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THE LOCATION PRODUCTION THEORY OF HETEROGENEOUS FIRMS

Ping-yen LAI

Department of Financial Management, Da-Yeh University, Changhua, Taiwan
Department of Economics, Tunghai University, Taichung, Taiwan

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Abstract: We discuss the strategic location interactions between two heterogeneous firms with different technologies in the context of Weber triangle. By assuming the transportation cost to be decreasing in the relative distance between these two firms, a special case of external economies of scale, we find the optimum location will not be independent of demand shock as long as one of the duopolistic firms has a nonlinear technology.

Key words: Duopoly; External Economies of Scale
JEL classification: R30; D23; L13

1. INTRODUCTION

Ever since Moses' (1958) work on the theory of location and production, based on the classical work of Weber (1929), there has been a continuing interest regarding the "location independent of output" issue in the context of the Weber triangle. The major concern is: under what kind of production conditions will the output be independent of demand shocks. One of the major approaches concerning this issue is the market structure approach. There are numerous articles focusing on the "independence" issue by assuming the output market to be either competitive or monopolistic, such as Bradfield (1971), Khalili et al. (1974), Miller and Jensen (1978), Eswaran et al. (1981), Shieh and Mai (1984), Hurter and Martinich (1989), and others. Recently, there are some growing interests on the oligopoly model, which can be identified as another aftermath of the imperfect competition revolution stemming from theory of industrial organization, as represented by Hwang and Mai (1990), Mai and Hwang (1992), Cheng et al. (1993), and Hwang et al. (1998). They concluded that optimum location is independent of a change in demand if the production function is constant returns to scale.

However, Mai and Hwang (1994) (henceforth MH) discussed a linear space model under duopoly and show a contrasting view. By assuming a two-stage game with two

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firms making location interactions at the first stage and output interactions at the second stage, they concluded that, under linear demand, each firm's optimum location is independent of a demand shift only if its own and the rival's production functions are constant returns to scale. Intuitively, the firm's technology can play a role through its effect on location choice, which in turn, will affect the other firm's location decision due to location interactions at the first stage. Such a result can be regarded as a by-product in MH (1994) because their focus is more on the exclusion theorem, the possibility of firms locating at the end point of the linear space, which is a central issue originated by Sakishita (1968) in the linear space model. Thus, the robustness of MH (1994) result in a two-dimensional space is more important because the independence issue, rather than the exclusion theorem, is a central issue in the Weber triangle. In fact, Hwang et al. (1998) examined the robustness of MH (1994) result in Weberian space and once again confirmed the independence proposition.

This paper discusses a one-stage duopoly model in the Weber triangle. We assume that two firms have different technologies such as economies of scale, or a different Cobb–Douglas weights on labor and capital and thus will locate at different points in the Weber triangle space. In addition, we try to capture external effect between firms in terms of transportation cost.[1] The transportation cost is assumed to be dependent on the relative distance between these two firms. The derivative could be positive. In that case, we are considering the case of external economies of scale. The derivative could also be negative. In that case, we are considering a traffic congestion effect.[2] Without lose of generality, we concentrate on the positive derivative case.

The agglomeration benefit or cost in transportation cost could stem from several sources. First, as firm's location is closer to each other, it becomes possible for a same transportation company to pull the inputs and output of these firms together on a same train or plane and ship them from/to a same destination. The possibility of joint arrangement in shipping cargoes allows the transportation cost to be reduced due to economies of scale. In fact, the proximity among firms also provides their workers with the opportunity of making co-ride arrangement and consequently reduce labor transportation cost. Second, the agglomeration of firms could imply a stronger incentive that the local government will provide local public goods on transportation, such as widening the road, setting up traffic sign, and increasing the frequency of public transit. Finally, once firms are too close to each other, one might expect neighborhood externalities such as traffic jams. Such a case is acceptable if we assume the deglomeration between firms due to congestion will not be so large that these two firms end up locating at the two sides of the triangle.

It follows that there is location interdependence and the MH (1994) result in linear space is demonstrated on Weberian space without restricting to the linear demand case.

[1] In Cheng et al. (1993), they discussed the relationship between the structure of transportation cost and the independence issue.

[2] The positive, as well as negative externalities, are captured by the shipping cost to output and input markets. This is a general setup of the external effect. One can, of course, limit the external effect to output market or input markets.

Output will not be independent of demand shock unless both firms' technologies are constant returns to scale. The assumption of external link is so crucial that it serves to generate the non-independence results, thus distinguishing our results from other discussions in Weber triangle, such as MH (1992) and Proposition 2 in Hwang et al. (1998).

The non-independence results have some interesting policy implications in international as well as in regional context. In fact, we do observe the exodus of labor-intensive firms from more developed regions to less developed regions, such as the movement from some East-Asia firms to mainland China and South-East Asia region. In addition, there are also movements from one developed country to another developed country, as witnessed by a recent influx of Japanese manufacturing plants into the United States. It is worth investigating whether external effect plays an important role in such cross-country, or cross-region movements. In an empirical study, Head et al. (1995) examined the location choices of 751 Japanese manufacturing plants built in the USA since 1980. Their conditional logit estimates support the hypothesis that industry-level agglomeration benefits play an important role in location decisions.

The format of this paper is as follows. In Section 2, we set up the model. Structurally, we follow the MH (1992) in setting up the model. Equations (1)–(9) are the same equations used in that paper, except for the additional external benefit assumption. The non-independence results are presented in Section 3. In Section 4, we draw the conclusions.

2. THE MODEL

Consider an oligopoly industry of two firms, 1 and 2 locating inside the Weber triangle as depicted in Figure 1, for the heterogeneous firms case. Each firm produces a homogeneous product while facing perfect competitive input markets. There are two inputs L and K, which are transported from A and B, respectively. The final output is shipped to point C, the market place. Thus, it is in each firm's interest to find out their optimum location E_i, for $i=1,2$ respectively since transportation costs involved in shipping the output, as well as inputs, are assumed to be significant. We denote the distance between E_i and C as h_i; s_i is the distance between E_i and A; z_i is the distance between E_i and B; a and b are the length of CA and CB; θ_i is the angle between CA and CE_i; $\tilde{\theta}$ is the angle between CA and CB. Since these two firms are different, we have to solve each firm's problem independently.

Each firm faces the same freight rate, which is specified as $T(y)$, $G(y)$, and $M(y)$ for shipping to the output, labor and capital markets, where y is the relative distance between these two firms. As mentioned earlier, the derivatives of the freight rate to y can be positive or negative. In the case of negative derivative, it is the congestion effect we wish to capture, while in the case of positive derivative, it is the external economies of scale effect we are targeting. Our major conclusions regarding the independence issue will hold true in either cases.

The production function of each firm is different and is specified as

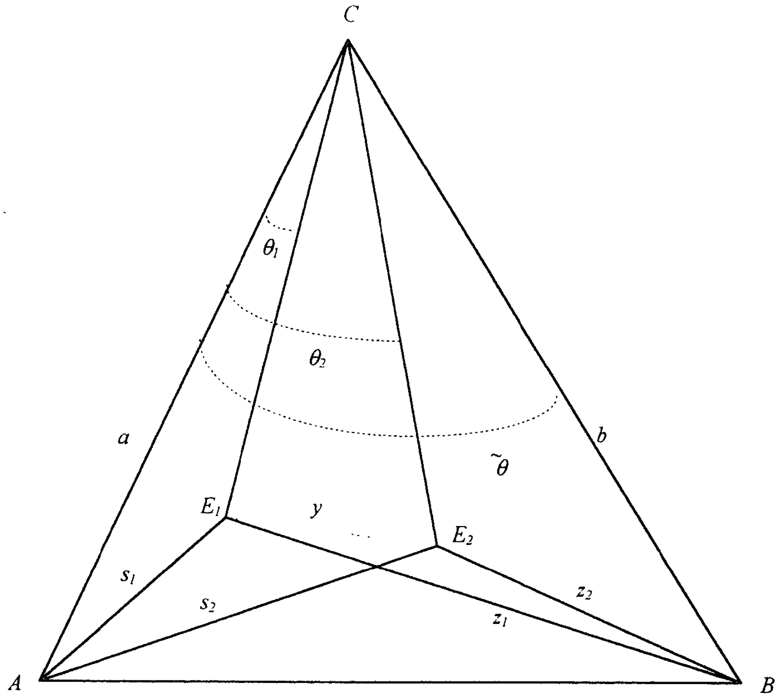

Fig. 1. Two Heterogeneous Firms on Weber Triangle.

$$q_i = f_i(L_i, K_i) \quad i=1,2 \tag{1}$$

and is assumed to be homothetic. Without lose of generality, we assume firm 2 will always choose its location with an angle θ_2 that is greater than θ_1 due to technological differences such as different Cobb-Douglas weights on labor and capital. Following MH (1992), we derive the cost function for each firm subject to a certain output level:

$$\min_{L_i, K_i} [w + G(y)s_i]L_i + [r + M(y)z_i]K_i \quad s.t. \quad q_i = f_i(L_i, K_i), \quad i = 1, 2 \tag{2}$$

where w and r denote the wage and rental rate and are assumed to be constant, s_i, z_i, and y can be expressed in terms of h_i and θ_i:

$$\begin{aligned} s_i &= \sqrt{a^2 + h_i^2 - 2ah_i \cos\theta_i} \\ z_i &= \sqrt{b^2 + h_i^2 - 2bh_i \cos(\tilde{\theta} - \theta_i)} \quad \text{for} \quad i = 1, 2 \\ y &= \sqrt{h_1^2 + h_2^2 - 2h_1 h_2 \cos(\theta_2 - \theta_1)} \end{aligned} \tag{3}$$

Since production function of both firms are assumed to be homothetic, we can decompose the cost function into the product of two functions: the function of factor prices and the function of output as:

$$C_i(q_i) = c_i(w + G(y)s_i, \quad r + M(y)z_i)H_i(q_i) \quad \text{for} \quad i = 1, 2 \tag{4}$$

It follows that the marginal and average cost functions can be expressed as

$$MC_i = C_i'(q_i) = c_i H_i' \quad \text{for} \quad i = 1, 2 \tag{5}$$

$$AC_i = \frac{C_i(q_i)}{q_i} = \frac{c_i H_i}{q_i} \quad \text{for} \quad i = 1, 2 \tag{6}$$

Equations (5) and (6) together imply the following relation:

$$\begin{aligned} &\frac{H_i}{q_i} > H_i' \quad \text{iff the production function exhibits increasing returns to scale} \\ &\frac{H_i}{q_i} = H_i \quad \text{iff the production function exhibits constant returns to scale} \\ &\frac{H_i}{q_i} < H_i' \quad \text{iff the production exhibits decreasing returns to scale } i{=}1, 2 \end{aligned} \tag{7}$$

The above relation will turn out to be crucial in obtaining the independence result. Finally, the inverse demand is assumed to be everywhere twice differentiable with the following properties:

$$P = P(Q, \alpha) = P(q_1 + q_2, \alpha) \quad P_Q < 0, \quad P_\alpha > 0, \quad P_{Q\alpha} = 0 \tag{8}$$

Based on the above assumptions, firm 1 and 2 make their output and location choices simultaneously, each faces the following profit maximization problem:

$$\max_{q_i, h_i, \theta_i} \pi^i = [P(q_1 + q_2, \alpha) - T(y)h_i]q_i - c_i(w + G(y)s_i, r + M(y)z_i)H_i(q_i) \qquad i{=}1, 2 \tag{9}$$

We then derive the following first order conditions:

$$\pi^1_{q_1} = P + P_Q q_1 - T h_1 - c_1 H_{1q_1} = 0 \tag{10}$$

$$\pi^2_{q_2} = P + P_Q q_2 - T h_2 - c_2 H_{2q_2} = 0 \tag{11}$$

$$\pi^1_{h_1} = -T q_1 - T_{h_1} h_1 q_1 - c_{1h_1} H_1 = 0 \tag{12}$$

$$\pi^2_{h_2} = -T q_2 - T_{h_2} h_2 q_2 - c_{2h_2} H_2 = 0 \tag{13}$$

$$\pi^1_{\theta_1} = -T_{\theta_1} h_1 q_1 - c_{1\theta_1} H_1 = 0 \tag{14}$$

$$\pi^2_{\theta_2} = -T_{\theta_2} h_2 q_2 - c_{2\theta_2} H_2 = 0 \tag{15}$$

Equations (12)–(15) are different from the model of symmetric firms without agglomeration benefit and deserve a careful look. Specifically, the second term of equations (12)–(13) and the first term of equations (14)–(15) will be missing without assuming external benefit. Expressions on equations (12)–(13) can be decomposed into four effects, the increase in output market shipping cost, and the change in output market related external cost, as captured by the first two terms, and the change in input markets shipping cost and related external cost, as captured by the final term. Expressions on equations (14)–(15) can be decomposed into three effects, the change in output market related external benefit, as captured by the first term, and the change in input markets shipping cost and related external cost, as captured by the final term. Thus, $c_{1_{\theta 1}}$, $c_{2_{\theta 2}}$ should be positive and negative if there is output market related external benefit. In MH (1992) $c_{1_{\theta 1}}$, $c_{2_{\theta 2}}$ are both equal to zero because: 1) there is no external benefit, and 2) the production function is homothetic.

We assume that the second order conditions are satisfied since studies in the past, such as Emerson (1973), Miller and Jensen (1978), Kusumoto (1984) all suggest that corner solution in the Weber triangle is not possible unless we make some drastic assumptions, such as zero transportation cost for output or inputs. Thus, equations (10)–(15) can be used to determine the equilibrium value of q_1, q_2, h_1, h_2, θ_1, θ_2. Furthermore, taking the total derivatives of (10)–(15) and using the Cramer's rule, we can derive the comparative statics results.

3. CHANGES OF OPTIMUM LOCATION UNDER DEMAND SHOCK

We follow Moses (1958) by first setting h fixed and examine the change of θ, the circumferential location, under the demand shock. By total differentiating equations (10), (11), (14), (15) with respect to q_1, q_2, θ_1, θ_2, and α, we can obtain the following expressions:

$$\left.\frac{d\theta_1}{d\alpha}\right|_{\bar{h}_i} = \frac{P_\alpha c_{2\theta_2}\pi^1_{\theta_1\theta_2}(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})\left(\frac{H_2}{q_2} - H_{2q_2}\right) + P_\alpha c_{1\theta_1}(D_{21} + D_{22})\left(\frac{H_1}{q_1} - H_{1q_1}\right)}{D_4}$$

where

$$D_{21} = \pi^1_{q_1\theta_2}\pi^2_{\theta_2q_2} - \pi^1_{q_1q_2}\pi^2_{\theta_2\theta_2}, \quad D_{22} = \pi^2_{q_2q_2}\pi^2_{\theta_2\theta_2} - \pi^2_{q_2\theta_2}\pi^2_{\theta_2q_2} \quad i = 1, 2 \tag{16}$$

$$\left.\frac{d\theta_2}{d\alpha}\right|_{\bar{h}_i} = \frac{P_\alpha c_{1\theta_1}\pi^2_{\theta_2\theta_1}(\pi^1_{q_1q_2} - \pi^2_{q_2q_2})\left(\frac{H_1}{q_1} - H_{1q_1}\right) + P_\alpha c_{2\theta_2}(D_{23} + D_{24})\left(\frac{H_2}{q_2} - H_{2q_2}\right)}{D_4}$$

where

$$D_{23} = \pi^2_{q_2\theta_1}\pi^1_{\theta_1q_1} - \pi^2_{q_2q_1}\pi^1_{\theta_1\theta_1}, \quad D_{24} = \pi^1_{q_1q_1}\pi^1_{\theta_1\theta_1} - \pi^1_{q_1\theta_1}\pi^1_{\theta_1q_1} \quad i = 1, 2 \tag{17}$$

A detailed derivation is contained in Appendix B. The D_4 in the above two equations are the relevant Hessian matrix. Technically, the effect of external benefit is implied in the first derivatives, such as the nonzero values of $c_{1_{\theta 1}}$, $c_{2_{\theta 2}}$, as well as in some second derivatives, such as the nonzero values of $\pi^1_{\theta_1\theta_2}$ and $\pi^2_{\theta_2\theta_1}$.

Clearly, equations (16) and (17) will not be equal to zero unless both firms have linear technologies. As long as one of the firms has a non-linear technology, one of the two terms in the numerators will not be zero and the independence result will not hold. According to equations (16) and (17), we can derive the following proposition.

PROPOSITION 1. *Holding all h_i as fixed and treating all θ_i as variables under the Cournot competition of two heterogeneous firms, the optimum location will be variant to demand shock even if the firm exhibits a constant returns to scale technology. As long as the other firm's technology is not constant returns to scale, both firms will move their locations along the arc IJ under demand shock.*

One of the key assumptions that lead to the above result is the decreasing transportation rate with respect to the relative distance between firms. In the absence of that assumption, both $\pi^1_{\theta_1\theta_2}$ and $\pi^2_{\theta_2\theta_1}$ in equations (16) and (17) will be equal to zero[3] and thus the whole expression will be equal to zero as long as the firm's own technology is linear, i.e., $c_{1\theta_1}$ and $c_{2\theta_2}$ are equal to zero. In MH (1992), the circumferential location is invariant under demand shock as long as the firm's technology is homothetic. Contrary to their finding, in this paper, even if the technology is linear, the firm's circumferential location will be variant to demand shift as long as the other firm's technology is nonlinear. The intuition behind the MH (1992) result is as follows: as long as technology is homothetic, the firm will keep using the same input ratio during expansion path, and the relative pull from both input markets will remain the same. However, in this case, there is an additional consideration for the firm, i.e., its strategic link with another firm, and it is that extra consideration that disturbs the original equilibrium. During a demand expansion, the firm with increasing returns technology most likely will find the importance of its transportation cost in output outweighs its transportation cost in input as compared with its previous equilibrium. Given its distance to the output market as fixed, the firm with increasing returns technology can save some of its transportation cost in output by enhancing it non-cooperative strategic link, i.e., by moving toward its competitor. As for the firm with constant returns technology, nothing has been changed except its strategic link with its competitor, it may be induced to move toward or away from its competitor depending on its spatial strategic relationship and output strategic relationship with the other firm.[4] In general, the firm with nonlinear technology is the engine that generates the circumferential location move.

A corollary related to Proposition 1 can be stated as the following:

COROLLARY 1. *Holding all h_i as fixed and treating all θ_i as variables in the heterogeneous firms model under Weber triangle, linearity of both firms' technology is the necessary and sufficient condition for the independence result.*

This is different from the conventional result which suggests homotheticity of firm's

[3] These two cross derivatives can be zero even if there is external benefit or cost. We will simply ignore that special case.

[4] Strategic output relationship is important because it may not be in one firm's best interest to provide the other firm with the opportunity of lowering its cost. See Lai (1996) for a mathematical derivation and more

technology is the necessary and sufficient condition for the independence result. Again, this is due to the assumed external benefit in output market. In the absence of that assumption, $c_{1\theta_1}$ and $c_{2\theta_2}$ will be zero because T_{θ_1} and T_{θ_2} in equations (14) and (15), which stand for output market related external benefit, will be missing. In that case, equations (14) and (15) are just like equation (11) in MH (1992) and the independence result is implied as long as technology is homothetic.

The usual explanation toward the swing on arc IJ is inputs substitution effect, as was suggested by Moses (1958) ever since. In contrast to previous research, we generate the swing on the arc IJ through the strategic effect. In MH (1992), the change of θ under demand shock is zero. The major difference is the nonzero value of the cross derivatives between θ_1 and θ_2, which measure the induced spatial strategic effects between firms. In contrast to MH (1994), our non-independence result is not restricted to the case of linear demand. Intuitively, we use the internal economies of scale technology in one firm as the driving force, which work through the channel of external economies of scale and the non-independence result is thus implied.

Next, we consider the case when both h_i and θ_i are variables. In this case, we have to deal with a 6×6 matrix. In order to obtain simple expressions, we assume firm 1 have a linear technology and obtain equations (18a), (19a), (20a), (21a). Alternatively, we assume firm 2 has a linear technology and obtain equations (18b), (19b), (20b), (21b). A detailed derivation is contained in Appendix A. The changes of location under demand shock are:

$$\frac{dh_1}{d\alpha} = \frac{P_\alpha(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})(\pi^2_{h_2q_2}D_{31} + \pi^2_{\theta_2q_2}D_{32})}{D_6} \tag{18a}$$

$$\frac{dh_1}{d\alpha} = \frac{P_\alpha(\pi^2_{q_2q_2} - \pi^1_{q_1q_2})(\pi^1_{h_1q_1}D_{33} + \pi^1_{\theta_1q_1}D_{34})}{D_6} \tag{18b}$$

$$\frac{dh_2}{d\alpha} = \frac{P_\alpha(\pi^1_{q_1q_1} - \pi^2_{q_2q_1})(\pi^2_{h_2q_2}D_{35} + \pi^2_{\theta_2q_2}D_{36})}{D_6} \tag{19a}$$

$$\frac{dh_2}{d\alpha} = \frac{P_\alpha(\pi^1_{q_1q_2} - \pi^2_{q_2q_2})(\pi^1_{h_1q_1}D_{37} + \pi^1_{\theta_1q_1}D_{38})}{D_6} \tag{19b}$$

$$\frac{d\theta_1}{d\alpha} = \frac{P_\alpha(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})(\pi^2_{h_2q_2}E_{31} + \pi^2_{\theta_2q_2}E_{32})}{D_6} \tag{20a}$$

$$\frac{d\theta_1}{d\alpha} = \frac{P_\alpha(\pi^2_{q_2q_2} - \pi^1_{q_1q_2})(\pi^1_{h_1q_1}E_{33} + \pi^1_{\theta_1q_1}E_{34})}{D_6} \tag{20b}$$

$$\frac{d\theta_2}{d\alpha} = \frac{P_\alpha(\pi^1_{q_1q_1} - \pi^2_{q_2q_1})(\pi^2_{h_2q_2}E_{35} + \pi^2_{\theta_2q_2}E_{36})}{D_6} \tag{21a}$$

$$\frac{d\theta_2}{d\alpha} = \frac{P_\alpha(\pi^1_{q_1q_2} - \pi^2_{q_2q_2})(\pi^1_{h_1q_1}E_{37} + \pi^1_{\theta_1q_1}E_{38})}{D_6} \tag{21b}$$

where D_{31} to D_{38}, E_{31} to E_{38} are 3×3 matrices defined in Appendix B. Of those 3×3 matrices, only D_{33}, D_{35}, E_{34}, E_{36} are the relevant Hessian matrices and are negative by the second order conditions. D_6 is also a Hessian matrix and is positive. Equations (18)–(21) will not be zero unless both firms' technologies are linear. Suppose firm 1 has a linear technology, we know from (18a) and (19a) that the optimum market distance of both firms will not be independent of demand shock since both equations are not equal to zero. Similar results will hold for the other three pairs. The intuition is the same as the case when h_i is held as fixed. We thus conclude with the following proposition.

PROPOSITION 2. *Treating h_i and θ_i as variables in the Cournot competition model of two heterogeneous firms, the optimum location will be variant to demand shock even if the firm has a constant returns to scale technology. As long as one firm's technology is not constant returns to scale, both firms will move their optimum location under demand shock.*

Again, Proposition 2 will not hold without assuming the transportation cost to be decreasing in relative distance. Equations (18a), (19b), (20a), (21b) are the expression of changes in optimum location while assuming the firms' own technologies are constant returns to scale. In the absence of the above assumption, all the Hessian matrices inside the parenthesis will be equal to zero due to the vanishing of cross derivatives with respect to h_1, h_2, θ_1, θ_2. Intuitively, the nonzero values of these cross derivative terms stand for strategic location interactions.

4. CONCLUSIONS

Unlike the Hotelling (1929) model, which has been a showcase of cooperative game theory, the strategic location interactions under Weber triangle have been largely ignored in literature. In this paper, we generate the strategic location interactions between firms by considering the heterogeneous firms under external economies of scale. It is shown that the independence result will not hold as long as one of the firms has a non-linear technology. Our findings are different from the independence result in MH (1992) and Hwang et al. (1998) because in the both cases, firms are identical and/or without external links. Thus, the MH (1994) result on the linear space and linear demand is robust under two-dimensional space and more generalized demand conditions. Furthermore, it is now nontrivial, under this model, to discuss the impacts of different policy, such as production tax and subsidy, on the agglomeration or deglomeration between firms.

APPENDIX A

This appendix contains the derivation of equations (18)–(21). A total differentiation of equations (10)–(15) with respect to q_1, q_2, h_1, h_2, θ_1, θ_2 will yield the following matrix:

$$\begin{bmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & \pi^1_{q_1h_2} & \pi^1_{q_1\theta_1} & \pi^1_{q_1\theta_2} \\ \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & \pi^2_{q_2h_2} & \pi^2_{q_2\theta_1} & \pi^2_{q_2\theta_2} \\ \pi^1_{h_1q_1} & \pi^1_{h_1q_2} & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2q_1} & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1q_1} & \pi^1_{\theta_1q_2} & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2q_1} & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{bmatrix} \begin{bmatrix} dq_1 \\ dq_2 \\ dh_1 \\ dh_2 \\ d\theta_1 \\ d\theta_2 \end{bmatrix} = \begin{bmatrix} -\pi^1_{q_1\alpha} \\ -\pi^2_{q_2\alpha} \\ -\pi^1_{h_1\alpha} \\ -\pi^2_{h_2\alpha} \\ -\pi^1_{\theta_1\alpha} \\ -\pi^2_{\theta_2\alpha} \end{bmatrix} d\alpha$$

where

$$\pi^1_{q_1q_1} = 2P_Q + P_{QQ}q_1 - c_1 H_{1q_1q_1}$$

$$\pi^1_{q_1q_2} = P_Q + P_{QQ}q_1$$

$$\pi^1_{q_1h_1} = \pi^1_{h_1q_1} = -T - T_{h_1}h_1 - c_{1h_1}H_{1q_1} = c_{1h_1}\left(\frac{H_1}{q_1} - H_{1q_1}\right) \quad \text{(by equation (12))}$$

$$\pi^1_{q_1h_2} = -T_{h_2}h_1 - c_{1h_2}H_{1q_1}$$

$$\pi^1_{q_1\theta_1} = \pi^1_{\theta_1q_1} = -T_{\theta_1}h_1 - c_{1\theta_1}H_{1q_1} = c_{1\theta_1}\left(\frac{H_1}{q_1} - H_{1q_1}\right) \quad \text{(by equation (14))}$$

$$\pi^1_{q_1\theta_2} = -T_{\theta_2}h_1 - c_{1\theta_2}H_{1q_1}$$

$$\pi^2_{q_2q_1} = P_Q + P_{QQ}q_2$$

$$\pi^2_{q_2q_2} = 2P_Q + P_{QQ}q_2 - c_2 H_{2q_2q_2}$$

$$\pi^2_{q_2h_1} = -T_{h_1}h_2 - c_{2h_1}H_{2q_2}$$

$$\pi^2_{q_2h_2} = \pi^2_{h_2q_2} = -T_{h_2}h_2 - T - c_{2h_2}H_{2q_2} = c_{2h_2}\left(\frac{H_2}{q_2} - H_{2q_2}\right) \quad \text{(by equation (13))}$$

$$\pi^2_{q_2\theta_1} = -T_{\theta_1}h_2 - c_{2\theta_1}H_{2q_2}$$

$$\pi^2_{q_2\theta_2} = \pi^2_{\theta_2q_2} = -T_{\theta_2}h_2 - c_{2\theta_2}H_{2q_2} = c_{2\theta_2}\left(\frac{H_2}{q_2} - H_{2q_2}\right) \quad \text{(by equation (15))}$$

$$\pi^1_{h_1q_2} = 0$$

$$\pi^1_{h_1h_1} = -2T_{h_1}q_1 - T_{h_1h_1}h_1q_1 - c_{1h_1h_1}H_1$$

$$\pi^1_{h_1h_2} = -T_{h_2}q_1 - T_{h_1h_2}h_1q_1 - c_{1h_1h_2}H_1$$

$$\pi^1_{h_1\theta_1} = \pi^1_{\theta_1h_1} = -T_{\theta_1}q_1 - T_{h_1\theta_1}h_1q_1 - c_{1h_1\theta_1}H_1$$

$$\pi^1_{h_1\theta_2} = -T_{\theta_2}q_1 - T_{h_1\theta_2}h_1q_1 - c_{1h_1\theta_2}H_1$$

$$\pi^2_{h_2q_1} = 0$$

$$\pi^2_{h_2h_1} = -T_{h_1}q_2 - T_{h_2h_1}h_2q_2 - c_{2h_2h_1}H_2$$

$$\pi^2_{h_2h_2} = -2T_{h_2}q_2 - T_{h_2h_2}h_2q_2 - c_{2h_2h_2}H_2$$

$$\pi^2_{h_2\theta_1} = -T_{\theta_1}q_2 - T_{h_2\theta_1}h_2q_2 - c_{2h_2\theta_1}H_2$$

$$\pi^2_{h_2\theta_2} = \pi^2_{\theta_2h_2} = -T_{\theta_2}q_2 - T_{h_2\theta_2}h_2q_2 - c_{2h_2\theta_2}H_2$$

$$\pi^1_{\theta_1q_2} = 0$$

$$\pi^1_{\theta_1h_2} = -T_{\theta_1h_2}h_1q_1 - c_{1\theta_1h_2}H_1$$

$$\pi^1_{\theta_1\theta_1} = -T_{\theta_1\theta_1}h_1q_1 - c_{1\theta_1\theta_1}H_1$$

$$\pi^1_{\theta_1\theta_2} = -T_{\theta_1\theta_2}h_1q_1 - c_{1\theta_1\theta_2}H_1$$

$$\pi^2_{\theta_2q_1} = 0$$

$$\pi^2_{\theta_2h_1} = -T_{\theta_2h_1}h_2q_2 - c_{2\theta_2h_1}H_2$$

$$\pi^2_{\theta_2\theta_1} = -T_{\theta_2\theta_1}h_2q_2 - c_{2\theta_2\theta_1}H_2$$

$$\pi^2_{\theta_2\theta_2} = -T_{\theta_2\theta_2}h_2q_2 - c_{2\theta_2\theta_2}H_2$$

$$\pi^1_{q_1\alpha} = P_\alpha,\ \pi^2_{q_2\alpha} = P_\alpha,\ \pi^1_{h_1\alpha} = 0,\ \pi^2_{h_2\alpha} = 0,\ \pi^1_{\theta_1\alpha} = 0,\ \pi^2_{\theta_2\alpha} = 0$$

By the Cramer's rule, we can obtain the following comparative statics results:

$$\frac{dh_1}{d\alpha} = \frac{\begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & -P_\alpha & \pi^1_{q_1h_2} & \pi^1_{q_1\theta_1} & \pi^1_{q_1\theta_2} \\ \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & -P_\alpha & \pi^2_{q_2h_2} & \pi^2_{q_2\theta_1} & \pi^2_{q_2\theta_2} \\ \pi^1_{h_1q_1} & 0 & 0 & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ 0 & \pi^2_{h_2q_2} & 0 & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1q_1} & 0 & 0 & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ 0 & \pi^2_{\theta_2q_2} & 0 & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}}{D_6}$$

$$= \frac{P_\alpha}{D_6}\left[\begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_2} & \pi^1_{q_1\theta_1} & \pi^1_{q_1\theta_2} \\ \pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ 0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix} - \begin{vmatrix} \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_2} & \pi^2_{q_2\theta_1} & \pi^2_{q_2\theta_2} \\ \pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ 0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}\right]$$

$$
\frac{dh_2}{d\alpha} = \frac{\begin{vmatrix}
\pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & -P_\alpha & \pi^1_{q_1\theta_1} & \pi^1_{q_1\theta_2} \\
\pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & -P_\alpha & \pi^2_{q_2\theta_1} & \pi^2_{q_2\theta_2} \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & 0 & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & 0 & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & 0 & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & 0 & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}
\end{vmatrix}}{D_6}
$$

$$
= \frac{P_\alpha}{D_6}\left[\begin{vmatrix}
\pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & \pi^2_{q_2\theta_1} & \pi^2_{q_2\theta_2} \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}
\end{vmatrix} - \begin{vmatrix}
\pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & \pi^1_{q_1\theta_1} & \pi^1_{q_1\theta_2} \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}
\end{vmatrix}\right]
$$

$$
\frac{d\theta_1}{d\alpha} = \frac{\begin{vmatrix}
\pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & \pi^1_{q_1h_2} & -P_\alpha & \pi^1_{q_1\theta_2} \\
\pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & \pi^2_{q_2h_2} & -P_\alpha & \pi^2_{q_2\theta_2} \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & 0 & \pi^1_{h_1\theta_2} \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & 0 & \pi^2_{h_2\theta_2} \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & 0 & \pi^1_{\theta_1\theta_2} \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & 0 & \pi^2_{\theta_2\theta_2}
\end{vmatrix}}{D_6}
$$

$$
= \frac{P_\alpha}{D_6}\left[\begin{vmatrix}
\pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & \pi^1_{q_1h_2} & \pi^1_{q_1\theta_2} \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2} \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2} \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2} \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2}
\end{vmatrix} - \begin{vmatrix}
\pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & \pi^2_{q_2h_2} & \pi^2_{q_2\theta_2} \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2} \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2} \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2} \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2}
\end{vmatrix}\right]
$$

$$
\frac{d\theta_2}{d\alpha} = \frac{\begin{vmatrix}
\pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & \pi^1_{q_1h_2} & \pi^1_{q_1\theta_1} & -P_\alpha \\
\pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & \pi^2_{q_2h_2} & \pi^2_{q_2\theta_1} & -P_\alpha \\
\pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & 0 \\
0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & 0 \\
\pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & 0 \\
0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & 0
\end{vmatrix}}{D_6}
$$

$$
= \frac{P_\alpha}{D_6}\left[\begin{vmatrix} \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2h_1} & \pi^2_{q_2h_2} & \pi^2_{q_2\theta_1} \\ \pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} \\ 0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix} - \begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1h_1} & \pi^1_{q_1h_2} & \pi^1_{q_1\theta_1} \\ \pi^1_{h_1q_1} & 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} \\ 0 & \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix}\right]
$$

Using the fact that $\pi^1_{q_1h_1} = \pi^1_{h_1q_1} = \pi^1_{q_1\theta_1} = \pi^1_{\theta_1q_1} = 0$ when firm 1 has a linear technology, we can derive equations (18a), (19a), (20a), (21a):

$$
\frac{dh_1}{d\alpha} = \frac{P_\alpha}{D_6}(\pi^1_{q_1q_1} - \pi^2_{q_2q_1})\begin{vmatrix} 0 & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2q_2} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ 0 & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}
$$

$$
= \frac{P_\alpha(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})(\pi^2_{h_2q_2}D_{31} + \pi^2_{\theta_2q_2}D_{32})}{D_6} \tag{18a}
$$

$$
\frac{dh_2}{d\alpha} = \frac{P_\alpha}{D_6}(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})\begin{vmatrix} 0 & \pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}
$$

$$
= \frac{P_\alpha(\pi^1_{q_1q_1} - \pi^2_{q_2q_1})(\pi^2_{h_2q_2}D_{35} + \pi^2_{\theta_2q_2}D_{36})}{D_6} \tag{19a}
$$

$$
\frac{d\theta_1}{d\alpha} = \frac{P_\alpha}{D_6}(\pi^1_{q_1q_1} - \pi^2_{q_2q_1})\begin{vmatrix} 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2} \\ 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2} \end{vmatrix}
$$

$$
= \frac{P_\alpha(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})(\pi^2_{h_2q_2}E_{31} + \pi^2_{\theta_2q_2}E_{32})}{D_6} \tag{20a}
$$

$$
\frac{d\theta_2}{d\alpha} = \frac{P_\alpha}{D_6}(\pi^2_{q_2q_1} - \pi^1_{q_1q_1})\begin{vmatrix} 0 & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} \\ \pi^2_{h_2q_2} & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} \\ 0 & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} \\ \pi^2_{\theta_2q_2} & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix}
$$

$$
= \frac{P_\alpha(\pi^1_{q_1q_1} - \pi^2_{q_2q_1})(\pi^2_{h_2q_2}E_{35} + \pi^2_{\theta_2q_2}E_{36})}{D_6} \tag{21a}
$$

where

$$D_{31}=\begin{vmatrix}\pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2}\\ \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2}\\ \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}\end{vmatrix},\quad D_{32}=\begin{vmatrix}\pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2}\\ \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2}\\ \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2}\end{vmatrix}$$

$$D_{35}=\begin{vmatrix}\pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2}\\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2}\\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}\end{vmatrix},\quad D_{36}=\begin{vmatrix}\pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2}\\ \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2}\\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2}\end{vmatrix}$$

$$E_{31}=\begin{vmatrix}\pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2}\\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2}\\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2}\end{vmatrix},\quad E_{32}=\begin{vmatrix}\pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2}\\ \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2}\\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2}\end{vmatrix}$$

$$E_{35}=\begin{vmatrix}\pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1}\\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1}\\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1}\end{vmatrix},\quad E_{36}=\begin{vmatrix}\pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1}\\ \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1}\\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1}\end{vmatrix}$$

Using the fact that $\pi^2_{q_2h_2}=\pi^2_{h_2q_2}=\pi^2_{q_2\theta_2}=\pi^2_{\theta_2q_2}=0$ when firm 2 has a linear technology, we can derive equations (18b), (19b), (20b), (21b):

$$\frac{dh_1}{d\alpha}=\frac{P_\alpha}{D_6}(\pi^2_{q_2q_2}-\pi^1_{q_1q_2})\begin{vmatrix}\pi^1_{h_1q_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2}\\ 0 & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2}\\ \pi^1_{\theta_1q_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2}\\ 0 & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}\end{vmatrix}$$

$$=\frac{P_\alpha(\pi^2_{q_2q_2}-\pi^1_{q_1q_2})(\pi^1_{h_1q_1}D_{33}+\pi^1_{\theta_1q_1}D_{34})}{D_6}\tag{18b}$$

$$\frac{dh_2}{d\alpha}=\frac{P_\alpha}{D_6}(\pi^1_{q_1q_2}-\pi^2_{q_2q_2})\begin{vmatrix}\pi^1_{h_1q_1} & \pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2}\\ 0 & \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2}\\ \pi^1_{\theta_1q_1} & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2}\\ 0 & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2}\end{vmatrix}$$

$$=\frac{P_\alpha(\pi^1_{q_1q_2}-\pi^2_{q_2q_2})(\pi^1_{h_1q_1}D_{37}+\pi^1_{\theta_1q_1}D_{38})}{D_6}\tag{19b}$$

$$\frac{d\theta_1}{d\alpha}=\frac{P_\alpha}{D_6}(\pi^2_{q_2q_2}-\pi^1_{q_1q_2})\begin{vmatrix}\pi^1_{h_1q_1} & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2}\\ 0 & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2}\\ \pi^1_{\theta_1q_1} & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2}\\ 0 & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2}\end{vmatrix}$$

$$= \frac{P_\alpha(\pi^2_{q_2q_2} - \pi^1_{q_1q_2})(\pi^1_{h_1q_1}E_{33} + \pi^1_{\theta_1q_1}E_{34})}{D_6} \tag{20b}$$

$$\frac{d\theta_2}{d\alpha} = \frac{P_\alpha}{D_6}(\pi^1_{q_1q_2} - \pi^2_{q_2q_2}) \begin{vmatrix} \pi^1_{h_1q_1} & \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} \\ 0 & \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} \\ \pi^1_{\theta_1q_1} & \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} \\ 0 & \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix}$$

$$= \frac{P_\alpha(\pi^1_{q_1q_2} - \pi^2_{q_2q_2})(\pi^1_{h_1q_1}E_{37} + \pi^1_{\theta_1q_1}E_{38})}{D_6} \tag{21b}$$

where

$$D_{33} = \begin{vmatrix} \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}, \quad D_{34} = \begin{vmatrix} \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}$$

$$D_{37} = \begin{vmatrix} \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}, \quad D_{38} = \begin{vmatrix} \pi^1_{h_1h_1} & \pi^1_{h_1\theta_1} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2h_1} & \pi^2_{h_2\theta_1} & \pi^2_{h_2\theta_2} \\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{vmatrix}$$

$$E_{33} = \begin{vmatrix} \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2} \\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2} \end{vmatrix}, \quad E_{34} = \begin{vmatrix} \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_2} \\ \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_2} \\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_2} \end{vmatrix}$$

$$E_{37} = \begin{vmatrix} \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} \\ \pi^1_{\theta_1h_1} & \pi^1_{\theta_1h_2} & \pi^1_{\theta_1\theta_1} \\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix}, \quad E_{38} = \begin{vmatrix} \pi^1_{h_1h_1} & \pi^1_{h_1h_2} & \pi^1_{h_1\theta_1} \\ \pi^2_{h_2h_1} & \pi^2_{h_2h_2} & \pi^2_{h_2\theta_1} \\ \pi^2_{\theta_2h_1} & \pi^2_{\theta_2h_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix}$$

APPENDIX B

This appendix contains the derivation of equations (16)–(17). A total differentiation of equations (10), (11), (14), (15) with respect to q_1, q_2, θ_1, θ_2 will yield the following matrix:

$$\begin{bmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1\theta_1} & \pi^1_{q_1\theta_2} \\ \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2\theta_1} & \pi^2_{q_2\theta_2} \\ \pi^1_{\theta_1q_1} & \pi^1_{\theta_1q_2} & \pi^1_{\theta_1\theta_1} & \pi^1_{\theta_1\theta_2} \\ \pi^2_{\theta_2q_1} & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2\theta_1} & \pi^2_{\theta_2\theta_2} \end{bmatrix} \begin{bmatrix} dq_1 \\ dq_2 \\ d\theta_1 \\ d\theta_2 \end{bmatrix} = \begin{bmatrix} -\pi^1_{q_1\alpha} \\ -\pi^2_{q_2\alpha} \\ -\pi^1_{\theta_1\alpha} \\ -\pi^2_{\theta_2\alpha} \end{bmatrix} d\alpha$$

where each element is defined the same way as in Appendix A. By the Cramer's rule, we can obtain the following comparative statics results:

$$\left.\frac{d\theta_1}{d\alpha}\right|_{\bar{h}_i} = \frac{\begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & -P_\alpha & \pi^1_{q_1\theta_2} \\ \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & -P_\alpha & \pi^2_{q_2\theta_2} \\ \pi^1_{\theta_1q_1} & 0 & 0 & \pi^1_{\theta_1\theta_2} \\ 0 & \pi^2_{\theta_2q_2} & 0 & \pi^2_{\theta_2\theta_2} \end{vmatrix}}{D_4}$$

$$= \frac{P_\alpha}{D_4}\left[\begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1\theta_2} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1\theta_2} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2\theta_2} \end{vmatrix} - \begin{vmatrix} \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2\theta_2} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1\theta_2} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2\theta_2} \end{vmatrix}\right]$$

$$= \frac{P_\alpha\left[\pi^1_{q_1\theta_2}\pi^1_{\theta_1q_1}\pi^2_{\theta_2q_2} - \pi^1_{q_1q_2}\pi^1_{\theta_1q_1}\pi^2_{\theta_2\theta_2} - \pi^1_{q_1q_1}\pi^1_{\theta_1\theta_2}\pi^2_{\theta_2q_2}\right]}{D_4}$$

$$- \frac{P_\alpha\left[\pi^2_{q_2\theta_2}\pi^1_{\theta_1q_1}\pi^2_{\theta_2q_2} - \pi^2_{q_2q_2}\pi^1_{\theta_1q_1}\pi^2_{\theta_2\theta_2} - \pi^2_{q_2q_1}\pi^1_{\theta_1\theta_2}\pi^2_{\theta_2q_2}\right]}{D_4}$$

$$= \frac{P_\alpha\pi^1_{\theta_1\theta_2}\left(\pi^2_{q_2q_1} - \pi^1_{q_1q_1}\right)\pi^2_{\theta_2q_2}}{D_4}$$

$$+ \frac{P_\alpha\left(\pi^1_{q_1\theta_1}\pi^2_{\theta_2q_2} - \pi^1_{q_1q_2}\pi^2_{\theta_2\theta_2} + \pi^2_{q_2q_2}\pi^2_{\theta_2\theta_2} - \pi^2_{q_2\theta_2}\pi^2_{\theta_2q_2}\right)\pi^1_{\theta_1q_1}}{D_4}$$

$$= \frac{P_\alpha\pi^1_{\theta_1\theta_2}\left(\pi^2_{q_2q_1} - \pi^1_{q_1q_1}\right)\left(\frac{H_2}{q_2} - H_{2q_2}\right) + P_\alpha(D_{21} + D_{22})\left(\frac{H_1}{q_1} - H_{1q_1}\right)}{D_4}$$

where

$$D_{21} = \pi^1_{q_1\theta_2}\pi^2_{\theta_2q_2} - \pi^1_{q_1q_2}\pi^2_{\theta_2\theta_2}, \quad D_{22} = \pi^2_{q_2q_2}\pi^2_{\theta_2\theta_2} - \pi^2_{q_2\theta_2}\pi^2_{\theta_2q_2} \quad i = 1, 2 \tag{16}$$

$$\left.\frac{d\theta_2}{d\alpha}\right|_{\bar{h}_i} = \frac{\begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1\theta_1} & -P_\alpha \\ \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2\theta_1} & -P_\alpha \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1\theta_1} & 0 \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2\theta_1} & 0 \end{vmatrix}}{D_4}$$

$$= \frac{P_\alpha}{D_4}\left[\begin{vmatrix} \pi^2_{q_2q_1} & \pi^2_{q_2q_2} & \pi^2_{q_2\theta_1} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1\theta_1} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix} - \begin{vmatrix} \pi^1_{q_1q_1} & \pi^1_{q_1q_2} & \pi^1_{q_1\theta_1} \\ \pi^1_{\theta_1q_1} & 0 & \pi^1_{\theta_1\theta_1} \\ 0 & \pi^2_{\theta_2q_2} & \pi^2_{\theta_2\theta_1} \end{vmatrix}\right]$$

$$= \frac{P_\alpha\left(\pi^2_{q_2\theta_1}\pi^1_{\theta_1q_1}\pi^2_{\theta_2q_2} - \pi^2_{q_2q_1}\pi^1_{\theta_1\theta_1}\pi^2_{\theta_2q_2} - \pi^2_{q_2q_2}\pi^2_{\theta_2\theta_1}\pi^1_{\theta_1q_1}\right)}{D_4}$$

$$- \frac{P_\alpha\left(\pi^1_{q_1\theta_1}\pi^2_{\theta_2q_2}\pi^1_{\theta_1q_1} - \pi^1_{q_1q_1}\pi^1_{\theta_1\theta_1}\pi^2_{\theta_2q_2} - \pi^1_{q_1q_2}\pi^2_{\theta_2\theta_1}\pi^1_{\theta_1q_1}\right)}{D_4}$$

$$= \frac{P_\alpha\pi^2_{\theta_2\theta_1}\left(\pi^1_{q_1q_2} - \pi^2_{q_2q_2}\right)\pi^1_{\theta_1q_1}}{D_4}$$

$$+ \frac{P_\alpha\left(\pi^2_{q_2\theta_1}\pi^1_{\theta_1q_1} - \pi^2_{q_2q_1}\pi^1_{\theta_1\theta_1} + \pi^1_{q_1q_1}\pi^1_{\theta_1\theta_1} - \pi^1_{q_1\theta_1}\pi^1_{\theta_1q_1}\right)\pi^2_{\theta_2q_2}}{D_4}$$

$$= \frac{P_\alpha\pi^2_{\theta_2\theta_1}\left(\pi^1_{q_1q_2} - \pi^2_{q_2q_2}\right)\left(\frac{H_1}{q_1} - H_{1q_1}\right) + P_\alpha(D_{23} + D_{24})\left(\frac{H_2}{q_2} - H_{2q_2}\right)}{D_4}$$

where

$$D_{23} = \pi^2_{q_2\theta_1}\pi^1_{\theta_1q_1} - \pi^2_{q_2q_1}\pi^1_{\theta_1\theta_1}, \quad D_{24} = \pi^1_{q_1q_1}\pi^1_{\theta_1\theta_1} - \pi^1_{q_1\theta_1}\pi^1_{\theta_1q_1} \quad i = 1, 2 \tag{17}$$

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