case of technical progress in manufacturing sector (X_M), we can obtain,

$$dY = W(dL + L_M d\theta_M) + rK_M d\beta_M \tag{28}$$

where θ_M and β_M are agents of technical charge in X_M. Here again in view of (19) through (21), the $dY > 0$ when $k_M < k_A$, $L_W > 0$ and $L_I \leq 0$. In this case, the results are ambiguous when $k_M > k_A$. All this leads to the following theorem.

THEOREM 3: *With equilibrium unemployment and a constant price ratio, technical progress may be immiserizing.*

Let us now turn to factor accumulation. Assuming that $d\beta = d\theta = 0$, it is obvious that with a constant P, the k_i $(i = A, M)$ are also constant. As was shown earlier, the equilibrium rate of unemployment is given by $\lambda = (L_2\bar{L}/O\bar{L})$. Therefore, the equilibrium level of unemployment is $L_2\bar{L} = \lambda\bar{L}$ and, hence, $L_x + L_y = (1 - \lambda)\bar{L}$. In view of this and differentiating (14) and (15) totally, we obtain,

$$dL_A = \left[\frac{1}{(k_M - k_A)}\right][k_M d\bar{L}(1 - \lambda) - d\bar{K}] \tag{29}$$

Totally differentiating (1) and (2) and using (29) and (30) we have

$$dL_M = \left[\frac{1}{k_M - k_A}\right][-k_A d\bar{L}(1 - \lambda) + d\bar{K}] \tag{30}$$

$$dX_A = \frac{f_A[k_M d\bar{L}(1 - \lambda) - d\bar{K}]}{(k_M - k_A)} \tag{31}$$

$$dX_M = \frac{f_M[-k_A d\bar{L}(1 - \lambda) + d\bar{K}]}{(k_M - k_A)} \tag{32}$$

From these equations it follows that when $d\bar{K} > 0$ and $d\bar{L} = 0$, we have $dX_A < 0$ and $dX_M > 0$ if $k_M > k_A$. Alternatively, when $d\bar{L} > 0$ and $d\bar{K} = 0$, we have $dX_A > 0$ and $dX_M < 0$ if $k_M > k_A$. The following result is then immediate.

THEOREM 4: *A rise in the supply of capital (labor) at a constant product-price ratio, raises the output of the capital-intensive sector (labor-intensive) at the expense of the output of the other sector.*

Let us now assume that both factors grow in the steady state (at the same rate); what is the effect of this change upon sectoral outputs? To see this, let us define steady state by $(d\bar{K}/d\bar{L}) = (K/L) = k$, when k is the overall capital labor ratio in the economy. Also, assume ρ_i $(i = A, M)$ to be the proportion of labor used in each sector. Then using (15) and (5)′ and incorporating (31) and (32), yields the following:

$$\begin{aligned}(dX_A/d\bar{L}) &= [f_A/(k_M - k_A)][k_M(1 - \lambda) - k] \\ &= \rho_A f_A = (X_A/\bar{L})\end{aligned} \tag{33}$$

and,

$$(dX_M/d\bar{L}) = [f_M/(k_M - k_A)][-k_A(1-\lambda) + k]$$
$$= \rho_M f_M = (X_M/\bar{L}) \tag{33}$$

From there it is fairly obvious that,

$$(dX_i/X_i) = (d\bar{L}/\bar{L}) = (d\bar{K}/\bar{K}) \tag{35}$$

So that if both factors grow at the same rate, then both outputs end up growing at the same rate. The following theorem is immediately available.

THEOREM 5: *In an economy characterized by equilibrium unemployment, at constant price ratio if both factors grow in the steady state, both outputs grow at the same rate.*

4. CONCLUDING REMARKS

This paper has studied the implications of equilibrium unemployment upon the theoretical effects of economic expansion in a small economy where the labor supply function is obtained from an optimization process wherein consumers choose between real income and leisure. The effects of labor-saving, capital saving, and Hicks-neutral technical progress, as well as factor accumulation upon the equilibrium rate of unemployment, sector output and national income were explored. Several important policy implications can be derived from the results.

We start by noting that although the model treats technical progress as exogenous, such progress can be influenced by governmental tax incentives for research and development as well as for capital investment, patent protection, and educational policies. That is, the government may tip technological progress towards being labor-saving or capital-saving.

Particularly, governments in many countries may be inclined to support technological change in industries that have been defined as leading sectors. Such technologically advanced sectors are likely to be capital-intensive. Given the current model, it is clear that a program designed to stimulate technological change in capital-intensive industries may cause a reduction in employment. Consequently, an administration that measures it success according to the number of jobs created may face a trade-off between channeling research dollars into already technologically advanced capital-intensive industries and stimulating additional labor force participation through reducing equilibrium unemployment. The simultaneous goal of achieving increases in national income combined with an increase in employment is more readily achieved by channeling technological change into labor-intensive industries. Unfortunately, such labor-intensive industries may not garner the necessary political support, because they fail to capture the political vision of a more technological advanced society.

An important extension of the model would be to examine the question of gains from trade. Another related issue, among others, concerns reexamination of the licensing of new technology versus direct foreign investment literature. Finally, the effect of many other trade policies on equilibrium rate of unemployment can be analyzed.

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