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EXISTENCE OF EQUILIBRIUM FOR COURNOT OLIGOPOLY-OLIGOPSONY

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Abstract: Oligopolists in a product market are more often than not oligopsonists in a factor market. No economist, however, has analyzed oligopolistic, oligopsonistic industry. This paper is the first attempt to tackle this type of industry characterized by product as well as factor market imperfection. Specifically we prove the existence of a unique equilibrium for Cournot oligopoly-oligopsony without product differentiation. We reduce the existence problem to a fixed-point problem for a function whose only variable is the industry output.

JEL Classification Nos: D43, L13.
Key words: existence of equilibrium, Cournot, oligopoly, oligopsony, market imperfection.

1. INTRODUCTION

In present day economies, oligopolists in product markets are very often oligopsonists in factor markets. To the best of the author's knowledge, however, no economist has formulated and analyzed oligopoly involving oligopsony. Since Theocharis (1959), many papers have appeared on the existence and stability of the Cournot equilibrium for oligopoly facing perfectly competitive factor market.[1] The common feature of these papers is the use of cost function, which is untenable if the factor market is imperfectly competitive. Naylor (1994) has analyzed Cournot oligopsony facing perfect competition in a product market. He has assumed constant elasticity of supply functions for two kinds of labor, one of which belongs to a discriminated group. He has, in addition, considered symmetric firms with identical linear production function and assumed that each firm maximizes its utility instead of its profits.

It this paper we will be concerned with the existence of Cournot equilibrium for oligopoly where firms face market imperfection in both product and factor markets. We assume that each firm uses labor and capital as its inputs and maximizes its profits under the Cournot assumption regarding its rival's factor

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[1] The first paper on the existence is Frank and Quandt (1963).

inputs. We will derive as set of sufficient conditions for the existence of a unique Cournot oligopoly-oligopsony with firms which are not necessarily symmetric. We will reduce the existence problem to a fixed point problem for a function involving the industry output as its sole variable. A similar approach has been adopted by Szidarovszky and Yakowitz (1977) in proving the existence of a unique Cournot oligopoly equilibrium in perfect factor market. It has been used extensively also by Okuguchi (1993) to analyze various Cournot models. If the factor market is perfectly competitive, any firm's cost becomes a function of its output. Hence in Cournot oligopoly facing perfectly competitive factor market, any firm maximizes its profit optimally adjusting a single variable, namely its output. On the other hand, if the factor market is imperfectly competitive and there are two factors of production, labor and capital, as in our model, any firm has to maximize its profits choosing optimal values for two variables, labor and capital, which necessarily makes our existence analysis more complicated than that for Cournot oligopoly in perfectly competitive factor market. However, our existence theorem provides as its special case an alternative proof for the existence of Cournot oligopoly equilibrium in the presence of perfectly competitive factor market.

2. THE MODEL AND ANALYSIS

Let there be n firms which are oligopolists in the product market and at the same time oligopsonists in the factor market. All firms are assumed to be producing one identical good with the help of labor and capital. Firm i's production function is given by

(1) $$x_i=f^i(L_i, K_i)\,, \quad i=1,2,\cdots,n\,,$$

where x_i, L_i, and K_i are its output, labor and capital inputs, respectively. In the following analysis we assume continuous differentiability of necessary orders of all the relevant functions.[2] We assume also the production function to be strictly concave. Let suffixes 1 and 2 to f^i represent partial derivatives of f^i with respect to the first argument L_i and the second one K_i, respectively. Then

(A.2.1) $$f^i_{11}<0\,, \quad f^i_{11}f^i_{22}-f^i_{12}f^i_{21}>0\,, \quad i=1,2,\cdots,n\,,$$

(A.2.2) $$(f^i_1)^2f^i_{22}+(f^i_2)^2f^i_{11}-2f^i_1f^i_2f^i_{12}<0\,, \quad i=1,2,\cdots,n\,,$$

where an A-prefix refers to the assumption. We will adopt the same convention also in the following analysis to distinguish the assumptions from definitions or claims.

Let $Q\equiv\Sigma x_i$ be the total industry output, and $p=p(Q)$, $p'<0$, be the inverse demand function for the output. Let $w=w(L)$ and $r=r(K)$ be the wage rate and

[2] Without this assumption the left-hand sides of (6.1) and (6.2) are not continuously differentiable, which prevents us from applying the implicit function theorem to derive (11.1) and (11.2) as continuously differentiable functions, consequently all the ensuing arguments based on (11.1) and (11.2) become invalid.

rental of capital, where L and K are the total labor and capital, respectively. If imperfect competition prevails in the product as well as in the factor markets, firm i's profit function π^i is defined by

$$\pi^i \equiv p\left(\sum_j f^j(L_j, K_j)\right) f^i(L_i, K_i) - wL_i - rK_i , \quad i=1, 2, \cdots, n . \tag{3}$$

Before proceeding further, we introduce the following assumptions.

(A.4.1) $\quad MR^i \equiv p + x_i p' > 0 , \quad i=1, 2, \cdots, n .$

(A.4.2) $\quad p' + x_i p'' < 0 , \quad i=1, 2, \cdots, n .$

(A.5.1) $\quad w' \geq 0 , \quad w'' \geq 0 .$

(A.5.2) $\quad r' \geq 0 , \quad r'' \geq 0 .$

If (A.4.1) holds, any firm's marginal revenue with respect to its own output is positive but decreasing with respect to an increase in any other firm's output[3] in light of (A.4.2). According to (A.5.1) and (A.5.2), the wage rate and rental are non-decreasing in L_i and K_i, respectively, and the marginal labor and capital costs are both non-decreasing. Assumptions (A.4.1) and (A.4.2) have been widely used in the stability analysis on the Cournot oligopoly equilibrium in the absence of factor market imperfection (see Okuguchi (1976), and Okuguchi and Szidarovszky (1990)).

Suppose now that each firm have Cournot-type expectations regarding other firms' factor inputs. Assuming away the corner solution,[4] we get the following first order conditions for profit maximization for firm i, where $f_1^i \equiv \partial f^i/\partial L_i$, $f_2^i = \partial f^i/\partial K_i$.

$$\begin{aligned} \partial\pi^i/\partial L_i &\equiv \pi_1^i \\ &= \{p(Q) + f^i(L_i, K_i)p'(Q)\} f_1^i(L_i, K_i) - (w(L) + L_i w'(L)) \\ &= 0 , \qquad i=1, 2, \cdots, n , \end{aligned} \tag{6.1}$$

[3] Note that under (A.4.2), $\partial(MR^i)/\partial x_i < 0$.

[4] Let $\pi_i = \pi_i(L_i, K_i, (L_i, K_i)_{-i})$ for the sake of notational simplicity, where

$$(L_i, K_i)_{-i} = (L_1, K_1 ; \cdots, L_{i-1}K_{i-1} ; L_{i+1}, K_{i+1} ; \cdots, L_n, K_n) .$$

The Cournot equilibrium is characterized as a set of vectors (L_i^*, K_i^*)s such that

$$\pi_i(L_i^*, K_i^* ; (L_i^*, K_i^*)_{-i} \geq \pi_i(L_i, K_i ; (L_i^*, K_i^*)_{-1}) \qquad \text{for all } L_i \text{ and } K_i, \quad i=1, 2, \cdots, n .$$

If the corner solution is allowed, the Cournot equilibrium is identical to the solution of the following non-linear complementarity problem. Find the solution to

$$\begin{aligned} &\partial\pi_i/\partial L_i \leq 0 , \quad L_i\partial\pi_i/\partial L_i = 0 , \quad L_i \geq 0 , \\ &\partial\pi_i/\partial K_i \leq 0 , \quad K_i\partial\pi_i/\partial K_i = 0 , \quad K_i \geq 0 , \end{aligned} \qquad i=1, 2, \cdots, n .$$

The complementarity problem approach has been adopted by Okuguchi (1983) to prove the existence of the equilibrium for Cournot oligopoly in the absence of factor market imperfection.

$$\partial \pi^i/\partial K_i \equiv \pi_2^i$$
$$(6.2) \qquad = \{p(Q)+f^i(L_i, K_i)p'(Q)\}f_2^i(L_i, K_i)-(r(K)+K_i r'(K))$$
$$=0, \qquad i=1, 2, \cdots, n.$$

Let $\partial(MR_i^i)/\partial x^i \equiv MR_i^i$. Then in light of (A.2), (A.4) and (A.5) we have

$$(7.1) \qquad \pi_{11}^i = MR_i^i(f_1^i)^2 + MR^i f_{11}^i - (2w' + L^i w'') < 0, \qquad i=1, 2, \cdots, n.$$

$$(7.2) \qquad \begin{vmatrix} \pi_{11}^i & \pi_{12}^i \\ \pi_{21}^i & \pi_{22}^i \end{vmatrix} = MR_i^i MR^i\{(f_1^i)^2 f_{22}^i + (f_2^i)^2 f_{11}^i - 2f_1^i f_2^i f_{12}^i\}$$
$$+(MR^i)^2(f_{11}^i f_{22}^i - (f_{12}^i)^2) - (MR_i^i(f_1^i)^2 + MR^i f_{11}^i)(2r' + K_i r'')$$
$$-(MR_i^i(f_2^i)^2 + MR^i f_{22}^i)(2w' + L_i w'') + (2w' + L_i w'')(2r' + K_i r'') > 0,$$
$$i=1, 2, \cdots, n.$$

where $f_{jj'}^i$ is the partial derivative of f_j^i with respect to the j'-th argument, $j, j'=1, 2$, and $\pi_{jj'}^i$ denotes the partial derivative of π_j^i (the partial derivative of π^i with respect to the j-th argument of f^i) with respect to the j'-th argument of f^i, $j, j'=1, 2$. Hence the second order condition is satisfied.

Observe that the variables L_i, K_i, L, K, and Q appear in (6.1) and (6.2), and totally differentiate them to get

$$(8.1) \qquad (p'(f_1^i)^2 + MR^i f_{11}^i - w')dL_i + (p'f_1^i f_2^i + MR^i f_{12}^i)dK_i$$
$$= -(p' + f^i p'')f_1^i dQ + (w' + L_i w'')dL, \qquad i=1, 2, \cdots, n,$$

$$(8.2) \qquad (p'f_1^i f_2^i + MR^i f_{21}^i)dL_i + (p'(f_2^i)^2 + MR^i f_{22}^i - r')dK_i$$
$$= -(p' + f^i p'')f_2^i dQ + (r' + K_i r'')dK, \qquad i=1, 2, \cdots, n.$$

The coefficient of dL_i in (8.1) and that of dK_i in (8.2) are negative, but the sign of the coefficient of dK_i in (8.1) and that of dL_i in (8.2) are indeterminate. We therefore assume that

$$(A.9) \qquad p'f_1^i f_2^i + MR^i f_{12}^i > 0, \qquad i=1, 2, \cdots, n.$$

This assumption holds if the product market is perfectly competitive. Let the determinant of the coefficient matrix of (8.1) and (8.2) be Δ_i. It is easy to see that

$$(10) \qquad \Delta_i = p'MR^i((f_1^i)^2 f_{22}^i + (f_2^i)^2 f_{11}^i - 2f_1^i f_2^i f_{12}^i) - (p(f_1^i)^2 + MR^i f_{11}^i)r'$$
$$-(p'(f_2^i)^2 + MR^i f_{22}^i)w' + (MR^i)^2(f_{11}^i f_{22}^i - (f_{12}^i)^2) + r'w' > 0,$$
$$i=1, 2, \cdots, n.$$

We can therefore solve (6.1) and (6.2) uniquely with respect to L_i and K_i as continuously differentiable functions of L, K, and Q, namely

$$(11.1) \qquad L_i \equiv g^i(L, K, Q), \qquad i=1, 2, \cdots, n,$$

$$(11.2) \qquad K_i \equiv h^i(L, K, Q), \qquad i=1, 2, \cdots, n,$$

where the partial derivatives of g^i and h^i have the following signs by virtue of (8), (A.2), (A.4) and (A.5).

(11.1') $$g^i_L \leq 0\,,\quad g^i_k \leq 0\,,\quad g^i_Q < 0\,,\qquad i=1,2,\cdots,n\,.$$

(11.2') $$h^i_L \leq 0\,,\quad h^i_K \leq 0\,,\quad h^i_Q < 0\,,\qquad i=1,2,\cdots,n\,.$$

We note that equalities in (11.1') and (11.2') hold if $w'=r'=0$, *i.e.* the factor market is perfectly competitive.

By definition, L_is and K_is must satisfy

(12.1) $$L=\sum_i g^i(L,K,Q)\equiv g(L,K,Q)\,,$$

(12.2) $$K=\sum_i h^i(L,K,Q)\equiv h(L,K,Q)\,,$$

where (11.1') and (11.2') yield

(12.1') $$g_L \leq 0\,,\quad g_K \leq 0\,,\quad g_Q > 0\,.$$

(12.2') $$h_L \leq 0\,,\quad h_K \leq 0\,,\quad h_Q < 0\,.$$

Given K and Q, the function $g(L, K, Q)$ can be depicted as a downward-sloping curve or a horizontal line as in Fig. 1. Hence there exists a unique L satisfying

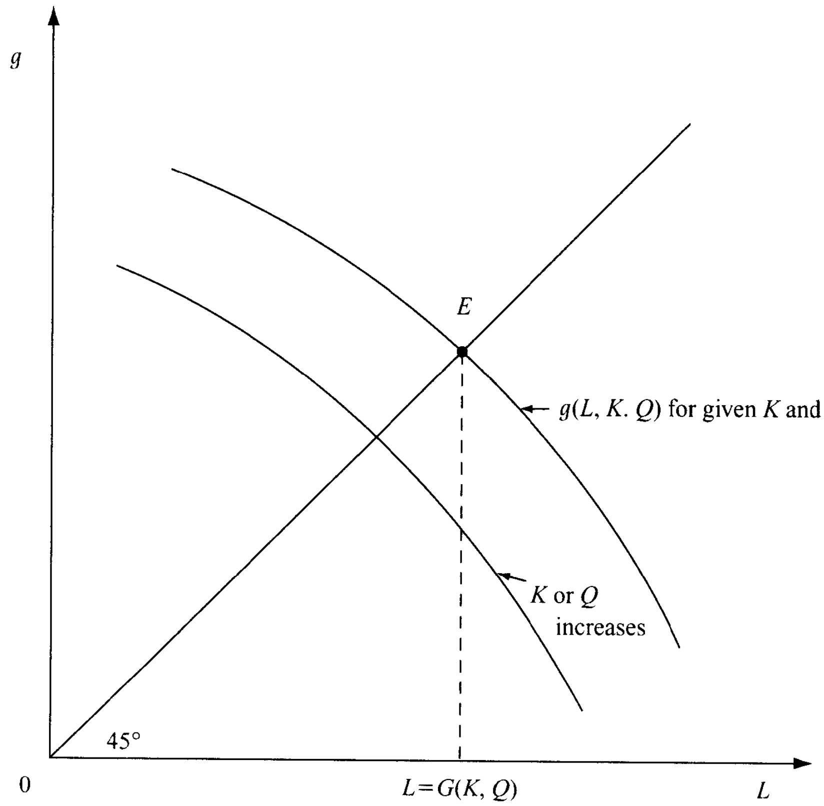

Fig. 1. Determination of L satifying (12.1).

(12.1) for given K and Q. This L corresponds to the intersection E of the 45 degree line and the curve as in Fig. 1. The curve shifts downward or does not move in the event of an increase in K, as a consequence of this, the intersection moves downward along the 45 degree line or does not move if K increases. If Q increases, the curve shifts downward and the intersection moves downward along the 45 degree line. Thus we have

(13.1) $$L \equiv G(K, Q), \quad G_k \leq 0, \quad G_Q < 0 .$$

Applying a similar argument to (12.2), we derive

(13.2) $$K \equiv H(L, Q), \quad H_L \leq 0, \quad H_Q < 0 .$$

Solving (13) with respect to L and K, we have

(14.1) $$L \equiv L^*(Q),$$

(14.2) $$K \equiv K^*(Q),$$

where we assume that

(A.15.1) $$dL^*/dQ = (g_Q(1-h_K) + h_Q g_K)/\delta < 0 ,$$

(A.15.2) $$dK^*/dQ = (h_Q(1-g_L) + g_Q h_L)/\delta < 0 ,$$

(A.16) $$\delta \equiv (1-g_L)(1-h_K) - g_K h_L > 0 .$$

It is clear that these three assumptions are satisfied if the factor market is perfectly competitive. To clarify the general validity of (A.16), introduce a sequential algorithm (17) for computing (14.1) and (14.2) for arbitrarily given Q.

(17.1) $$dL/dt = \alpha_1(g(L, K, Q) - L) ,$$

(17.2) $$dK/dt = \alpha_2(h(L, K, Q) - K)$$

where t denotes time, and a_1 and a_2 are positive constants. According to the theorem of Olech (1963), L and K converges globally to $L^*(Q)$ and $K^*(Q)$, respectively if $g_L - 1 < 0$ and if, in addition, (A.16) holds. The first condition certainly holds in the light of the first inequality in (12.1').

Define a function $F(Q)$ by

(18) $$F(Q) \equiv \sum_i f^i(g^i(L^*(Q), K^*(Q), Q), h^i(L^*(Q), K^*(Q), Q)) .$$

Then the Cournot equilibrium industry output is identical to the solution of a fixed point problem

(19) $$Q = F(Q) .$$

Differentiating $F(Q)$ with respect to Q and arranging, we have in light of (A.15),

(20) $$F'(Q) \equiv \{(g_Q(1-h_K) + h_Q g_K)/\delta\} \sum_i (f_1^i g_L^i + f_2^i h_L^i)$$

$$+\{(h_Q(1-g_L)+g_Qh_L)/\delta\}\sum_i(f_1^ig_K^i+f_2^ih_K^i)$$
$$+\sum(f_1^ig_Q'+f_2^ih_Q')<0\,.$$

The sign of F' is indeterminate in general since $f_1^ig_m^i+f_2^ih_m^i<0$, $m=L, K, Q$ and the two expressions between the braces are negative in light of (A.15.1) and (A.15.2). If F' is negative (the case of Fig. 2), the 45 degree line and the downward-sloping curve for $F(Q)$ intersect uniquely at E, establishing the existence of a unique solution of (19). Even if $F'=0$ or $0<F'<1$, the curve for $F(Q)$ has a unique intersection with the 45 degree line, hence there exists a unique solution of (19). Summarizing, we have the following

THEOREM: *Under our assumptions, if $F'<1$, there exists a unique Cournot oligopoly-oligopsony equilibrium.*

Consider next a simple case where $f^i=f$ for all i, i.e. each firm has an identical production function. In this case, (20) is simplified as

(21) $$F'(Q)=\{g_Q(1-h_K)+h_Qg_K\}/_\delta+\{h_Q(1-g_L)+g_Qh_L\}/_\delta<0\,.$$

Hence, the condition for the theorem is satisfied, leading to

COROLLARY 1: *If all firms have identical production functions, there exists a unique Cournot equilibrium under our assumptions.*

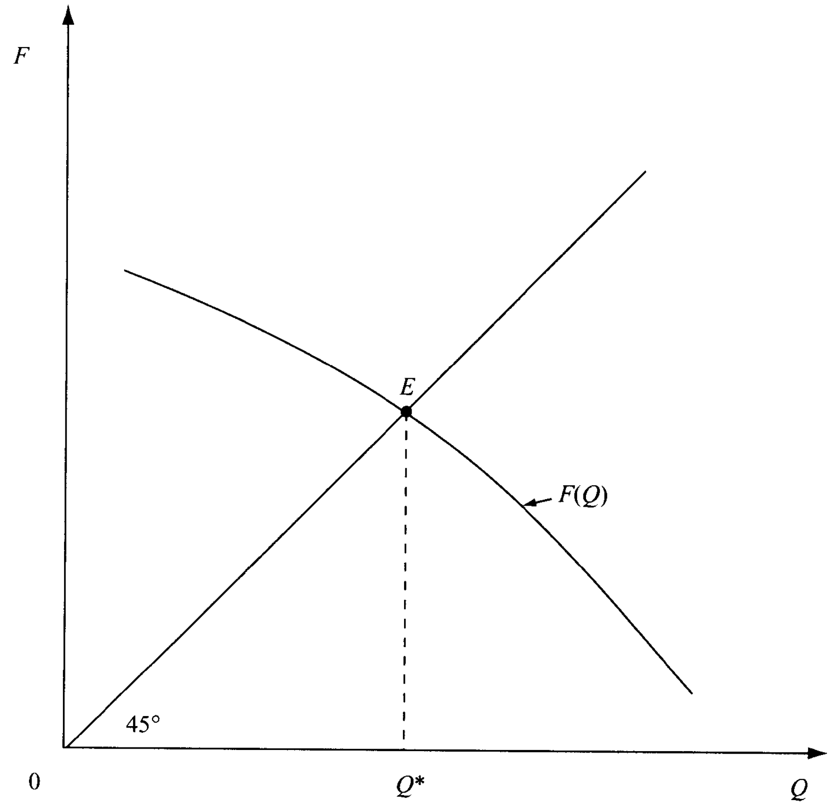

Fig. 2. Unique Cournot industry equilibrium output.

If the factor market is perfectly competitive,

(22) $$F'(Q)=\sum(f_1^i g_Q^i + f_2^i h_Q^i)<0\ .$$

Hence the following.

COROLLARY 2: *If the factor market is perfectly competitive, there exists a unique Cournot equilibrium under our assumptions even if the firms' production functions are not identical.*

The existence of the Cournot oligopoly equilibrium has been proved by Frank and Quandt (1963), Szidarovszky and Yakowitz (1977) and Okuguchi (1976), among others, using the cost function, which is valid only if the factor market is perfectly competitive. Our Corollary 2 provides an alternative proof of the existence of the Cournot oligopoly equilbrium, which does not depend on the cost function.

3. CONCLUSION

In this paper we have proved that under a set of assumptions, there exists a unique equilbrium for Cournot oligopoly-oligopsony. We have reducted the existence problem to a fixed point problem for a function which contains the industry output as its only variable. Our main result is stated as Theorem in Section 2, whose general validity is not clear at first glance. To avoid this difficulty, we have considered two simple cases, in one of which all firms' production functions are identical and in the other, the factor market is perfectly competitive. The latter case provides an alternative proof of the existence of the Cournot oligopoly equilibrium under perfectly competitive factor market. We have not analyzed in this paper the stability of the Cournot oligopoly-oligopsony equilibrium. The reader is referred to Chiarella and Okuguchi (1996, 1997) for this analysis for Cournot duopoly-duopsony with only one factor of production.

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