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COURNOT VS STACKELBERG: THE CASE OF LABOR-MANAGED DUOPOLY

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Abstract: First, the equilibrium properties are compared for general Cournot and Stackelberg two-person games with symmetric payoff functions. Second, labor-managed Cournot and Stackelberg duopoly equilibria are compared. Under our assumptions, the Cournot firm's output is larger than the Stackelberg follower's output, which in turn is larger than the leader's output. Dividend per unit of labor is larger for the Stackelberg follower than for the Stackelberg leader, which in turn gets larger dividend per unit of labor than the Cournot firm.

Key words: Cournot, Stackelberg, labor-managed duopoly.

1. INTRODUCTION

Stackelberg (1934) considered three duopoly models, in one of which two firms in duopoly both strive for leadership; in the second one firm becomes a leader and the other a follower; in the third case two firms both want to be followers. The second case is usually known as Stackelberg duopoly. Until several years ago, most economists took it for granted that the leader would gain more profits than the follower in Stackelberg duopoly. However, Krelle (1976) showed by way of a simple model that if duopolists' products are differentiated and if, in addition, duopolists are price setters rather than output setters, the follower which sets its price in response to the leader's price will get larger profits than the leader. Gal-Or (1985) derived a general condition for the first mover (leader) to be more advantageous or disadvantageous than the second mover (follower) in a general Stackelberg game with symmetric payoff functions, which are not necessarily profit functions.

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A labor-managed firm tries to maximize dividend per unit of its labor, which is the sum of the prevailing market wage rate and profit per unit of its labor. Labor-managed firms were prevalent in the former Yugoslavia. They are found also in many European countries and even in the United States. Large Japanese firms are also considered to be labor-managed since their profits are partly distributed as bonuses and payments in kind in various forms to their employees.[1] Okuguchi (1993c) analyzed relative advantage or disadvantage of the Stackelberg leader in symmetric labor-managed duopoly with product differentiation. He has found that under reasonable assumptions, the follower is more advantageous than the leader in both quantity-setting and price-setting labor-managed Stackelberg duopolies with product differentiation. Neither Gal-Or nor Okuguchi, however, compared equilibria for Cournot and Stackelberg games or duopolies. In this paper we will fill this gap. In Section 2, we consider a general two-person game with symmetric payoff functions. In Section 3 and 4, we consider labor-managed duopoly without product differentiation, where the firm's payoff is dividend per unit of its labor. In Section 3, we will prove the follower's relative advantage on the basis of Gal-Or's result. In Section 4, we will compare the equilibria for labor-managed Cournot and Stackelberg duopolies. Section 5 concludes. In the following analysis we have to introduce a few assumptions which play key roles in deriving our results. We will indicate in footnote 5 which assumption is responsible for which result.

2. COURNOT AND STACKELBERG

Let there be two players with symmetric payoff functions. Let, as in Gal-Or (1985), the first and second players be the leader (first mover) and follower (second mover), respectively, in a Stackelberg game and $g(x_1)$ be the reaction function for the follower and

$$G(x_1) \equiv \pi^1(x_1, g(x_1)) = \pi^1(x_1, x_2)\,, \tag{1}$$

where π^i and x_i are the payoff function and strategy for the i-th player, respectively. We denote the Stackelberg equilibrium by S over a variable. Then the first order condition for the leader is

$$G'(x_1^s) = 0\,. \tag{2}$$

We assume that[2]

$$G''(x_1) < 0\,, \tag{A.3}$$

[1] See Ireland and Law (1982), and Bonin and Putterman (1987) for comprehensive treatments of labor-managed firms. See also Okuguchi (1993a) for the most recent development.

[2] As Gal-Or (1985) observes, the general validity of the second order condition is not clear. As pointed out by a referee, this assumption may be replaced with the assumption of the uniqueness of the Stackelberg equilibrium. However, if (A.3) is dropped, the results (i)–(iv) below are not derivable.

where "A" in (A.3) refers to an assumption. We use similar convention in the following analysis. Under (A.3), the second order condition for the leader is satisfied.

Next, we consider the Cournot game, whose equilibrium is denoted by C over a variable. The first order conditions for the Cournot equilibrium are

$$\pi_1^1(x_1^C, x_2^C)=0\,, \tag{4.1}$$

$$\pi_1^2(x_2^C, x_1^C)=0\,, \tag{4.2}$$

where $x_1^C=x_2^C$, $x_1^C=g(x_2^C)$ and $x_2^C=g(x_1^C)$ by symmetry of the two players.

Differentiating (1) with respect to x_1, evaluating at $x_1=x_1^C$, and taking into account (4.1), we have

$$\begin{aligned} G'(x_1^C) &= \pi_1^1(x_1^C, x_2^C)+\pi_2^1(x_1^C, x_2^C)g'(x_1^C) \\ &= \pi_2^1(x_1^C, x_2^C)g'(x_1^C)\,. \end{aligned} \tag{5}$$

Noting that $g'(x_1^C)\gtrless 0$ according as $\pi_{12}^2(x_1^C, x_2^C)\gtrless 0$, and taking into account (2) and (A.3), we get the following results.

(i) If $\pi_{12}^2(x_1^C, x_2^C)>0$ and $\pi_2^1(x_1^C, x_2^C)>0$, $x_1^C<x_1^S$.
(ii) If $\pi_{12}^2(x_1^C, x_2^C)>0$ and $\pi_2^1(x_1^C, x_2^C)<0$, $x_1^C>x_1^S$.
(iii) If $\pi_{12}^2(x_1^C, x_2^C)<0$ and $\pi_2^1(x_1^C, x_2^C)>0$, $x_1^C>x_1^S$.
(iv) If $\pi_{12}^2(x_1^C, x_2^C)<0$ and $\pi_2^1(x_1^C, x_2^C)<0$, $x_1^C<x_1^S$.

We can assert regarding the payoffs that

$$\pi^1(x_1^S, x_2^S)=\pi^1(x_1^S, g(x_1^S))=G(x_1^S)>G(x_1^C)=\pi^1(x_1^C, g(x_1^C))=\pi^1(x_1^C, x_2^C)\,, \tag{6}$$

where the inequality is the consequence of the maximizing property of x_1^S for $G(x_1)$. Hence, the leader's payoff is larger than that of the Cournot player regardless of whether he has the first mover advantage or not. Note also that (6) holds regardless of whether $x_1^C<x_1^S$ or $x_1^C>x_1^S$. All we need to derive (6) is the uniqueness of the Stackelberg equilibrium, which is ensured by (A.3).

3. FOLLOWER'S ADVANTAGE

Let there be two symmetric labor-managed firms, and let

$$l_i=h(x_i)\,,\quad 0=h(0)\,,\quad h'>0\,,\quad h''>0\,,\quad i=1,2\,, \tag{7}$$

be firm i's labor demand function, where l_i and x_i are its labor and output, respectively. Furthermore, let $p=f(x_1+x_2)$, $f'<0$, be the inverse market demand function, where p is the product price, and k be the fixed cost. The dividend per unit of labor for firm i is defined by

$$v^i\equiv(f(x_1+x_2)x_i-k)/h(x_i)\,,\qquad i=1,2\,. \tag{8}$$

Let, without loss of generality, firm 1 and 2 be the leader and follower, respectively. Assume the maximum to be interior. Then, the first order condition for the follower is given by

$$v_2^2 \equiv \partial v^2/\partial x_2 \equiv [h(x_2)\{f(x_1+x_2)+x_2 f'(x_1+x_2)\} - h'(x_2)\{f(x_1+x_2)x_2-k\}]/h^2(x_2) = 0 . \tag{9}$$

The second order condition is $v_{22}^2 \equiv \partial^2 v^2/\partial x_2^2 < 0$, or

$$h(x_2)\{2f'(x_1+x_2)+x_2 f''(x_1+x_2)\} - h''(x_2)\{f(x_1+x_2)x_2-k\} < 0 . \tag{10}$$

We assume that

$$f'(x_1+x_2)+x_i f''(x_1+x_2) < 0 , \qquad i=1,2 . \tag{A.11}$$

This assumption means that the marginal revenue of one firm decreases if the other firm increases its output. The assumption has been widely used in the analysis of static and dynamic Cournot oligopolies (see Okuguchi (1976) and Okuguchi and Szidarovszky (1990)). If firm 2 is viable as a leader-managed firm, $f(x_1+x_2)x_2-k>0$ must hold. Hence in light of $f'<0$ and $h''>0$, (10) holds under assumption (A.11).

Solving (9) with respect to x_2, we have

$$x_2 = g(x_1) , \tag{12}$$

where

$$g'(x_1) = -\{x_2(h/x_2-h')f'+x_2 h f''\}/\{h(2f'+x_2 f'')-h''(fx_2-k)\} . \tag{13}$$

The denominator of the RHS of (13) is negative by virtue of (10). However, the numerator may take any sign. We therefore assume that

$$x_2(h/x_2-h')f'+x_2 h f'' > 0 . \tag{A.14}$$

The first term of the above inequality of positive in light of $0=h(0)$, $h''>0$ and $f'<0$. Hence (A.14) holds if $f''\geq 0$. It holds also if the inverse demand function is not too concave.

Under (A.14), we have $g'>0$ in light of (10). Let the superscript S denote the Stackelberg equilibrium, and let

$$U(x_1) \equiv v^1(x_1, g(x_1)) . \tag{15}$$

Then (x_1^S, x_2^S) is the Stackelberg equilibrium if and only if

$$U(x_1^S) \geq U(x_1) \qquad \text{for all } x_1 , \tag{16.1}$$

$$x_2^S = g(x_1^S) . \tag{16.2}$$

Taking into account (A.14),

$$v_{12}^2 = \{x_2(h/x_2-h')f'+x_2 h f''\}/h^2 > 0 . \tag{17}$$

Since $f'<0$,

(18) $$v_2^1 = f' x_1 / h < 0 \,.$$

We can therefore assert in light of Lemma 1 in Gal-Or (1985) that

(19) $$x_1^S < x_2^S \,.$$

We get from Proposition 1 in Gal-Or (1985), (17), and (10) which holds under (A.11) the following inequality.

(20) $$v^{1S} < v^{2S} \,.$$

Inequality (20) shows that under our assumptions, being the follower is more advantageous than being the leader in labor-managed Stackelberg duopoly without product differentiation.

4. COURNOT VS STACKELBERG

In view of (12), the leader's dividend per unit of labor is rewritten as

(21) $$U(x_1) \equiv \{f(x_1 + g(x_1))x_1 - k\}/h(x_1) \,.$$

Hence its equilibrium output x_1^S satisfies

(22) $$\begin{aligned} U'(x_1^S) = [&h(x_1^S)\{f(x_1^S + x_2^S) + x_1^S f'(x_1^S + x_2^S) \cdot (1 + g'(x_1^S))\} \\ &- h'(x_1^S)\{f(x_1^S + x_2^S)x_1^S - k\}]/h^2(x_1^S) \\ = 0 \,, & \end{aligned}$$

where $x_2^S \equiv g(x_1^S)$. The second order condition is $U''(x_1^S) < 0$. We assume that[3]

(A.23) $$U''(x_1) < 0 \,.$$

Now consider labor-managed Cournot duopoly and let its equilibrium be denoted by superscript C. Since two firm are symmetric,

(24) $$x_i^C = g(x_j^C) \,, \qquad i \neq j \,, \quad i, j = 1, 2 \,,$$

holds. Note that $x_1^C = x_2^C$ due to symmetry of the firms. Taking into account (17), (18), (A.23) and the result (ii) in Section 2, we can claim that

(25) $$x_1^C > x_1^S \,.$$

Under (10) and (A.14), $g' > 0$ as shown above. Hence (19) and (25) yield

(26) $$x_1^C = x_2^C = g(x_1^C) > g(x_1^S) = x_2^S > x_1^S \,.$$

[3] The second derivative of $U(x_1)$ is

$$\begin{aligned} U''(x_1) = [&h'\{f + x_1 f'(1 + g')\} + h(2f' + x_1 f'' + f'g' + x_1 f''g' + x_1 f'g'') \\ &- h''(x_1 f - k) - h'(f + x_1 f')]/h^2 - 2[h\{f + x_1 f'(1 + g')\} - h'(x_1 f - k)]/h^3 \,. \end{aligned}$$

Concrete cases where $U''(x_1) < 0$ is satisfied are hard to find. We are therefore obliged to introduce (A.23) as a mathematical assumption.

Furthermore, we can assert from (6) in light of (A.23) that[4]

(27) $$v^{1S} \equiv v^1(x_1^S, x_2^S) > v^1(x_1^C, x_2^C) \equiv v^{1C}.$$

Hence, we conclude in view of (20) and (27) that

(28) $$v^{2S} > v^{1S} > v^{1C} = v^{2C}.$$

This proves that for labor-managed duopoly, the dividend per unit of labor is larger for the Stackelberg follower than that for the leader, which in turn is larger than that for the Cournot firm. Since in labor-managed Stackelberg duopoly the follower is more advantageous than the leader, both firms strive for to be a follower. As a consequence, both firms' dividends per unit of labor are likely to become less than what is obtained when one of the firms acts as a leader.

5. CONCLUSION

In this paper we have first compared equilibria for general Cournot and Stackelberg two-person games with symmetric payoff functions. We have then applied our general results to labor-managed Cournot and Stackelberg duopolies with quantity strategy and without product differentiation, and derived the fundamental results (26) and (28). Under our assumptions,[5] the Cournot firm's output is larger than the Stackelberg follower's output, which in turn is larger than the leader's output. The Stackelberg follower's dividend per unit of labor is larger than the leader's dividend per unit of labor, which in turn is larger than the Cournot firm's dividend per unit of labor.

[4] Note that (27) holds because of the uniqueness of the labor-managed Stackelberg duopoly equilibrium ensured by (A.23). The same inequality holds provided the Stackelberg equilibrium were unique even without (A.23). However, (25) is not derivable without (A.23). See also footnote 2.

[5] As was pointed out by a referee, it is important to show clearly how our assumptions (A.3), (A.11), (A.14), (A.23) and the uniqueness of the Stackelberg equilibrium were used in deriving our results. A closer examination of our analysis will reveal the following logical implications.

(A.3)→Results (i)–(iv) in Section 2.

(A.3) or uniquness→(6).

$f'' < 0$ and (A.14)→(19).

(A.11) and (A.14)→$g' > 0$, hence (20).

$f'' < 0$, (A.14) and (A.23)→(25).

(A.11) and (A.14)→$g' > 0$, hence (∗) $x_2^C = g(x_1^C) > g(x_1^S) = x_2^S$ by (25).

(A.23) or uniqueness→(27).

(19), (25), and (∗)→(26).

Hence,

$f'' < 0$, (A.11), (A.14) and (A.23)→(26).

(20) and (27)→(28).

Hence,

(A.11), (A.14) and (A.23) or uniqueness→(28).

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