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CONTESTABILITY AND BERTRAND EQUILIBRIUM:
A UNIFIED APPROACH

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Abstract: We seek to unify the standard Bertrand duopoly outcome and the contestable outcome in a single model with price competition. We demonstrate that for small values of the fixed cost, the standard Bertrand duopoly outcome emerges as the equilibrium. For a large enough fixed cost, the unique equilibrium of this game yields the contestable outcome. Interestingly enough, for intermediate values of the fixed cost, duopoly outcomes may co-exist with the contestable one. Furthermore, we find that even under decreasing returns to scale a contestable outcome may exist.

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1. INTRODUCTION

In this paper we seek to provide an unified framework for the analysis of the contestable, as well as the standard Bertrand duopoly outcome.

Under the contestable approach, first developed by Baumol et al. (1982), we look for sustainable market outcomes such that no potential entrant can enter and, taking the existing price as given, make a strictly positive profit. The authors demonstrate that in a single product industry with increasing returns, the contestability theory predicts that there would be a unique operating firm making a profit of zero, and that average cost pricing would prevail. It is striking that increasing returns does not turn out to be a barrier to competitive pricing if the firms behave in a contestable manner. (See Tirole (1988) for a succinct summary of the contestability literature.)

There have been several attempts at providing non-cooperative foundations for the contestable outcome. In fact, Baumol et al. (1982) themselves suggest that the contestable outcome can be looked at as a generalisation of Bertrand competition to markets with increasing returns to scale. This suggests that the following should hold. Consider a model of price competition where the cost structure has been

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appropriately parametrised so that for different parameter specifications the cost function is of the increasing returns to scale, or of the decreasing returns to scale type. Then, under increasing returns to scale, the outcome should be contestable, whereas it should yield the standard Bertrand duopoly outcome if the cost function is of the decreasing returns to scale type. It is the aim of this paper to formalise this intuition.

To this end we look at a model of Bertrand competition where the firms simultaneously announce prices, and then supply the whole of the demand coming to them. Notice that in this formulation prices are completely flexible. This is of interest because the hit-and-run entry story usually propounded to justify the contestable outcome (see Baumol et al. (1982)), requires that the price charged by the incumbent should be rigid for a given time period during which the potential entrants can enter and undercut freely. In fact, the model of two stage Bertrand competition developed by Tirole (1988) also requires a form of price rigidity. Since it is generally accepted that prices are much more flexible compared to other economic variables, such price rigidity limits, to some extent, the practical relevance of the theory of contestability. (See Schwartz (1986) for arguments along these lines.) It is therefore of some importance to formulate a model that is not subject to the above criticism.

We then study how the equilibrium outcome of such a model is affected as the level of the fixed cost is allowed to change. Notice that with an increase in the fixed cost, the cost structure moves towards the increasing returns case, and thus our analysis essentially examines the impact of changes in the returns to scale properties of the cost structure. For technical reasons we also assume that prices vary over a grid, and then look at the limiting outcome as the size of the grid becomes very small.

Let us now summarise our results. For small values of the fixed cost there is an interval of prices such that all prices belonging to this interval can be sustained as a Bertrand duopoly outcome (where both the firms supply a positive amount). For larger values of the fixed cost, the contestable outcome emerges as the unique equilibrium. Interestingly enough, for intermediate values of the fixed cost, there may be a case where the standard duopoly outcomes co-exist with the contestable outcome. Thus we find that as the cost function moves towards the increasing returns to scale case, the existence of a contestable outcome becomes more likely. Furthermore, we demonstrate that increasing returns to scale is not a necessary condition for the existence of a contestable outcome i.e. decreasing returns to scale may be compatible with contestability.

Finally, we relate our work to the existing literature on the subject. Maskin and Tirole (1988) provides a justification of the contestable outcome based on capacity commitments, rather than price rigidities. Grossman (1981), on the other hand, develops a model where the firms announce supply schedules rather than prices. While both these papers are of undoubted importance in providing alternative foundations for the contestable outcome, they do move away from the Baumol...
et al. (1982) intuition whereby the contestable outcome is seen as a generalisation of Bertrand equilibrium to markets with increasing returns. It is the aim of this paper to stick to this original intuition while trying to provide an appropriate foundation for the contestable outcome.

The paper closest to our own is by Ray Chaudhuri (1995) who examines a model of price competition with linear cost functions to demonstrate that the contestable outcome is obtained as the unique outcome. Ray Chaudhuri (1995), however, suffers from two limitations. First it ignores the interesting case of convex costs. Secondly, the model always yields the contestable outcome. There is no level of fixed cost that leads to the standard Bertrand duopoly outcome. The present paper rectifies both these problems, thus allowing us to provide an unified framework where either outcome may, depending on the cost structure, emerge as the equilibrium.

Furthermore, this paper also derives some additional results of interest. Namely we demonstrate that in some cases both kinds of outcomes may turn out to be equilibria. Moreover, we show that in this framework diminishing returns to scale may be consistent with contestability.

2. THE MODEL

Consider an industry comprising of two firms, 1 and 2. The market demand is $q = D(p)$, and the cost functions of the two firms are:

$$C_i(q_i) = \begin{cases} f + c(q_i), & \text{if } q_i > 0, \\ 0, & \text{otherwise}. \end{cases}$$

We make the following assumptions on the demand and the cost functions.

A.1: The demand function $D(p)$ is twice continuously differentiable, and intersects both axes, i.e. there are $p^{\max}$ and $q^{\max}$ such that $D(p^{\max})=0$ and $D(0)=q^{\max}$.

A.2: The variable cost $c(q)$ is twice continuously differentiable and strictly convex with $c(0)=0$ and $c'(q) \geq 0$.

We assume that the two firms play a Bertrand game, where the firms simultaneously announce their prices, and then meet the whole of the demand coming to them.

We then consider the demand going to the ith firm, $D_i(p_1, p_2)$, where the announced price vector is $(p_1, p_2)$. We define:

$$(1) \quad D_i(p_1, p_2) = \begin{cases} 0, & \text{if } p_i > p_j, \\ D(p)/2, & \text{if } p_i = p_j = p, \\ D(p_i), & \text{if } p_i < p_j. \end{cases}$$

Let us now define the profit functions $\pi(p)$ and $\hat{\pi}(p)$ as follows:
\[
\pi(p) = D(p)p - c(D(p)) ,
\hat{\pi}(p) = D(p)p/2 - c(D(p)/2) .
\]

From equation (1) we can see that \(\pi(p)\) denotes the gross profit of a firm that has undercut the other firm by charging a price of \(p\). Clearly the net profit is \(\pi(p) - f\). Similarly, \(\hat{\pi}(p)\) denotes the gross profit of a firm that exactly matches the price charged by the other firm, the net profit being \(\hat{\pi}(p) - f\).

We then assume that prices vary over a grid. Given that there is a smallest possible unit beyond which money is not divisible, this assumption is perhaps not too unrealistic. From a purely technical point of view, however, this assumption allows us to avoid some open set problems associated with this game. Let the set \(\mathcal{P} = \{p_0, p_1, \ldots\}\), where \(p_0 = 0\) and \(p_i = p_{i-1} + x\), constitute the set of permissible prices. We look at the limiting outcome of this game as \(x\) becomes small. The objective therefore is to approximate the more standard continuous formulation by a discrete one, and then examine the limiting outcome as the difference between the two formulations become small.

We begin by making a few more definitions.

(i) \(\hat{p}(f)\) denotes the minimum \(p\) such that \(\pi(p) = f\),
(ii) \(\check{p}(f)\) is the minimum \(p\) such that \(\check{\pi}(p) = f\),
(iii) \(p(f)\) is the maximum \(p\) such that \(\pi(p) = f\),
(iv) \(p(x)\) denotes the minimum \(p\) such that \(\pi(p) = \pi(p + x)\),
(v) \(p^*(x)\) denotes the minimum \(p \in \mathcal{P}\) such that \(\pi(p) - f \geq 0\), and
(vi) \(p^M = \text{argmax } \pi(p)\).

In order to simplify the algebra we assume, without loss of generality, that none of these critical values defined above belongs to \(\mathcal{P}\). Figure 1 provides a convenient diagrammatic representation of the above definitions.

A.3: The functions \(\pi(p)\) and \(\hat{\pi}(p)\) are strictly concave in \(p\). Moreover, \(\pi(p) \geq \hat{\pi}(p)\) if and only if \(p \geq p(0)\).

Notice that assumption A.3 is satisfied in the case where the demand function is linear and the cost function is quadratic. Clearly assumption A.3 ensures that \(p^M\) is unique and that there are no other \(p\), apart from \(\hat{p}(f)\) and \(\check{p}(f)\), such that \(\hat{\pi}(p) = f\). We then collect together, for the sake of ready reference, some of the results in Dastidar (1995) into Lemma 1.

**Lemma 1.**
(i) \(\hat{p}(0) < \check{p}(0) < p(0)\).
(ii) If \(f = 0\), then all firms quoting a price \(p \in [\check{p}(0), p(0)] \cap \mathcal{P}\) constitute a Bertrand equilibrium.

Notice that lemma 1(i) and 1(ii) follow from lemma 6 and Proposition 1 respectively in Dastidar (1995). Let us briefly sketch the proof of lemma 1(ii) here. Consider any \(p \in [\check{p}(0), p(0)] \cap \mathcal{P}\). Notice that at this \(p\) both the firms make non-negative profits. (See Fig. 1.) Thus deviation by charging a higher price is
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not profitable for either firm since it makes a profit of zero. Similarly undercutting is not profitable either. Suppose one of the firms undercuts by charging a price \( q < p \). Then its profit is \( \pi(q) < \pi(p) \leq \pi(p) \).

Lemma 2 below establishes some useful properties of \( \hat{p}(f) \), \( p(x) \) and \( p^*(x) \) that we use in the proofs of Propositions 1 and 2 (to follow).

**Lemma 2.** Assume that \( f > \pi(p(0)) \). Then there exists some \( \hat{x} > 0 \), such that \( \forall x \leq \hat{x} \):

(i) \( p(x) \) is such that \( \pi(p) \geq \tilde{p}(p + x) \), if and only if \( p \geq p(x) \),

(ii) \( p(x) < \hat{p}(f) \), and

(iii) \( \pi(p^*(x)) - f < 0 \).

Notice that Lemma 2(i) follows from assumption A.3. We then consider Lemmas 2(ii) and 2(iii). Let us first consider the case where \( p^M > p(0) \). In this case, for \( x \) small, \( p(x) > p(0) \) and \( p(x) \) goes to \( p(0) \) as \( x \) goes to zero. Since for \( f > \pi(p(0)) \), \( p(f) > \hat{p}(f) \), Lemma 2(ii) follows. Next notice that for \( p^M > p(0) \) and \( f > \pi(p(0)) \), it is the case that \( \hat{p}(f) > \hat{p}(f) \). Thus for \( \pi(p(f) - \hat{p}(f) \), we have that \( \hat{p}(f) \leq p^*(x) < \hat{p}(f) \), and hence Lemma 2(iii) follows. Next consider the case where \( p^M < p(0) \). In this case for \( x \) small, \( p(x) < p(0) \). Since in this case \( p(0) < \hat{p}(f) \), Lemma 2(ii) follows. Finally observe that in this case \( \pi(p) \) is negatively sloped for all \( p \geq p(0) \). Since \( p^*(x) \geq p(0) \), Lemma 2(iii) follows.
We are finally in a position to state and prove the main results of this paper. The cases where \( p^M > p(0) \) and \( p^M < p(0) \) are considered separately. Which one of these cases prevails depends on the demand and the cost parameters. In the example where \( q = b(1 - p) \) and \( C(q) = f + eq^2 \), it is easy to see that \( p^M = (1 + bc)/(2 + bc) \) and \( p(0) = 3bc/(2 + 3bc) \). Thus in this case \( p^M < p(0) \) if and only if \( 2 < bc \).

In Proposition 1 we examine the case where \( p^M > p(0) \).

**Proposition 1.** Assume that \( p^M > p(0) \) and define \( \hat{\alpha} \) as in Lemma 2. Then, for all \( x < \hat{\alpha} \),

(i) if \( f < \pi(\hat{\alpha}) \), all prices \( p \in [\hat{\alpha}, p(0)] \cap \mathcal{P} \), can be sustained as a Bertrand duopoly equilibrium, and

(ii) if \( f > \pi(\hat{\alpha}) \), then the outcome involves one of the firms (say firm 1) charging a price of \( p^*(x) \) and supplying the whole of the demand, while the other firm (say firm 2) charges a price of \( p^*(x) + x \) and obtains no demand. (See Fig. 1.)

**Proof.** (i) The proof simply mimics that of Lemma 1(ii).

(ii) Existence: We first consider firm 1. Notice that under the outcome specified in the proposition the profit of firm 1 is \( \pi(p^*(x)) - f \geq 0 \). We then examine if firm 1 can deviate and gain. Clearly, if it charges a price of \( p^*(x) + x \), then its profit is \( \hat{\pi}(p^*(x) + x) - f \), which is strictly less than its equilibrium profit, \( \hat{\pi}(p^*(x)) - f \). (This follows from Lemma 2(i) and the fact that \( p^*(x) \geq \hat{p}(f) > p(x) \), where the last inequality is given by Lemma 2(ii).) If firm 1 charges an even higher price then its profit is zero. If, however, it charges a price, \( p \), that is strictly less than \( p^*(x) \) then its profit is \( \pi(p) - f \), which, from the definition of \( p^*(x) \), is strictly negative.

Next consider firm 2. Clearly its profit under the given outcome is zero. If it charges any price greater than \( p^*(x) + x \), even then its profit is zero. If it charges a price of \( p^*(x) \) then its profit is \( \hat{\pi}(p^*(x)) - f \), which from Lemma 2(iii) is strictly negative. Finally, if it charges a price \( p \), that is strictly less than \( p^*(x) \), then its profit is \( \pi(p) - f < 0 \).

**Uniqueness.** We begin by arguing that there are no equilibria where both the firms charge the same price. Suppose to the contrary that there is, and let the price charged by \( p^d \). Clearly, \( p^d > p^*(x) \). Because if not, then both the firms have a negative profit. This follows because \( \hat{\pi}(p^*(x)) - f < 0 \), and \( \forall p < p^*(x), \hat{\pi}(p) < \hat{\pi}(p^*(x)) \). But if \( p^d > p^*(x) \), then one of the firms can undercut and make a gain. This follows since \( p^d > p^*(x) > \hat{p}(f) > p(x) \) and from Lemma 2(i) this implies that \( \pi(p^d - x) > \hat{\pi}(p^d) \).

We then examine the case where the two firms charge different prices. Without loss of generality let firm 1 charge the lower price, \( p_1 \), and let firm 2 charge the higher price, \( p_2 \). Clearly, \( p_1 \geq p^*(x) \), since otherwise firm 1 will have a negative profit. First consider the case where \( p_1 > p^*(x) \). But then firm 2 can undercut by charging \( p_1 - x \), and earn a strictly positive profit, whereas its profit from charging \( p_2 \) is zero. Next consider the case where \( p_1 = p^*(x) \), but \( p_2 > p^*(x) + x \). If \( p_2 \leq p' \), where \( p' = \text{argmax} \pi(p) \), then firm 1 can charge \( p' - x \), and increase its profit. Similarly
if $p_2 > p'$, then firm 1 can increase its price to a level that is less than $p_2$ but yields the maximum possible profit in the interval $(p^*(\alpha), p_2)$, and increase its profit.

Thus for low values of f, any price $p$ in the interval $[\hat{p}(f), p(0)] \cap \mathcal{P}$ can be sustained as a Bertrand equilibrium. Since as $f$ increases, $\hat{p}(f)$ is increasing in $f$, this interval continues to shrink until, for $f = \pi(p(0))$, the interval reduces to the single point $p(0) (= \hat{p}(f))$. For any $f > \pi(p(0))$, the contestable outcome emerges as the unique equilibrium. Hence as $f$ increases i.e. as the cost curve moves towards the increasing returns case, the market equilibrium moves towards the contestable outcome.

Out next proposition is concerned with the case where $p^M < p(0)$.

PROPOSITION 2. Assume that $p^M < p(0)$ and define $\hat{\alpha}$ as in Lemma 2. Then for all $\alpha < \hat{\alpha}$,

(i) if $f < \pi(p(0))$, all prices $p \in [\hat{p}(f), p(0)] \cap \mathcal{P}$, can be sustained as a Bertrand duopoly equilibrium,

(ii) if $\pi(p(0)) < f < \hat{\pi}(p^M)$, then all $p \in [\hat{p}(f), \hat{p}(f)] \cap \mathcal{P}$ can be sustained as Bertrand duopoly. Moreover there is a contestable outcome where one of the firms charges a price of $p^*(\alpha)$ and supplies the whole of the demand, while the other firm charges a price of $p^*(\alpha) + \alpha$, and obtains no demand, and

(iii) if $f > \hat{\pi}(p^M)$, then the unique equilibrium involves one of the firms charging
a price of \( p^a(z) \) and supplying the whole of the demand, while the other firm charges a price of \( p^a(z) + z \), and obtains no demand. (See Fig. 2.)

The proof is similar to that of Proposition 1. The Bertrand duopoly outcomes can be sustained in a manner similar to that of Lemma 1(ii). The existence proof of the contestable outcome in both the cases is the same as in Proposition 1.

Next notice that as \( z \) goes towards zero, both \( p^a(z) \) and \( p^a(z) + z \) goes towards \( \tilde{p}(f) \). Thus is case of Proposition 2(ii) in the limit (as \( z \) goes to zero) the net profit under the contestable outcome is zero. Whereas the net profit under Bertrand duopoly is strictly positive for all prices in the interval \( [\tilde{p}(f), \bar{p}(f)] \) except at the two end-points \( \tilde{p}(f) \) and \( \bar{p}(f) \), where the profits are zero. Thus for \( z \) close to zero the Bertrand duopoly outcome generally payoff dominates the contestable outcome. Thus if the firms can communicate among themselves we can, perhaps, expect that a duopoly outcome would emerge.

Thus for a large enough \( f \), as \( z \) goes to zero, there is average cost pricing, with a single firm supplying the whole of the demand and earning a profit of zero. This vindicates the argument that contestability can be justified as a generalisation of Bertrand competition to markets with increasing returns to scale.

In our next proposition we formally relate the cost structure of the two firms to the existence of a contestable outcome. We begin with a definition.

**Lemma 1.** Let \( q(f) = D(p(f)) \). We say that a cost function satisfies the **Scale Property (SP)** if, for \( q = y(f) \), \( \frac{d(A \cdot C)}{dq} < 0 \), i.e. if the average cost curve \( AC \) is negatively sloped at the maximum output level for which \( p = AC \).

**Proposition 3.** SP is a sufficient condition for the existence of a contestable outcome. However, it is not necessary.

**Proof.**

**Sufficiency:** We begin by proving the sufficiency condition. Since \( AC(q) = f/q + c(q)/q \), \( d(AC)/dq = \frac{-f-c(q)+q \cdot c'(q)}{q^2} \). We then calculate \( d(AC)/dq \) at the output level \( q(f) \). Notice that at this output level \( f = p(f) \cdot y(f) - c(q(f)) \) and hence \( d(AC)/dq < 0 \), if and only if \( p(f) > c'(y(f)) \).

Next define \( q(0) = D(p(0)) \). Since \( p(0) \) satisfies the relation

\[
\pi(p(0)) = \pi(p(0)), \text{ we can write:}
\]

\[
p(0) = \frac{c(q(0)) - c(q(0)/2)}{[q(0)/2]}.
\]

Since \( c(q) \) is a differentiable convex function we can also write:

\[
\frac{c(q(0)) - c(q(0)/2)}{[q(0)/2]} < c'(q(0)).
\]

Combining the above two equations we have \( p(0) < c'(q(0)) \). We then claim that \( \tilde{p}(f) > p(0) \). Suppose not i.e. let \( \tilde{p}(f) \leq p(0) \). But this implies that \( \tilde{q}(f) \geq q(0) \), and since the cost function is convex, \( c'(\tilde{q}(f)) \geq c'(q(0)) \). Thus we have that

\[
p(0) < c'(q(0)) \leq c'(\tilde{q}(f)) < p(0) .
\]

But this is a contradiction. Hence we must have that \( \tilde{p}(f) > p(0) \), which, from
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Propositions 1 and 2 implies that a contestable outcome exists.

Non-necessity: The proof is through an example. Let us assume that the demand function is \( q = (1 - p) \), and the cost function is \( f + cq^2 \). Straightforward calculations now yield that

\[
p(0) = \frac{3c}{2 + 3c}, \quad \text{and} \quad q(0) = 1 - p(0) = \frac{2}{2 + 3c}.
\]

Evaluating at the point \( q = q(0) \), we can write that

\[
d(AC)/dq = \frac{-f + c(q(0))^2}{(q(0))^2}.
\]

Thus if \( f = p(0) \cdot q(0) - c(q(0))^2 \), then

\[
d(AC)/dq = \frac{[2cq(0) - p(0)]/q(0)}{c/2 > 0}.
\]

Notice that for any \( f \) slightly greater than \( p(0) \cdot q(0) - c(q(0))^2 \), we have that \( p(f) > p(0) \), so that a contestable outcome exists. Furthermore, from continuity, \( d(AC)/dq > 0 \) would hold, so that the scale property (SP) does not hold.

Thus this proposition formalises the idea that the contestable outcome is a generalisation of Bertrand competition to the case of increasing returns to scale. The non-necessity part is somewhat surprising. Tirole (1988), for example, argues that (page 310) if increasing returns to scale does not hold then a contestable outcome may not exist. On reflection, however, the discrepancy between the two results can be traced to the assumption that the firms have to meet the whole of the demand coming to them. Notice that our assumption makes undercutting more costly (since with convexity costs rise disproportionately with an increase in output), thus making for existence. (Let us notice that a similar approach is taken in Grossman (1981) where he assumes that the strategies of the firms consist of supply schedules rather than prices, which in effect, makes undercutting more difficult.) This suggests that this part of the result may not go through under an alternative model of Bertrand competition where the firms can supply less than the amount demanded.

3. CONCLUSION

Our analysis succeeds in unifying the standard Bertrand duopoly outcome and the contestable outcome under a single framework. We demonstrate that depending on the level of fixed costs either kind of market structure may emerge as the outcome. The model used here involves convex costs, as well as flexible prices, thus demonstrating that both these assumptions are consistent with contestability. Furthermore, we demonstrate that our formulation allows us to sustain a contestable outcome even under decreasing returns to scale, thus showing that contestability may be robust to decreasing returns to scale.

Finally recall that in our model the firms supply the whole of the demand coming to them. What happens when we allow the firms to supply less than the
quantity demanded is an open question. However, we conjecture that for ‘f’ large enough, the contestable outcome would again emerge as the outcome. This is because for a large enough ‘f’, the cost structure (over the relevant range) would be of the increasing returns type. Thus the firms would prefer to supply the whole of the demand, and our analysis would apply. Moreover, notice that the assumption that firms supply the whole of the demand, could be justified by appealing to reputational effects, or to government regulations. Alternatively we may assume that the pricing mechanism could be approximated by a sealed bid auctions process.

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