Title	STOCHASTIC LEARNING BY DOING IN NEW GROWTH THEORY
Sub Title	
Author	SENGUPTA, Jati K.
Publisher	Keio Economic Society, Keio University
Publication year	1998
Jtitle	Keio economic studies Vol.35, No.2 (1998.) ,p.9- 35
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19980002-0009

KEIO ECONOMIC STUDIES 35(2), 9-35 (1998)

STOCHASTIC LEARNING BY DOING IN NEW GROWTH THEORY

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First version received June 1997; final version accepted July 1998

Abstract: Modeling the learning process in new growth theory involves an understanding of the stochastic sources implicit in the diffusion process of modern technology and its spillover across the global trade. Stochasticity of this diffusion process and its impact on instability are discussed here in a theoretical setup. The empirical relevance of this type of analysis is also discussed in relation to the rapid growth episodes of newly industrializing countries of Southeast Asia. Various aspects of the stochastic learning process as they affect economic growth are discussed and their economic implications analyzed both theoretically and empirically.

1. INTRODUCTION

New growth theory refers to the recent developments in endogeneous models of economic growth, which purport to explain the rate of sustained growth of per capita income in the long run. From an applied perspective three types of forces have played an active dynamic role in this growth process. One is the technology and innovarion as the engine of sustained growth. The endogeneity of technological progress is mostly due to the direct and intentional investment by profit-seeking entrepreneurs, who held a forward looking view of the state of the world. Schumpeter (1934), Solow (1994) and many others, e.g., Grossman and Helpman (1994) have emphasized this dynamic role of technology for future sustained growth. A second important factor is the dynamic externalities due to the international diffusion of '*knowledge capital*' and the rapid advance of information technology. According to Lucas (1993) this knowledge spillover effect may be the most significant factor explaining the large differences in marginal productivity of capital between a less developed and a fast developing or developed economy, when the concept of capital is broadened to include human capital. The third important source of endogenous growth, as evidenced by the rapid growth

Acknowledgment. The author is greatly indebted to a referee for valuable suggestions on an earlier version of this paper. The detailed suggestions have considerably helped improve the presentation of the economic implications of the stochastic learning process and its relation to economic growth. The usual disclaimer applies.

episodes of the newly industrializing countries (NICs) of southeast Asia is the openness in trade and its impact on sectoral growth of output. Thus Lucas (1993) has strongly emphasized that the diffusion of spillover research technology implies the strong connection we observe between rapid productivity growth and trade or openness. Consider for example two small economies like Korea and the Philippines in 1960. Suppose now as Lucas argues that Korea shifts its work-force into producing new goods intended for the world market, but Philippines continues to produce the traditional goods. Then according to a learning-based growth theory, Korean production would grow more rapidly. Thus a large volume of trade is essential for this type of growth episode.

Modeling the learning process in the framework of new growth theory and empirically applying it over time series data have posed several challenges before the researchers. Two basic reasons may be cited for this. One is that the learning process has a core component of stochasticity. This is evidenced both in the inception of R & D technology and its diffusion across industries and over international boundaries. The second is the adjustment process, linking future expectations and gradual policy adjustments in the short run. The gap between the myopic and the steady state optimal paths of the policy variables has several stochastic components, which are important in a policy framework.

Learning in growth models may take several forms. Here we restrict ourselves to three types of learning phenomena. One is called learning by doing in the capital goods industry, where the productive efficiency of each producer depends on the cumulative aggregate output of capital goods. Arrow's model exploited this aspect of learning, which is unrelated to both research and new inventions. The second view of learning involves adopting newer and more efficient technology. This leads to scale economies and a decline in the level of minimum average cost over time. In endogenous growth theory this comes about through different sources such as knowledge spillover across domestic sectors and through international trade by means of either final products embodying new technology, or intermediate inputs bearing the blueprints of more efficient technology. Romer (1990), Grossman and Helpman (1991) and Jovanovic (1997) have explored this type of learning as a source of persistent growth of an economy. Finally, learning involves a process of dynamic adjustment in producer behavior, which is influenced both by past history and future expectations. Whereas 'history' accounts for the initial resource endowments, preferences and the existing technology, the 'expectations' refer to the innovations and investments for newer technologies which have a future goal. Thus history emphasizes a backward looking view, whereas expectations presume a more active role by the forward looking entrepreneurs who take a global view of the world market in future much in the Schumpterian tradition of a capitalist innovator. This type of learning phenomena has been explored by Kenan (1979), Krugman (1991) and Sengupta and Okamura (1996).

The impact of stochasticity of the learning process in growth is examined in this paper by comparing two types of formulation as follows:

(a) deterministic and stochastic environment with learning, and
(b) the stochastic environment with and without learning.

Section 2 discusses the comparative formulations of the deterministic and stochastic framework of the Solow model, both under learning by doing. To emphasize endogeneity of this learning process this section starts from the formulation due to Nordhaus (1967) and others, where the savings rate and the rate of technical change are determined by the producer's optimizing objective. The risk averse attitude of the producer and its consequences for the optimal growth path of output are analyzed here in terms of the variance characteristics of the optimal trajectory. In general it is shown that the temporal fluctuations in variance tend to be positively correlated with the growth trend of mean output. For nonlinear dynamics this may give rise to a chaotic instability. A producer who is averse to the high rate of fluctuations in the variance process will tend to prefer a lower output trajectory in a stochastic environment than in a deterministic environment.

Section 3 discusses openness in international trade and its stochastic impact on overall growth under conditions of positive learning and no learning. Here knowledge spillover occurs through the interaction of two groups of sectors, one oriented to exports and international trade and the other to the domestic economy. Recently Helpman (1997) analyzed the empirical data of about 100 countries over the period 1971–90 and found substantial R & D spillovers. For example a developing country that has an import share of foreign machinery and equipment of about 7% enjoys a TFP (toal factor productivity) elasticity with respect to foreign R & D capital of about 0.06, which is quite substantial. Since the international transmission of knowledge capital and know-how occurs mainly through the human capital, it is useful here to empirically evaluate the role of human capital accumulation in the context of specific NICs in Asia, which exhibited rapid economic growth episodes. Hence Section 3 attempts to empirically estimate for South Korea the growth trend in human capital and the influence of past history and future expectations.

Finally, Section 4 presents the stochastic framework of a dynamic limit pricing model, where risk aversion is directly incorporated into the objective function of a representative producer, who is competing in the international market. Here learning takes two forms: one is in the form of cost declines due to improved technology adoption and scale and the other in the form of new entry to an oligopolistic market which is sensitive to new technology and the persistence of high profits in the short run. Whereas the first type of learning emphasizes the role of first-time innovators in the Schumpterian process of 'creative destruction', the second type shows in a direct fashion the impact of stochastic market growth on the two groups of Cournot producers. In the latter case the impact of increased or decreased risk aversion on the price and output trajectories can be directly evaluated. Hence one can more directly compare the optimal paths for deterministic and stochastic versions.

2. STOCHASTIC SOLOW MODELS

The earliest form of learning by doing is due to Arrow (1962), who modeled the technical innovation process in terms of the experience of the airframe industry. Here the experience is measured by cumulated gross investment $K(t)$:

$$K(t)=\sum_{-\infty}^{t} I(v)dv \tag{1}$$

and the production function is specified as

$$Y(t)=F[K(t), A(t)L(t)] \tag{2}$$

where the current efficiency of labor is measured by

$$A(t)=[K(t)]^{\gamma}, 0<\gamma<1 . \tag{3}$$

Note that even if the production function $F(K, AL)$ has constant returns to scale in the two inputs K and AL, as in the neoclassical model, the overall function exhibits homogeneity of degree greater than one, e.g., in the Cobb–Douglas case

$$F(K, AL)=K^{\beta}(AL)^{1-\beta}=K^{\beta+\gamma(1-\beta)}L^{1-\beta}, 0<\beta<1 . \tag{4}$$

Assume a constant ratio (s) of savings to output. Then the ratio $k(t)$ of output to augmented labor ($k(t)=Y(t)/AL(t)$) follows the differential equation

$$\dot{k}=s(1-\gamma)f(k)-nk \tag{5}$$

where $\dot{L}/L=n$ is the fixed growth rate of labor. Let k^0 be the capital-labor ratio defined by

$$f(k^0)/k^0=n(s(1-\gamma))^{-1} \tag{6}$$

then by the property of the production function $f(k)>0, f'(k)>0, f''(k)<0$ for all $0<k<\infty$ it follows that starting from any arbitrary initial stock of capital k_0 the unique solution of (5) tends to k^0, i.e., $\lim_{t\to\infty} k(t)=k^0$, where k^0 is the balanced growth capital-labor ratio corresponding to a fixed savings rate.

It is clear that in a state of balanced growth with fixed k and a fixed level of savings per capita, the process of learning by doing follows the following path:

$$\dot{A}/A=\gamma(\dot{K}/K)=\frac{\gamma}{1-\gamma}\left(\frac{\dot{k}}{k}+n\right)$$

$$\text{i.e.,}\quad A(t)=[k(t)]^{\frac{\gamma}{1-\gamma}}\exp\left(t\left(\frac{\gamma n}{1-\gamma}\right)\right). \tag{7}$$

Clearly the higher the level of γ, the higher is the learning curve effect in raising output. Also this effect is enhanced by increasing the level of $k(t)$ itself, i.e., capital augmenting.

A second approach to learning by doing is to allow this effect through both labor and capital, i.e.,

$$Y = F(\gamma K, \mu L)$$

where the rates of factor augmentation are assumed to follow the rule

$$\dot{\gamma}/\gamma = g(\dot{\mu}/\mu) \tag{8}$$

where $g' < 0$ and $g'' < 0$.

This approach is due to Kennedy (1964), who has interpreted the $g(\cdot)$ function as an innovation possibility curve. With a fixed savings ratio s and the following per capita variables $y = Y/L$, $k = K/L$ and $x = \gamma K/\mu L$. Nordhaus (1967) has analyzed the time path of capital labor ratio as follows

$$\dot{k} = s\mu f(x) - nk\ . \tag{9}$$

To determine the optimal trajectory of technical change, Nordhaus assumes a planning authority which controls the aggregate savings rate s and the direction of technical change $= \tau = \dot{\mu}/\mu$. The objective is to maximize the discounted stream of per capita consumption, i.e.,

$$\text{Max}\, J = \int_0^{\infty} \exp(-\rho t)[(1-s)\mu f(x)]\mathrm{d}t\ . \tag{10}$$

Forming the Hamiltonian

$$H = \exp(-\rho t)\left[(1-s)\mu f\left(\frac{\lambda k}{\mu}\right) + p_1\left\{s\mu f\left(\frac{\lambda k}{\mu}\right) - nk\right\} + p_2 e^{ht} g(\tau)\lambda + p_3 \tau\mu\right]$$

and applying Pontryagin principle, the optimal trajectories must satisfy the following conditions on the continuous adjoint variables $p_i(t)$ $(i = 1, 2, 3)$ as follows:

$$\begin{aligned} \dot{p}_1 &= (\rho + n)p_1 - f'(x)\gamma\lambda \\ \dot{p}_2 &= (\rho - h - g(\tau))p_2 - f'(x)k\gamma e^{-ht} \\ \dot{p}_3 &= (\rho - \tau)p_3 - \gamma[f(x) - xf'(x)] \end{aligned} \tag{11}$$

where $\gamma = 1 - s + sp_1$, h is the solution to the equation $g(\tau) = 0$, prime denotes derivatives and a dot over a variable denotes its time derivative. In addition, the optimal trajectories of $s(t)$ and $\tau(t)$ must satisfy at each time point the conditions

$$\begin{aligned} &s(t) \text{ maximizes } \{1 - s + sp_1(t)\} \\ &\qquad \text{and} \\ &\partial H/\partial\tau = 0 = p_2(t)g'(\tau)\lambda e^{ht} + p_3(t)\mu \\ &\partial^2 H/\partial\tau^2 \leq 0 \text{ and } g(\tau) \text{ is concave in } \tau\ . \end{aligned} \tag{12}$$

It is clear that these optimizing conditions may be interpreted in two different ways. One is to view it as a central planner's problem, where knowledge capital is in the public domain and both the savings rate and the direction of the public innovation process are endogenously determined as an optimal choice problem. This is very different from the Solow model, where these two variables are more or less exogenous. Secondly, it may be viewed as a monopolistically competitive market equilibrium solution, where the private firms undertake research innovations in order to exploit the dynamic profits and rents over time until new entry occurs with improved innovations. Aghion and Howitt (1992) have analyzed this second aspect in a framework of vertical innovations, when the amount of research in any period depends upon the expected amount of research in the next period and furthermore the productivity of research or R & D investment is measured by a parameter indicating the effect of research on the Poisson arrival rate of innovation. We discuss in Section 3 an alternative formulation of a two-tier model of technical diffusion, where the productivity shock is reflected in terms of a stochastic parameter. Note that this model of optimal technical innovation has several flexible features. First of all, the innovation possibility function (8) links capital augmentation to the efficiency of labor or human capital and this is very much in line with the modern theory of 'knowledge capital'. Secondly, the use of $\tau = \dot{\mu}/\mu$ as the control variable by the planning authority suggests that R & D expenditures have to be optimally allocated, since the ratio μ/λ must satisfy the optimality rule given in (12):

$$\mu/\lambda = (-e^{ht} g'(\tau)) p_2/p_3$$

where $-g'(\tau)$ is positive. However the condition $g''(\tau) < 0$ which is required for optimality may not hold if the production set is nonconvex and the competitive market equilibrium has to obtain. Thus there is a direct conflict between the planned economy setup and the competitive equilibrium. In endogeneous growth theory this conflict is handled in two ways, e.g., either one replaces the competitive framework by monopolistic competition, or the objective function (10) is replaced by a discounted profit functional for a private producer. In the latter case a two-tier framework of directing technical innovation is of central importance. In the domestic front the producer acts jointly with the state support like a large quasi-monopoly firm, which exploits all the scale economies, whereas in the international front it acts more like a price taker. This aspect will be discussed later in section 3. Finally, the steady state level of k determined by (9) yields

$$f(x^0)/x^0 = n(s\lambda) . \tag{13}$$

This may be compared with the equation (6) when learning by doing occurs only through labor augmentation. Clearly when μ rises, τ falls and this leads to an increase in the output-capital ratio in efficiency units given by the left hand side variable in (13). For the labor augmenting variety of learning by doing, a rise in γ in (6) leads to an increase in output-capital ratio. Recently Binder and Pesaran (1996) have empirically investigated the degree to which stochasticity in technological progress and the labor input can contribute to differences in steady state capital output ratios across countries. In the framework of this model (8), the parameters λ and μ would be stochastic in character, which would affect the transient and the steady state behavior of the capital-output ratio in the extended Solow model. The sources of this stochasticity are two-fold. One is the uncertainty associated with the R & D investments, which not only generates new information embodied in new products or new services, but also enhances the firm's ability to exploit existing information. Recently Cohen and Levinthal (1989) have empirically analyzed R & D investment data of 1302 business units representing 151 lines of business from the FTC's data file and found that learning or absorptive capacity represents an important part of a firm's ability to create new knowledge or a new product. This explains why firms may conduct basic research, e.g., for reasons that they are more able to identify and exploit useful scientific and tecknowledge, whereby they may gain a first-mover advantage in exploiting new technologies. A second source of uncertainty involves adjusting labor and capital stocks to their desired levels. For example a firm which finds that its current stocks of capital and labor are inconsistent with the long run equilibrium implied by current factor prices and their expected changes in future, will generally spread the planned adjustment to long run equilibrium over time. This imparts a stochasticity to the changes in labor and capital. Treadway (1974), Kennan (1979) and Gregory et al. (1993) have analyzed such problems.

So far we have discussed learning by doing through the labor and capital inputs, a third type arises through the Hicksian technical progress fuction, where the production function is specified as

$$Y(t)=F[K(t), L(t), t] .$$

Here t on the right hand side represents a time trend variable used as a proxy for neutral technological progress, e.g., a shift in production function. Recently Norsworthy and Jang (1992) have discussed the disadvantages of this type of specification and suggested other types of explanatory variables such as 'cumulative output' which has a learning curve effect of economies of scale. Assuming a Cobb–Douglas form, the production function here takes the form

$$Y=\dot{Z}=AZ^{\theta}L^{a_1}K^{a_2};\ \alpha_i>0\ , \quad 0<\theta<1 \tag{14}$$

where $Z(t)=\int_0^t Y(\tau)\mathrm{d}\tau$ is cumulative output representing the embodied form of all knowledge capital and cumulative experience. With the other assumptions of the Solow model, i.e., a fixed savings ratio s and the growth of labor as $n=\dot{L}/L$, one could easily derive the logistic equation in terms of the variable $u=\dot{Z}/Z$ as follows

$$\dot{u}/u=n\alpha_1\left(1-\theta-\frac{s\alpha_2 Z}{sZ+K_0}\right).$$

This yields the reduced form

$$\dot{u}/u = r(m-u) \tag{15}$$

where $r = 1-\alpha_2-\theta$, $m = n\alpha_1(1-\theta-\alpha_2)^{-1}$ and K_0 is set to zero as a starting point. This can also be written in a convenient form as follows:

$$\dot{u}/u = b(1-u/\bar{u}) \tag{16}$$

where b is a suitable positive parameter and $\bar{u}$ is the maximum level of u, e.g., $b = rm$ and $\bar{u} = 1/m$ in the case above.

Two important types of economic interpretation may be emphasized in terms of the learning capability parameter θ in (15), when one compares the deterministic version of this model with the stochastic version. One type of stochastic version assumes that the parameter $m = m(t)$ in (16) which incorporates the learning capability parameter θ follows a Gaussian process $m(t) = m_0 + \gamma W(t)$ with $W(t)$ being white noise, then the mean and variance processes take the simple forms

$$\mu(x) = x(\alpha - x), \quad \sigma^2(x) = \gamma^2 x^2 .$$

Clearly $\partial\mu(x)/\partial\sigma^2(x) < 0$, which implies that increased volatility tends to lower the mean level of output growth in the long run. Thus the deterministic model (15) implies that countries with high learning ability will evidence high rate of growth, since $\partial(\dot{u}/u)/\partial\theta = u$ is positive. But the stochastic analogue of this model would imply in the long run that this process may not be self sustained due to the negative correlation of the mean and the variance. Note also that the time varying propagation of the u-process would be markedly different in a stochastic framework from that of a deterministic framework, due to the existence of the variance process $\sigma^2(x)$. So long as this variance $\sigma^2(x)$ is positive for any time t, the mean of the $u(t)$ process would differ from the deterministic solution. This has the further implication for the risk averse agents involved in the macrodynamic innovation and investment decisions. Since higher risk aversion implies more sensitivity to fluctuations measured by variance, these agents would prefer lower levels of mean growth rates in investment and hence output. Unlike consumption risk this stochastic impact implies a high cost of economic fluctuations on the firms whose production technology entails commitments that are costly to reverse. Since most of these fluctuations are unanticipated, they cannot be incorporated into the firm's production plans, hence volatility due to variance $\sigma^2(x)$ leads to lower mean output as a consequence of ex post technological inefficiency. This issue also remains very important in econometric estimation of the output trajectory, since the specification of the deterministic model must incorporate some form of cost due to unanticipated fluctuations. Now consider two types of stochasticity in the logistic process models (15) and (16). One arises through the variations in the random parameters m in (15) and b in (16). The sources of randomness may be due to the Hicksian technical progress function. A second type of stochasticity arises throught interpreting $u(t)\mathrm{d}t$

as a stochastic process satisfying a birth and death process for example. Here we consider the probability that $u(t)$ takes a particular value at time t, when the transitions to other states are Markovian.

Consider the first case and assume that the parameter m varies randomly, i.e., $m = m_0 + \gamma(t)$ around the mean value m_0 with $\gamma(t)$ as a white noise component with mean zero and variance σ^2. One can then derive the appropriate Fokker–Planck equation for the population probability distribution $f(u, t)$ by following the methods outlined in Bharucha-Reid (1960). The steady state probability distribution $f^*(u)$ then takes the limiting form

$$f^*(u) = c \exp(-2\mu/\sigma^2)$$

where c is the normalization constant, making the integrated probability unity. The mean $M = E(u)$ and the root mean square relative fluctuation R can be derived as

$$M = M_0[1 - (\sigma^2/2m_0)]$$

$$R = \frac{\sqrt{E(\mu - M)^2}}{M} = \left[\frac{\sigma^2/2m_0}{1 - (\sigma^2/2m_0)}\right]^{1/2}. \tag{17}$$

Note that relative fluctuations become increasingly severe as σ^2 increases towards $2m_0$, beyond which no equilibrium or steady state solution exists. Thus the mean variance ratio m_0/σ^2 taking the value 0.50 provides a critical level of volatility measured by the relative fluctuations.

Consider now the second form of the logistic equation (16) and assume that the parameter $b = b_0 + \sigma\xi(t)$ fluctuates like a Gaussian variable around the mean value b_0 with a variance σ^2 where $\xi(t)$ represents white noise. This yields a Gaussian delta continuous process for $u(t)$, which satisfies the Fokker–Planck equation

$$\partial P/\partial t = -\frac{\partial}{\partial u}[a(u)P] + \frac{1}{2}\frac{\partial^2}{\partial u^2}[c(u)P] \tag{18}$$

where $P = p[u \mid y, t]$ is the conditional probability that the random variable U will take the value u at time t given that it takes a value y at time zero and $a(u)$, $c(u)$ are defined as follows

$$a(u) = \sigma(u) + \frac{1}{4}\frac{\partial}{\partial u}(\beta(u)^2);\ \alpha(u) = b_0 u\left(1 - \frac{u}{\bar{u}}\right)$$

$$c(u) = [\beta(u)]^2;\ \beta(u) = \sigma u\left(1 - \frac{u}{\bar{u}}\right)$$

and

$$du/dt = \alpha(u) + \beta(u)\xi(t)\,.$$

Again one can show that such stochastic processes, which are completely determined by the coefficients $a(u)$ and $c(u)$ of the Fokker–Planck equation have unlimited state spaces if $c(u)>0$ and $a(u)$ is infinite. In such cases the relative fluctuations enter an explosive phase as in (17). For the normalized variable $z=\exp(\sigma u)$ the mean Ez and variance $V(z)$ can be derived as follows

$$Ez=\exp\left(\frac{\sigma^2 t}{2}\right)\exp\left[\sigma\left(u_0+\frac{b_0 t}{\sigma}\right)\right], \quad u_0=u(t) \quad \text{at} \quad t=0$$

$$\mathrm{Var}\, z=(Ez)^2[e^{t\sigma^2}-1]$$

$$\sqrt{var\, z}/Ez=(e^{t\sigma^2}-1)^{1/2}\to\infty \quad \text{as} \quad t\to\infty\,.$$

Note also that the deterministic model is

$$\mathrm{d}u/\mathrm{d}t=b_0 u(1-u/\bar{u})$$

which completely ignores the effects of $\beta(u)\xi(t)$, which combines the noise terms σ^2 and $\xi(t)$. Thus the effect of variance σ^2 may be stabilizing when it tends to reduce the mean level of u as t gets larger; or it may be destabilizing when it tends to make the u-process explosive. The zones of stability and chaotic instability are thus characterized by the stochastic interpretation of the growth model subject to the Hicksian type of neutral technological progress. The empirical estimation by Binder and Pesaran (1996) of the volatility of the capital-output ratio over 72 countries (1960–92) shows the importance of such issues for stochastic growth models. The steady state probability density function $p(u)$ of the process defined by (18) is of the form

$$p(u)=Cu^{(2b_0/\sigma^2)-1}(1-u/\bar{u})^{-(2u/\sigma^2)-1} \tag{19}$$

which shows that if $2b_0/\sigma^2$ is less than one, then the density function is U-shaped, indicating that it approaches zero or $\bar{u}$. But if $2b_0>\sigma^2$ then the density is monotonically increasing in a J-shaped form; which suggests the existence of explosive regions where the steady state equilibrium may not exist at all.

There is an alternative way of looking at the deterministic growth equation (15) for $u(t)$. One can rewrite it with an additive error term $\mathrm{d}\varepsilon$:

$$\mathrm{d}u=[gu-hu^2)dt]+\mathrm{d}\varepsilon\,, \tag{20}$$

where the first term under square bracket on the right hand side represents the systematic part of the stochastic changes $\mathrm{d}u$ and the error term $\mathrm{d}\varepsilon$, has a mean zero in the limit with a variance $(gu-hu^2)\mathrm{d}t$. Note that the parameters g and h are functions of the parameters r and m defined before in (15), i.e., the learning and experience effects in the R & D processes. Let μ be the asymptotic mean of the $u(t)$ process and $X(t)=u(t)-\mu$. Then

$$X(t+\Delta t)-X(t)=[gu(t)-hU^2(t)]\mathrm{d}t+\mathrm{d}\varepsilon(t)\,.$$

This implies

$$E[X(t+\Delta t)-X(t)]=[gE(u(t)-hE(u^2(t))]\mathrm{d}t+E(\mathrm{d}\varepsilon)\,.$$

Letting $\Delta t \to 0$ this yields

$$(g/h)\mu-\mu^2=\sigma_u^2 \tag{21}$$

where σ_u^2 is the asymptotic variance of μ. This shows very clearly that

$$\partial\sigma_u^2/\partial u=(g/h)-\mu \gtreqqless 0, \text{ according as } \mu = \lesseqqgtr (g/h)\,.$$

Hence $\partial\mu/\partial\sigma_u^2<0$ if $\mu>g/h$.

Thus as the mean income level μ increases above the level set by g/h, higher variance of u leads to a lower mean level of u, i.e., higher volatility tends to have lower means. But for the other case when $\mu<g/h$, the correlation betweeen μ and σ_μ^2 is expected to be strongly positive.

Now we consider a stochastic birth and death process model for $u(t)$ and assume that the transition probability $p_u(t)=\mathrm{Prob}[u(t)=u]$ for $u=0, 1, 2, \cdots$ satisfies the standard Markovian assumptions, e.g., (i) assumption of stationary independent increments, i.e., the transition from u to $u+1$ is given by $\lambda_u\Delta t+0(\Delta t)$ and from u to $u-1$ by $\mu\Delta t+0(\Delta t)t)$, where $0(\Delta t)$ denotes infinitesimals of order two or higher which can be neglected for $\Delta t \to 0$ and (ii) the probability of no transition is $(1-\lambda_u-\mu_u)\Delta t+0(\Delta t)$ and (iii) the probability of transition to a value other than the neighboring value is $0(\Delta t)$. Under these assumptions the transition probability $p_u(t)$ of $u(t)$ taking a value u at time t satisfies the following Chapman–Kolmogorov equation (see, e.g., Tintner and Sengupta (1972):

$$\mathrm{d}p_u/\mathrm{d}t=\lambda_{u-1}p_{u-1}(t)+\mu_{u+1}p_{u+1}(t)-(\lambda_u+\mu_u)p_u(t) \tag{22}$$

where the parameters λ_u, μ_u which depend on the level of u are called birth and death rate parameters, since the former leads to positive growth (e.g., effect of experience) and the latter to decay (e.g., obsolescence due to the introduction of new technology). Now assume that the birth rate parameter λ_u declines with increasing u but the death rate parameter μ_u remains proportional to u^2, i.e.,

$$\lambda_u=ua_1(1-u)\,,\qquad \mu=a_2u^2$$

where a_1, a_2 are positive constants. Then the mean value function $m(t)=E[u(t)]$ would follow the trajectory as follows:

$$\mathrm{d}m(t)/\mathrm{d}t=(a_1+a_2)\left[\frac{a_1}{a_1+a_2}m(t)-m^2(t)-v(t)\right] \tag{23}$$

where $v(t)$ is the variance function for the income process $u(t)$. For the deterministic growth model the differential equation (23) reduces to a simpler form

$$\dot{u}(t)=a_1u(t)-u^2(t) \tag{24}$$

if we normalize as $a_1+a_2=1.0$. The stochastic case however is of the form

$$\dot{m}(t)=a_1 m(t)-m^2(t)-v(t)\,. \tag{25}$$

On comparing the deterministic and stochastic forms of the growth equations (24) and (25), one may derive some useful results. First of all, in the steady state one obtains

$$\partial m/\partial v<0 \text{ if and only if } m>(1/2)a_1$$

whereas $\dot{u}(t)$ is zero at the level $u(t)=a_1$ and positive for $u(t)<a_1$. Otherwise the higher variance would tend to be associated with a larger mean. Clearly some countries would correspond with the latter case, as Goodwin (1990) has shown in his nonlinear model of economic growth. Secondly, the presence of a positive variance function in (25) implies that the deterministic trajectory $u(t)$ in (24) would tend to be shifted downward in the stochastic growth model. Due to this downward bias the steady state value of the mean m^0 would be less than the steady state deterministic value of u, i.e., $m^0<a_1=u^0$. Note that the shift in the mean value process $m(t)$ can be empirically analyzed through the econometric tests on the variance process $v(t)$, i.e., whether it follows random walk or other Arch processes. Recently Sengupta and Zheng (1995) have empirically estimated mean variance models of stock market volatility, where chaotic behavior could not be ruled out. Finally, one may consider a discrete time variant of the logistic model (24) as $u_t=f(u_{t-1})$ and analyze the stability of the map $f(\cdot)$ given an initial point u_0. The sequence of points u_0, $f(u_0)$, $f^2(u_0)$, $\cdots$ is called the orbit of the map where the iterates $f^n(u_0)$ are defined by $f^{n+1}(u_0)=f[f^n(u_0)]$ and $f^0(u_0)=u_0$. The following classification for the stability of the map $f(\cdot)$ is often used in chaos theory:

$$\begin{aligned}
|f'|>1 &: \text{ linearly unstable}\\
|f'|\le 1 &: \text{ linearly stable}\\
|f'|<1 &: \text{ strongly stable}\\
|f'|=1 &: \text{ marginally stable}\\
f'=0 &: \text{ superstable}
\end{aligned}$$

where f' is the slope of the map f at a fixed point with $|f'|$ denoting its absolute value. Note that the equation in discrete time form:

$$u_t/u_{t-1}=a(1-u_{t-1})$$

has its critical parameter a acting as a bifurcation parameter in the sense that the qualitative behavior of u_t suddenly changes for different value of a. For example, the range $1<a<2$ defines monotonic growth of u_t converging to the steady state $u^0=(1-1/a)$, but for $a>3$ the steady state becomes unstable and a two-period cycle emerges. In fact the simulation studies by Lorenz (1963) showed that as an increases beyond $1+\sqrt{6}=3.45$, higher and higher even-order cycles emerge; beyond 3.57 he found that very higher odd-period cycles appear and so on. This type of chaotic behavior may sometimes be reduced or aggravated by the variance process in the stochastic process model.

3. TRADE AND ECONOMIC GROWTH

The dynamic effects of openness in trade have been strongly emphasized in new growth theory. Export growth and the impact of the export-intensive sectors on the other sectors of the economy have played a very critical role in the rapid growth episode of countries like the NICs in Southeast Asia. In new growth theory openness in trade has been viewed as a catalytic mechanism which alleviates the bottlenecks that impede the steady growth of the less developed countries (LDCs). Empirical studies of the growth of exports have revealed two broad trends for the successful NICs. Thus Bradford (1987) examined empirical data for more than a dozen countries over 1965–80 covering the link between structural change and economic growth, where the index of structural change was derived from 16 manufacturing sectors and concluded that high rates of growth and rapid structural change are closely associated with those countries which are the successful NICS. Moreover for some countries like South Korea the pace of rapid structural change was also associated with a change in export-mix in response to world competition. The shift from traditional to R & D intensive products in export growth has been remarkable for the four successful NICs in Asia as follows:

TABLE 1. Percentage distribution of exports to the US.

Product-group	Hong Kong		Korea		Taiwan		Singapore	
	1966	1986	1966	1986	1966	1986	1966	1986
1. Traditional	67.9	62.2	56.5	52.7	44.6	49.1	73.6	13.9
2. R & D intensive (general)	9.8	23.8	2.0	19.2	15.8	22.3	0.0	58.2
3. R & D intensive (sophisticated)	17.5	29.5	3.9	29.6	20.3	29.2	0.20	78.1

This analysis by Kellman and Chow (1989) also showed another important characteristic, e.g., the traditional export items mainly from the primary sector were very dissimilar in pattern across the four countries above and also very insensitive to changes in relative prices but all the four countries were found to be very similar in the pattern of exports of certain sophisticated R & D intensive products such as electrical machinery, optical equipment and telecommunications and computer equipment and consumer electronics; also these R & D intensive products were found to be strongly responsive to international competition. In the more recent decade 1986–1996 this tendency has intensified due to two main reasons, e.g., (a) the impact of innovation in the semiconductor industry and its spillover effects on other sectors, and (b) the globalization of trade which has increased the competitive efficiency.

Several key channels have been identified in modern growth theory in order to explain the close association by the openness in trade via exports and the rapid overall growth. One channel is the "trade-knowledge externality" which was originally emphasized by Alfred Marshall. The benefits of this trade-knowledge cannot be fully appropriate internally. As Caballero and Lyons (1992) have interpreted, the expression of 'trade-knowledge' includes according to Marshall not only R & D but also knowledge along the lines of process innovation and best practice technology in general. In Romer's (1990) endogeneous growth theory, the capital input embodies this trade knowledge externality. The export sector's externality spills over to the other sectors and generates a feedback effect. A second channel of interaction why the export sector plays a leading role is that it is more productive and more intensive in modern technological inputs. Hence the export sector exerts a strong positive influence on the rest of the economy. A direct empirical test of this dominance effect of the export sector may be made by means of a two-sector model with outputs X and N for the export and the non-export sector subject to two production functions

$$N = F(K_N, L_N, X)$$
$$X = G(K_X, L_X, N) \tag{26}$$

where $K = K_N + K_X$ is total capital and $L = L_N + L_X$ is total labor. It follows therefore

$$\Delta N = F_K \Delta K_N + F_L \Delta L_N + F_X \Delta X \tag{27}$$
$$AX = G_K \Delta K_X + G_L \Delta L_X + G_N \Delta N$$

where the subscripts on F and G denote the marginal productivity of the two inputs in the two sectors. A direct empirical estimate of the two sector model (27) from Korean national income statistics data produced the following results.

	1964–83	1964–86	1969–86	1970–90
F_X	1.92	1.00	0.99	0.89
G_N	0.28	0.31	0.32	0.30
F_X/G_N	6.90	3.20	3.11	2.96

It is clear that the dynamic interdependence effect from the export to the non-export sector is roughly between three and seven times larger than the reverse effect from the non-export to the export sector. The estimates for other successful NICs in Asia analyzed by Sengupta (1993) confirm this dominance hypothesis for the export sector. Finally, Lucas (1993) has emphasized the knowledge spillover effect as the most significant factor which explains the difference in capital productivity between an LDC and a developed economy. Thus the external benefits of human capital in his theory can be captured by specifying the production function for sector i ($i=1, 2$) as

$$y_i = A_i x_i^{\beta_i} h_i^{\gamma_i}; \quad \Delta h_i = h_i(1-u_i)\theta_i \tag{28}$$

where the three variables y, x and h denote output, physical and human capital per effective worker. The term $h_i^{\gamma_i}$ is interpreted as an externality which multiplies the productivity of a worker at any skill level just as the shift factor A_i. For the export-intensive sector ($i=1$, e.g.) we have higher h_1 and higher γ_1 then the other sector. Also h_1 tends to grow faster over time, since the proportion $(1-u_1)$ devoted to human capital accumulation and its productivity effect (θ_i) is likely to be higher for the export sector ($i=1$).

There is an alternative way of modelling the knowledge spillover effect through human capital in the global economy. This follows the approach of Grossman and Helpman (1991) and more recently the learning models due to Jovanovic (1997), where endogeneous quality increments follow the process of learning through research. Here there are invention costs but no adoption costs and the output of research is designs, which are sold by innovators to intermediate-goods producers. Specifically the use of the new intermediate good augments the productivity parameter A in (28). This formulation allows the direct introduction of the Schumpeterian innovation process which is sufficiently important to affect the entire economy, as has been shown by Aghion and Howitt (1992).

We now consider an empirical econometric application of the learning mechanism, as it influences the behavior of the representative producer as the dynamic agent. The model, which follows the formulations of Kennan (1979) and Gregory et al. (1993) involves a two-step decision process. In the first step the producer decides on the optimal inputs given by the vector x_t^* say by minimizing a steady state cost function. The second step then postulates an optimal adjustment or learning rule towards the optimal target levels of input x_t^* and output $Y_t^* = F(X_t^*)$. The learning mechanism in the second step explicitly assumes a short-run adjustment behavior of the producer, who finds his current factor uses are inconsistent with the steady state equilibrium path (X_t^*, Y_t^*) determined in the first step by the relative factor prices and their expected changes in the future. In order to resolve or reduce the inconsistency problem the producer minimizes an intertemporal adjustment cost function, which includes the expected cost of deviations from the steady state equilibrium levels. Recently Sengupta and Okamura (1996) have introduced a quadratic adjustment cost function involving two types of costs, e.g., disequilibrium costs due to deviation of X_t from X_t^* and risk aversion costs due to input fluctuations. Thus the two components of the expected adjustment costs illustrate the learning behavior through adaptivity. This dynamic adjustment model was applied to explain the trend of economic fluctuations in Japan over the period 1965–90 for the time series of input and output growth. The specific adjustment model minimizes the expected value of a quadratic loss function as follows:

$$\underset{x_t}{\text{Min}}\, E_t L$$

where

$$L=\sum_{t=0}^{\infty} r^t[(x_t-x_t^*)'\Lambda(x_t-x_t^*)+(x_t-x_{t-1})'\psi(x_t-x_{t-1})] \tag{29a}$$

where $E_t(\cdot)$ is expectations of time $t, r=(1+\rho)^{-1}$ is the exogenous rate of discount, $x_t=\ln X_t$, $x_t=\ln X_t^*$ and Λ, ψ are matrices of nonnegative weights. The first component of the cost function is an inefficiency cost due to deviations of short run input levels from their optimal long run levels. The second component reflects the cost of successive movement towards the optimal input combination. On applying the necessary condition for minimization of the expected loss function above, one could easily derive the optimal linear decision rule as follows:

$$[-(1/r)(P+(1+r)I_m)Z+(1/r)Z^2+I_m]E_t x_{t-1}=(-1/r)E_t x_t^* \tag{29b}$$

where $P=\psi^{-1}\Lambda$, I_m = identity matrix of order m for m inputs and Z is a backward lag operator. It is well known that the characteristic equation of this difference equation system (29b) will have half of the roots stable and half unstable. Let μ be the square matrix of stable roots of this system. Then one could define a long run input demand vector as d_t:

$$d_t=(I_m-r\mu)\sum_{s=0}^{\infty} r^s\mu^s x_{t+s}^* .$$

Based on this estimated demand vector, one could derive the partial adjustment rule as the final estimating equation. For example with one input this would yield

$$\Delta x_t=\phi(d_t-x_{t-1})+\varepsilon_t \tag{29c}$$

where $\phi=1-\mu_1$, $d_t=(1-r\mu_1)E_t[\sum_{s=0}^{\infty} r^s\mu_1^s x_{t+s}^*]$ and it is assumed that the rational expectations (RE) hypothesis holds as

$$E_t(x_{t+1})=x_{t+1} \tag{29d}$$

i.e., the expected value of the x_{t+1} equals the observed value.

If the error component ε_t is a white noise process and d_t estimated consistently by the instrument variable mechod as Kennan has done, then this dynamic adjustment equation (29d) which incorporates active learning by the producer can be estimated by a statistically consistent way.

Several important features of the learning mechanism are to be specifically noted. First, if there is no learning, the producer would have no adjustment, so that the observed input and output paths would exhibit more fluctuations due to the uncertainty in the exogenous variables and the various unanticipated externality effects of international trade. With some learning through the adjustment cost, the optimal producer behavior is more risk averse. As in portfolio theory it tends to reduce the variance of fluctuations around the selected decision rule. Secondly, the rational expectations (RE) hypothesis implies that the estimated input demand turns out to be equal to the observed level on the average. This is the perfect foresight condition of the stochastic control model. If this condition is not fulfilled or fulfilled only partially, there would occur more divergence of the optimal from the actual trajectory. Finally, the objective function (29a) involves only the minimization of the expected loss function. A more general risk sensitive rule would be to minimize a weighted combination of mean and variance as follows:

$$\text{Min}\, E_t L + w \,\text{Var}\, L\,.$$

Risk sensitive optimal decision rules of this type have been recently applied by Sengupta and Fanchon (1997).

Finally, the optimal linear decision rule equation (29b) may be directly used to test which of the two forces: past history or future expectations played a more dominant role in the optimal capital expansion policies of the producers in Japan. Since future expectations are forward looking and hence more oriented to dynamic learning, its significance in an estimated equation would inducate the presence of active learning. The backward looking view represented by the past history and its trend would be comparison represent the less learning and less adjustment.

The detailed empirical model of dynamic adjustment was estimated by Sengupta and Okamura (1996) for Japan (1965–90) based on aggregate time series data. We report here two major findings that are relevant for learning-based growth of inputs. Based on a Cobb–Douglas production function

$$Q_t = B(A_t, V_t) K_t^a L_t^b$$

with the technical progress variable B as a function of export V_t and an external shock variable A_t for international transmission of knowledge, the linear decision rules for optimal demand for labor (L_t) and capital (K_t) appear as follows:

$$\Delta \ln L_t = \phi_L (d_t^L - \ln L_{t-1}) + \text{error}$$

$$\Delta \ln K_t = \phi_k (d_t^K - \ln K_{t-1}) + \text{error}\,.$$

Note that we could derive here a relationship between the speed of adjustment parameter ϕ_i and the ratio θ_i of the weights on two cost components in (29a):

$$\theta_i = \phi_i r + \phi_i (1 - \phi_i)^{-1}\,, \qquad i = L, K\,.$$

Since the target demand d_t^i can be estimated in terms of either the past lagged instrument variables (backward looking view), or the expected values of future instrument variables (forward looking view), we may derive two types of estimates as follows:

TABLE 2. Estimate of the speed of adjustment parameters (ϕ_i) and the weight ratio $\theta_i = A_i/\psi_i$ ($i = K, L$)

Output ($\ln Y_t$)	Input ($\ln X_{it}$)		ϕ_i	θ_i	ch. root $\mu_i = 1 - \phi_i$
1. GNP	backward looking	K	0.118*	0.016	0.882*
		L	0.898*	7.873	0.102
	forward looking	K	0.082*	0.007	0.918*
		L	0.837*	4.281	0.163**
2. GDP	backward looking	K	0.120	0.016	0.880
		L	0.924*	11.231	0.076*
	forward looking	K	0.086*	0.008	0.914*
		L	0.569*	0.753	0.431

Note: 1. One and two asterisks denote significant t-values at 5% and 1% levels.
2. The stable characteristic roots μ_i are only reported.

Clearly labor adjusts much faster than capital. Thus the ratio ϕ_L/ϕ_K varies from 6.6 to 10.2. The implications of a slow adjustment speed for capital are two-fold. First, the characteristic root for capital is very close to unity. Secondly, much of capital expansion is in the form of capital deepening, thus reflecting a stronger role of future expectations. Also the forward looking estimates of the characteristic root $\mu_K = 1 - \phi_K$ are higher than the backward looking ones. A more direct estimate of the optimal demand equations produced the following results:

$$\Delta l_t = 2.241 - 0.545 \ln_{t-1} + 0.432\tilde{l}_{t+1} \quad R^2 = 0.372, \quad DW = 3.02$$
$$(t = -3.47) \quad (2.53)$$

$$\Delta k_t = -0.026 - 0.511 k_{t-1} + 0.512 k_{t+1} \quad R^2 = 0.893, \quad DW = 2.67$$
$$(-0.21)(-8.62) \quad (8.00)$$

$$\Delta y_t = 0.039 - 0.505 y_{t-1} + 0.500\tilde{y}_{t+1} \quad R^2 = 0.735, \quad DW = 2.73.$$
$$(0.21)(-6.47) \quad (5.45)$$

Here the lower case letters are in logs of labor, capital and output and the t-values are in parentheses. The tilda over a variable denotes its estimated value at $t+1$ and this is used as a regressor in order to reduce the bias due to autocorrelation of errors. It is clear that the future variables $\tilde{l}_{t+1}$, $\tilde{k}_{t+1}$ and $\tilde{y}_{t+1}$ play a more dominant and significant role than the past denoted by l_{t-1}, k_{t-1} and y_{t-1}.

Similar estimates for South Korea over the period 1971–1990 produced very similar results as follows:

$$\Delta l_t = 0.099 - \underset{(-4.58)}{0.510 l_{t-1}} + \underset{(3.94)}{0.504 \tilde{l}_{t+1}} \qquad R^2 = 0.61\,, \quad DW = 2.49$$

$$\Delta k_t = -0.110 - \underset{(-7.41)}{0.536 k_{t-1}} + \underset{(6.79)}{0.544 \tilde{k}_{t+1}} \qquad R^2 = 0.86\,, \quad DW = 1.64$$

$$\Delta y_t = -0.023 - \underset{(-5.13)}{0.504 y_{t-1}} + \underset{(4.83)}{0.573 y_{t+1}} \qquad R^2 = 0.65\,, \quad DW = 2.24\,.$$

These results seem to support the hypothesis that learning-based future expectations play a more dynamic positive role in the growth of inputs and output in the Asian NICs.

Consider now the stochastic view of learning through the bivariate interaction of the two sectors in the growth process model in the form (26). Assume a bivariate birth and death process satisfying the Chapman–Kolmogorov equations as before. Then the system of differential equations for the transition probability $p_{x,n}(t)$ at time t can be written as

$$\begin{aligned} \mathrm{d}p_{x,n}(t) = {} & -(\lambda_x + \mu_x + \lambda_n + \mu_n) p_{x,n}(t) + \lambda_{x-1} p_{x-1,n}(t) \\ & + \mu_{x-1} p_{x+1,n}(t) + \lambda_{n-1} p_{x,n-1}(t) + \mu_{n+1} p_{x,n+1}(t) \end{aligned} \tag{29}$$

for $x, n = 0, 1, 2, \cdots$. We now consider the application of the above system of differential equations to two special cases. The first case occurs when there is no death rate, i.e., $\mu_x = 0 = \mu_n$ and the deterministic system in $x(t)$ and $n(t)$ follows one way interdependence as follows

$$\mathrm{d}x(t)/\mathrm{d}t = \dot{x}(t) = b_1 v x(t)$$

$$\mathrm{d}n(t)/\mathrm{d}t = \dot{n}(t) = a_2 n(t) + b_2(1-v)x(t)\,; \qquad 0 < v < 1\,.$$

Here $n(t)$ depends on $x(t)$ for its growth, whereas $x(t)$ grows due to the high proportion of total output $x(t) + n(t)$ devoted to human capital, i.e., high level of v which implies a low level allocated for the growth of the $n(t)$ sector. The means (M_x, M_n) and variances (V_x, V_n) may then be calculated as

$$M_x(t) = \exp(b_1 v t)$$

$$V_x(t) = v(2-v)^{-1} \exp(b_1 v t)[\exp(b_1 v t) - 1]$$

where the initial value $x(0)$ is set equal to one. However the non export sector output follows a different mean variance structure. The mean is

$$M_n(t) = b_2(1-v)(b_1 v - a_2)^{-1} \exp(b_1 v t) + \exp(a_2 t)$$

but the variance is a more complicated function with a dominant term proportional to $\exp(2a_2 t)$. It is clear from these mean variance relationships that

$$\partial M_x(t)/\partial V_x(t) < 0 \quad \text{as} \quad M_x(t) < 1.0$$

i.e., countries with a higher volatility of export output would tend to have a lower mean export level, otherwise $\partial M_x/\partial V_x > 0$ as $M_x(t) > 1.0$. Secondly, the allocation ratio v can be used directly by public policy favoring the export sector. The national governments in NICs in Asia have always stressed these policy measures. For example the government planners in Japan and Korea have consistently allocated a growing share of domestic and foreign resources through credit rationing and other export subsidy measures to capital-intensive industries and also consumer electronics. Finally, as $t \to \infty$, the coefficient of variation (CV) measuring the relative level of fluctuations tends to settle down in both sectors, $\mathrm{CV}_x \sim (v/(2-v))^{1/2}$. This implies that the CV_x ratio increases as v rises.

A second case of the strochastic process model (28) occurs when the sectoral interdependence takes the following form

$$\lambda_x = \lambda_1 x,\ \mu_x = xf(x, n) = x(\mu_{11}x + \mu_{12}n)$$
$$\lambda_n = \lambda_2 n,\ \mu_n = nf(x, n) = n(\mu_{21}x + \mu_{22}n)$$

with $f(x, n) = \alpha_1 x + a_2 n$ denoting the interaction term. This form allows various types of interaction effects through the functions $f(\cdot)$ and $g(\cdot)$. One could derive from this system differential equations involving the first two moments of the stochastic process as follows:

$$\dot{m}_{11}(t) = \lambda_1 m_{11}(t) - \mu_{11} m_{12}(t) - \mu_{12}\bar{m}_{11}(t) \tag{30}$$
$$\dot{m}_{21}(t) = \lambda_2 m_{21}(t) - \mu_{21}\bar{m}_{11}(t) - \mu_{22} m_{22}(t)\,.$$

Here $m_{11}(t) = E\{X(t)\}$, $m_{12}(t) = E\{X^2(t)\}$, $m_{21}(t) = E\{N(t)\}$, $m_{22}(t) = E\{N^2(t)\}$ and $\bar{m}_{11}(t) = E\{X(t)N(t)\}$ and the dot over a variable denotes its time derivative. Clearly if there is no interaction between the sectors, then the two sectoral outputs $m_{11}(t)$ and $M_{21}(t)$ grow at the exponential rates λ_1 and λ_2 respectively. But if $\mu_{12} = 0 = \mu_{21}$, and both μ_{11} and μ_{22} are negative, then both sectoral outputs tend to grow expoentially. The deterministic system corresponding to (30) may be specified as

$$\dot{X}(t) = \lambda_1 X(t) - \mu_1\alpha_1 X^2(t) - \mu_1\alpha_2 X(t)N(t)$$
$$\dot{N}(t) = \lambda_2 N(t) - \mu_2\alpha_1 X(t)N(t) - \mu_2\alpha_2 N^2(t)\,.$$

Note that this system has a logistic time profile for the export sector output if $\mu_1\alpha_2$ is negligibly small, i.e.,

$$\dot{X}(t) = \lambda_1 X(t) - \mu_1\alpha_1 X^2(t)$$

with steady state values

$$X^* = \lambda_1/(\mu_1\alpha_1)\,, \qquad N^* = \frac{\lambda_2}{\alpha_2\mu_2} - \frac{\lambda_1}{\alpha_2\mu_1}\,.$$

The stability of these steady state values depends of course on the underlying characteristic roots. However the stability of the steady state values m_{11}^*, m_{21}^* of the stochastic system (30) depends on much more restrictive conditions. As May

(1973) has shown that the probability of unstable steady states is much higher and there exist biological systems where this type of instability phenomenon is persistent. For example consider the second equation of the Lucas model (28) and assume a two-sector interacting framework as

$$\dot{h}_1 = h_1(t) f_1(h_1, h_2) \qquad (31)$$
$$\dot{h}_2 = h_2(t) f_2(h_1, h_2)$$

where

$$f_1(\cdot) = k_1 - h_1(t) - a h_2(t)$$
$$f_2(\cdot) = k_2 - h_2(t) - a h_1(t) .$$

Here the coefficient a measures the symmetric competition between the two sectors for the common pool of human capital in the population and k_1, k_2 are the sector-specific parameters, which are constants in the deterministic case but random around a mean in a stochastic environment. In the steady state of the deterministic model the levels of h_1^* and h_2^* are found by putting the growth rates $\dot{h}_1 = \dot{h}_2$ to zero and then the linearized version around these steady states has the coefficient matrix

$$A = \begin{bmatrix} -h_1^* & -a h_2^* \\ -a h_1^* & -h_2^* \end{bmatrix} .$$

Clearly both the eigenvablues of this matrix are negative if and only if the coefficient $a < 1.0$. This is the well known Gauss–Lotka–Volterra criterion for a stable bivariate population. Now let us consider a stochastic framework:

$$k_1 = k_0 + \gamma_1(t)$$
$$k_2 = k_0 + \gamma_2(t)$$

where the random parameters k_1, k_2 have a common mean value k_0 and two independent white noise random variables with zero mean and a common variance α^2. Let R denote the root mean square measure of relative fluctuation as is used in (17) before. Then it holds approximately that

$$R^2 \sim \frac{\sigma^2}{k_0(1-a)} .$$

This result derived by May (1973) has two important implications. One is that the system is stable so long as $a < 1.0$. But as soon as the magnitude of a increases up to 1.0 the interaction dynamics provide a weaker and weaker stabilizing influence and in the limit R^2 tends to be explosive. Secondly, the common parameter k_0 is only assumed for simplicity, the result would hold even if it is different for the two sectors. Clearly as k_0 decreases to lower and lower values and satisfies the inequality $k_0(1-a) < \sigma^2$, then the fluctuations measured by R^2 in (32) would tend to be higher and higher. This implies a tendency favoring increased random fluctuations in the $h_i(t)$ process and hence in the output process $y_i(t)$ defined in the Lucas model (28).

4. INDUSTRIAL ORGANIZATION AND DYNAMIC GAMES

Rapid growth episodes in the NICs in Asia have been closely associated with some key trends in these countries. One is the persistence of scale economies which tends to produce oligopolistic firms even if there are no formal barriers to entry under competitive world trade. In this framework the output of each firm is given by a Cournot–Nash equilibrium. Openness in world trade implying a larger market will tend to reduce the oligopolistic mark-up of price (p) over marginal cost (c). A second trend is the learning curve effect of cumulative experinece of knowledge capital, which is undergoing an international spillover. For example Norsworthy and Japan (1992), have empirically shown this effect to be substantial in Japan and other Asian NICs in microelectronics and semiconductor indudstries in particular. In a dynamic limit pricing model discussed, e.g., by Sengupta (1983) and more recently by Sengupta and Fanchon (1997) the entry into the market by other oligopolistic firms depends on the mark-up of price over marginal cost, where the actual price lies somewhere between the short-run monopoly price and the competitive price, the exact positioning depending on the barriers to entry, risk aversion and the impact on cost reduction through cumulative experience and learning. The third trend in the rapidly growing NICs in Asia is to capture the cost savings over time due to building capacity ahead of demand and to adopt flexible manufacturing practices. In this set-up it is important to distinguish between current ouptut ($\dot{y}=\mathrm{d}y/\mathrm{d}t$) and cumulative output $y(t)$ in the production function $F(y(t), x(t))$ where $x(t)$ denotes the variable inputs, e.g., the functional form

$$\dot{y}(t)=Ay^{\delta}x(t)^{1\gamma}$$

summarizes the dynamic process of producing the joint products of learning and output from resources and experiences. Assume now a duopolistic framework with two producers producing output $\dot{y}_1(t)$, $\dot{y}_2(t)$ at prices $p_1(t)$, $p_2(t)$. The NICs may represent one producer and the rest of the world as the second producer. The dynamic optimization model for the first producer may then take the form

$$\begin{aligned}\operatorname{Max} J_1 &= \int_0 \mathrm{e}^{-rt}(p_1(t)-c_1(t))\dot{y}_1\,\mathrm{d}t \\ &\text{subject to } \dot{y}_i(t)=F_i(y_1(t), y_2(t), p_1(t), p_2(t))\,; \qquad i=1,2 \qquad (32)\end{aligned}$$

On using the current value Hamiltonian $H=\mathrm{e}^{-rt}\{(p_1-c_1+\lambda_1)F_1+(p_2-c_2+\lambda_2)F_2\}$ and assuming the regularity conditions for the existence of an optimal trajectory the Pontryagin maximum principle specifies the following necessary conditions for optimality (for $i=12,$):

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial H}{\partial y_i}$$
$$\dot{y}_i = F_i(y_1, y_2, p_1, p_2) \qquad (33)$$
$$\partial H/\partial p_i = 0 \qquad \text{for all} \quad t$$

and

$$\lim \mathrm{e}^{-rt}\lambda_i(t) = 0 \text{ (transversality)} .$$

By using $[\mu_{ii} = (\partial F_i/\partial p_i)(p_i/F_i)$ and $\mu_{ji} = (\partial F_j/\partial p_i)(p_i/F_j)$ when $1 \neq j$ as the own price elasticity and cross elasticity of demand, the optimal price rule can be written as

$$p_1 = (1 + \mu_{11})^{-1}[\mu_{11}(c_1 - \lambda_1) - \lambda_2\mu_{21}] \qquad (34)$$

with the optimal trajectory for $\lambda_1(t)$ as

$$\dot{\lambda}_1 = r\lambda_1 + (c_1 - \lambda_1)F_{1y_1} - p_1F_{1y_1} + c_{1y}F_1$$
$$\text{i.e.,} \quad \lambda(t) = \int_t^{\infty} \mathrm{e}^{-r(\tau-1)}\left[\left(\frac{p_1}{\mu_{11}} + \frac{\lambda_2\mu_{21}}{\mu_{11}}\right)F_{1y_1} + c_{1y_1}F\right]\mathrm{d}\tau \qquad (35)$$

where

$$F_{1y_1} = \partial F_1/\partial y_1$$
$$c_{1y_1} = \partial c_1/\partial y_1 < 0 , \quad \text{i.e., future cost decline}$$
$$\mu_{11} < 0 , \quad \mu_{21} > 0 .$$

Clearly the optimal pricing rules involve the trajectories of both the current price $p_1(t)$ and the shadow prices $k_1(t)$. The current pricing rule (34) shows the price to be much lower than the monopoly price $(1+\mu_{11})^{-1}\mu_{11}(c_1-\lambda_1)$ since λ_2 is usually negative, since more competition hurts the market position of y_1 more. Secondly, the extent of future cost declines $(-c_{1y_1} > 0)$ tends to reduce the dynamic shadow price $\lambda_1(t)$. Thirdly, if the demand function in (33) is a function of prices alone, i.e., $\dot{y}_i = F(p_1p_2)$ then the sign of $\dot{p} = \mathrm{d}p/\mathrm{d}t$ may be shown as

$$\operatorname{sign}(\dot{p}) = \operatorname{sign}(-r\lambda_1 + c_{1t} - \lambda_2\mu_{21})$$

where $c_{1t} = \partial c_1/\partial t$ is the decline of cost over time due to learning and experience since $c_{y_1} < 0$. This shows a strong pressure for price declines over time. This has happened exactly in the semi-conductor and R & D intensive industries such as electronics, telecommunications and personal computers. Recently Helpman (1997) analyzed the empirical data of about 100 countries over the period 1971–90 and found substantial impact of R & D investment through foreign capital stock.

A simpler form of the decision model (32) results when we assume one average market price $p(t)$ for a homogeneous product and a dynamic Cournot model with two outputs $\dot{y}_1$ and $\dot{y}_2$ as decision variables. In this case we reformulate the model as

$$\text{Max}\, J_1 = \int_0^{\infty} \mathrm{e}^{-rt}(p(t) - c_1(t))\dot{y}_1 \mathrm{d}t$$

$$\text{s.t.} \quad \dot{p}(t) = k(\tilde{p} - p), \quad \tilde{p} = a - b(\dot{y}_1 + \dot{y}_2) \tag{36}$$

where $\tilde{p} = \tilde{p}(t)$ is the demand price expected and p is the market price. Assuming quadratic cost functions, i.e., $c_1\dot{y}_1 = w\mu_1 + 1/2u_1^2$ where $u_1 = \dot{y}_1$ and the parameter w declines over time due to learning and experience, the above is a linear quadratic control model and hence the optimal feedback strategies $u_i^*(t)$ can be easily calculated as

$$u_i^*(t) = [1 - bkh(t)]p(t) + bkm(t) - w \tag{37}$$

with

$$h(t) = (6k^2b^2)^{-1}[r + 4bk + 2k - \{(r + 4bk + 2k)^2 - 12k^2b^2)\}^{1/2}]$$

$$m(t) = (r - 3b^2k^2h(t) + k + 2bk)^{-1}[w - \text{as}\, h(t) - 2bkwh(t)]\,.$$

Here zero conjectural variation on the part of each player is assumed. Clearly this linear feedback form of the conjectural equilibrium output path in (36) yields a steady state price level p^* as

$$p^* = [2b(1 - bkh^* + 1]^{-1}[a + 2b(w - bkm^*)]$$

where h^*, m^* are the steady state values of $h(t)$ and $m(t)$ respectively.

Stochasticity in this framework may now be introduced in two simple ways. One is in the LQG (linear quadratic Gaussian) framework with the dynamic price equation rewritten as

$$\mathrm{d}p(t) = k(\tilde{p} - p)\mathrm{d}t + \mathrm{d}v(t)$$

where $v(t)$ is a zero mean Gaussian process with stationary independent increments and a constant variance σ^2. The objective function now is to maximize the expected value of J_1 in (36). In this case the optimal $h(t)$ which is called the Kalman gain in filtering theory is directly influenced by the variances σ^2 of the error term and hence the steady state price level p^* changes due to σ^2. Generally the price level p^* gets higher with higher a. Secondly, the conjectural variation assumption that $\partial u_i/\partial u_j = 0$ for $i \neq j$ may not hold due to the presence of random noise in the market demand equation. In this case there may be stochastic instability in the convergence process and the steady state p^*, u_i^* may not be stable. In such cases the duopolists may see implicit cooperation by 'subjective random devices' as proposed by Aumann (1974). As Ohyama and Fukushima (1995) have recently shown that in dualistic market structures of the NICs in Asia, the Asian producers adopt a two-tier policy. In the domestic front they act more like a monopoly and tend to exploit all scale economies due to learning and experinece, whereas in the international market they attempt to build up implicit cooperation with the US producers, since they do not have a large base in R & D investment.

Finally, one could directly introduce risk aversion into the objective function by rewriting it as:

$$\int_0^{\infty} e^{-rt}[E\pi(t)-(1/2)\text{var}\,\pi(t)]dt$$

where $\pi(t)=\tilde{p}(t)-c_1(t)$ with E and V denoting the mean and variance of profits. Market growth can be postulated as $\tilde{p}(t)=ae^{nt}-b(\dot{y}_1+\dot{y}_2)$. The stochastic model then would incorporate learning through risk adjustment and influencing market growth through knowledge spillover and uncertain entry. This type of model analyzed by Sengupta (1983) elsewhere compares the effects of learning on the steady state levels of optimal price, quantity and the market share. If S^* denotes the steady state optimal market share and p^* the optimal price, then the following results illustrate the impact of learning:

$$\partial S^*/\partial n>0\,,\quad \partial S^*/\partial k<0$$

$$\partial S^*/\partial r<0\,,\quad \partial p^*/\partial n<0\,,\qquad \text{if}\quad n>r/2$$

$$\partial p^*/\partial r>0\quad\text{and}\quad \partial p^*/\partial k>0\,,\qquad \text{if}\quad n>r$$

These results show that increased market growth increases the market share of the dominant firm and reduces the market price. An increase in risk aversion tends to increase the steady state optimal price and also reduce the market share of the dominant firm. But it imparts more stability to the optimal trajectories. Since a risk averse optimal strategy tends to minimize variance of profits, it yields a more robust policy of price and output, when the conditions of entry are uncertain.

5. CONCLUDING REMARKS

The impact of learning by doing and human capital in the modern endogenous growth theory has been mostly deterministic. Yet the stochastic aspects arise very naturally in this framework through learning about the furture outcome of current R & D investment, expectations about future demand and even projecting a long run state of the world market. Three basic sources of uncertainty and their economic implications for instability are briefly discussed here. First, we consider learning through the optimizing process of the producer deciding on the optimal direction of technical change. Stochastic versions of the Solow model are here compared with the new growth theory formulations. Secondly, we consider learning by knowledge spillovers through international trade and empirically estimate a linear decision rule for Japan, where learning-based expectations about the future play a more dynamic role than the past history. Finally, a dynamic limit pricing model of uncertain entry is considered for a firm facing international competition and impact of learning through risk adjustment and uncertain market growth is analyzed.

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