<table>
<thead>
<tr>
<th>Title</th>
<th>ON SOME IMPLICATIONS OF DEBT FINANCING WITH LIMITED LIABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub Title</td>
<td></td>
</tr>
<tr>
<td>Author</td>
<td>DASTIDAR, Krishnendu Ghosh</td>
</tr>
<tr>
<td></td>
<td>SENGUPTA, Kunal</td>
</tr>
<tr>
<td>Publisher</td>
<td>Keio Economic Society, Keio University</td>
</tr>
<tr>
<td>Publication year</td>
<td>1998</td>
</tr>
<tr>
<td>Jtitle</td>
<td>Keio economic studies Vol.35, No.1 (1998.)  ... 45-65</td>
</tr>
<tr>
<td>Abstract</td>
<td>This paper extends the results of Brander and Lewis (1986).</td>
</tr>
<tr>
<td></td>
<td>It derives the reaction functions when firms are debt</td>
</tr>
<tr>
<td></td>
<td>financed with limited liability. It shows with the help of</td>
</tr>
<tr>
<td></td>
<td>an example that reaction functions which are downward</td>
</tr>
<tr>
<td></td>
<td>sloping when firms are completely equity financed may</td>
</tr>
<tr>
<td></td>
<td>become upward sloping if firms are debt financed with</td>
</tr>
<tr>
<td></td>
<td>limited liability. Then it shows that for firms with limited</td>
</tr>
<tr>
<td></td>
<td>liability certain standard results do not follow. In this</td>
</tr>
<tr>
<td></td>
<td>respect it recasts Dixit (1980) in a limited liability</td>
</tr>
<tr>
<td></td>
<td>framework and shows that firms can keep excess capacity to</td>
</tr>
<tr>
<td></td>
<td>deter entry even if the products are strategic substitutes</td>
</tr>
<tr>
<td></td>
<td>when they are completely equity financed. A couple of</td>
</tr>
<tr>
<td></td>
<td>examples are provided to show that imposition of tariff</td>
</tr>
<tr>
<td></td>
<td>may lead to either an increase in domestic country's imports</td>
</tr>
<tr>
<td></td>
<td>or to no change in imports if the foreign firm is debt</td>
</tr>
<tr>
<td></td>
<td>financed with limited liability. Had the foreign firm been</td>
</tr>
<tr>
<td></td>
<td>completely equity financed then in both the examples</td>
</tr>
<tr>
<td></td>
<td>imposition of tariff would have decreased imports as</td>
</tr>
<tr>
<td></td>
<td>normally expected.</td>
</tr>
<tr>
<td>Notes</td>
<td></td>
</tr>
<tr>
<td>Genre</td>
<td>Journal Article</td>
</tr>
</tbody>
</table>

The copyrights of content available on the Keio Associated Repository of Academic Resources (KOARA) belong to the respective authors, academic societies, or publishers/issuers, and these rights are protected by the Japanese Copyright Act. When quoting the content, please follow the Japanese copyright act.
ON SOME IMPLICATIONS OF DEBT FINANCING WITH LIMITED LIABILITY

Krishnendu Ghosh DASTIDAR

Harvard University, Cambridge, U.S.A.
and Jawaharlal Nehru University, India

and

Kunal SENGUPTA

Jawaharlal Nehru University, India

First version received February 1997; final version accepted May 1998

Abstract: This paper extends the results of Brander and Lewis (1986). It derives the reaction functions when firms are debt financed with limited liability. It shows with the help of an example that reaction functions which are downward sloping when firms are completely equity financed may become upward sloping if firms are debt financed with limited liability. Then it shows that for firms with limited liability certain standard results do not follow. In this respect it recasts Dixit (1980) in a limited liability framework and shows that firms can keep excess capacity to deter entry even if the products are strategic substitutes when they are completely equity financed. A couple of examples are provided to show that imposition of tariff may lead to either to an increase in domestic country’s imports or to no change in imports if the foreign firm is debt financed with limited liability. Had the foreign firm been completely equity financed then in both the examples imposition of tariff would have decreased imports as normally expected.

INTRODUCTION

There is a growing literature in Industrial Organisation theory regarding the effect of financial structure of a firm on its output market decisions. This literature mostly stems out of the pioneering work of Brander and Lewis (1986). They showed that any firm competing in a Cournot framework with an exogenous

Acknowledgement. The authors are indebted to Sudipta Dasgupta, Anjan Mukherji, Debraj Ray and a referee of this journal for their comments. The paper was revised while K. G. Dastidar was visiting Harvard University. He gratefully acknowledges the Grant from the Ford Foundation (fellowship No. 15976021 and Grant No. 890–0264-1). The usual disclaimer applies.
demand shock would always find it optimal to become leveraged and, consequently, behave more aggressively in the product market despite this implying a decrease in market prices and lower industry profits. In recent years quite a few papers like Maksimovic (1988), Glazer (1994) and Showalter (1995) have formalized the ways in which product market decisions may both influence and be influenced by corporate financing decisions (see the survey by Harris and Raviv, 1991). Dastidar (1993) and Dastidar and Sengupta (1993) have extended the Brander and Lewis model and derived the best reaction correspondence of a firm in an output game where the firm is debt financed with limited liability. We have later come to know that Campos (1995) have also arrived at similar results independently. In this paper we intend to analyse some implications of limited liability effect of a debt financed firm. We will see that certain standard conclusions need not follow if firms are debt financed with limited liability. The plan of the paper is as follows.

In the first Section we will spell out the Brander and Lewis (1986) (hence forth called B—L) story. In Section II we will provide the model of our exercise. In this section we will also discuss about the slope of the reaction functions of firms with limited liability. In this respect we will give a specific example where the reaction function of a debt financed firm is upward sloping with infinite states of the world; whereas it would have been downward sloping had the firm been completely equity financed. In Section III we will portray the reaction function of a debt financed firm when there are finite states of the world. In Section IV we will consider a model of entry deterrence where the incumbent can finance its capital installation costs through borrowing. It will be shown that an entry deterring incumbent may install a level of capacity, a part of which is not going to be utilised ex post. It is interesting to note that had the incumbent been completely equity financed it would never had held excess capacity (see Dixit, 1980). The important thing to emphasise here is that our analysis is not tethered to any restrictions on the nature of the demand functions. Though our example in this section is based on the assumption of finite states of the world the result can easily be generalised to the case of infinite states of the world. This is clear from the example of upward sloping reaction function given in Section II, where it is shown that strategic complementarities can arise with debt financing. Lastly in Section V we will provide a couple of examples (one with finite states and the other with infinite states) to show how the imposition of tariff may lead to either increase in imports or no change in imports if the foreign firm is debt financed with limited liability.

The standard literature indicates that any outcome in oligopolistic interaction depends crucially on whether products are strategic substitutes or complements (see Bulow et al., 1985b). For example we can say that for output competition, reaction functions are downward sloping if the products are strategic substitutes whereas they are upward sloping for strategic complements. The shapes of the reaction functions in general will depend on the primitives of the models including the nature of competition (that is price or quantity competition), the form of the
demand curves, the cost structures and others. Our analysis will suggest that such classification may no longer remain valid when one considers the possibility of debt financing by the firms. This can create problems for the analysts since the extent of debt financing typically cannot be taken as a primitive of any model but has to be solved for in the content of a particular model. In this paper we will provide specific examples to show how the limited liability nature of debt may lead to counterintuitive results. We give below the B–L story which we take to be our standard framework.

I. THE B–L STORY

Consider a homogeneous product duopoly where each firm is owned by a group of risk neutral shareholders protected by limited liability, who may turn to outside investors to finance production instead of only using equity capital. It is assumed that firms can raise funds in a competitive capital market, where a group of debtholders with identical outside options is willing to supply firm \( i \) with a loan with face value \( D_{fi} \) which is payable once the product market profits have been called in. In stage 1 each firm chooses a level of debt in order to maximise its expected market value, given by the potential returns it could generate not only to its shareholders but also to its debtholders. Debt is understood, in general, as any kind of (homogeneous) monetary obligation which the firm must pay back before dividends can be distributed to shareholders. In the second stage the firms choose output levels taking as given the debt levels chosen in the first stage. Here B–L assume that the manager of the firm is free to choose whatever output level he desires after debt is issued. In the second stage output is chosen to maximise expected returns to the shareholders. The equilibrium concept is sequentially rational Nash equilibrium in debt levels and output levels. In other words, the second stage outcome is a Cournot equilibrium in output which is correctly anticipated by firms when choosing debt levels in the first stage. The output decisions of firms are made before the realisation of a random variable reflecting variation in demand. Once profits are determined, firms are obliged to pay debt claims out of operating profits, if possible. If profits are insufficient to meet the debt obligations, the firm goes bankrupt and its assets are turned over to the debtholders. In this set up and under certain general assumptions B–L derives two basic results. (i) In the financial stage of the game, both firms will always select a positive level of debt because it has a strategic effect on rival’s output. (ii) The second result is precisely this strategic effect: as a consequence of the protection offered to shareholders by limited liability, the behaviour of a leveraged firm in the product market is more aggressive relative to that of unleveraged firm.¹

We give below the specific model of our exercise which will closely follow the

¹ Campos (1995) has shown that the first result is not necessarily true. He constructs an example where zero debt is the optimal choice.
II. THE MODEL

Consider a homogeneous product duopoly with symmetric costs given by
\[ C_i(Q_i) = wQ_i + f \]
where \( Q_i \) is the output of the \( i \)th firm, \( w \) is the constant marginal cost and \( f \) is the fixed cost.

Let \( R_i(Q_1, Q_2, z) \) denote the revenue function of the \( i \)th firm, where \( z \in [z, \bar{z}] \) is a random variable with distribution function \( F(z) \) and density function \( f(z) \).

The following assumptions are made:
(a) \( R_i(Q_1, Q_2, z) \) is concave in \( Q_i \) given \( Q_j \) and \( z \), for \( \forall i \) and \( j \neq i \).
(b) \( R_i(Q_1, Q_2, z) \) and \( MR_i(Q_1, Q_2, z) \) are decreasing in \( Q_j \) and increasing in \( z \), for \( \forall i \) and \( j \neq i \). The uncertainty is resolved only after the output decisions have been made by the firms.

Let \( D_{fi} \) and \( D_{mi} \) be the face value and market value (respectively) of the debt raised by firm \( i \). In other words \( D_{mi} \) is the amount which the bondholders give to the firm which in turn promises to pay back \( D_{fi} \).

(c) Our formulation assumes that the firm has a limited liability contract vis-a-vis the debtholders, that is the debtholders received whatever is left over from the revenue, net of the costs incurred in the production stage up to a maximum of \( D_{fi} \). Here it may be noted that for simplicity we have assumed that the asset value of the firm is zero, as if assets are completely used up in the production of output. Creditors can therefore, collect only current operating profits if the firm becomes insolvent.

Let us denote the profit of the \( i \)th firm by \( \pi_i(Q_1, Q_2, z) \). Then
\[ \pi_i(Q_1, Q_2, z) = R_i(Q_1, Q_2, z) - wQ_i - f \]

For any output pair \( (Q_1, Q_2) \) the expected payoff to the firm \( i \) is given by
\[ V_i = E\left[ \max\{\pi_i(Q_1, Q_2, z) - D_{fi}, 0\}\right] + D_{mi} \]

It may be noted that
\[ V_i = \int_{\overline{z}}^{\bar{z}} \left[ \pi_i(Q_1, Q_2, z) - D_{fi} \right] f(z) dz + D_{mi} \quad (1) \]

where \( \overline{z} \) is the critical threshold determined by the following:
\[ \pi_i(Q_1, Q_2, \overline{z}) = D_{fi} \quad (2) \]

One may note that the \( i \)th firm's reaction function, given by \( \phi_i(Q_j) \), is the solution in \( Q_i \) of the following.
\[ \frac{\partial V_i}{\partial Q_i} = 0 \quad \text{(FOC)} \quad \text{and} \quad \frac{\partial^2 V_i}{\partial Q_i^2} < 0 \quad \text{(2OC)} \]

Now we have
\[
\frac{\partial V_i}{\partial Q_i} = \int_{z_i}^{z} \left[ \frac{\partial \pi_i(Q_1, Q_2, z)}{\partial Q_i} \right] f(z) \, dz - \left[ \frac{d\pi_i}{dQ_i} \right] [\pi_i(Q_1, Q_2, z_i) - D_{fi}] f(z_i)
\]

From (2) we get that \( \pi_i(Q_1, Q_2, z_i) = D_{fi} \). Therefore we have

\[
\frac{\partial V_i}{\partial Q_i} = \int_{z_i}^{z} \left[ \frac{\partial \pi_i(Q_1, Q_2, z)}{\partial Q_i} \right] f(z) \, dz = 0 \quad \text{(FOC)}
\]

From (2) we also have

\[
\frac{d\pi_i}{dQ_i} = -\frac{\partial \pi_i}{\partial Q_i} \frac{\partial Q_i}{\partial z_i}
\]

With a little bit of manipulation we get that,

\[
\frac{d^2 V_i}{dQ_i dQ_j} = \int_{z_i}^{z} \left[ \frac{\partial^2 \pi_i}{\partial Q_i \partial Q_j} f(z) \, dz + f(z_i) \frac{\partial \pi_i}{\partial Q_j} \frac{\partial \pi_i}{\partial Q_i} \right] \frac{\partial \pi_i}{\partial Q_i} \frac{\partial \pi_i}{\partial Q_j} f(z) \frac{\partial \pi_i}{\partial Q_i} \frac{\partial \pi_i}{\partial Q_j}
\]

Note that in the second term above, all the derivatives are evaluated at \( z_i \). Now from the assumptions of the model it follows that \( \frac{\partial \pi_i}{\partial Q_i} < 0 \). Also \( \frac{\partial \pi_i}{\partial Q_i} \) evaluated at \( z_i \) must be negative. This follows from the first order condition \( \frac{\partial V_i}{\partial Q_i} = 0 \) (see above) and the assumption that \( \frac{\partial \pi_i}{\partial Q_i} \) is increasing in \( z \). Also we know that \( \frac{\partial \pi_i}{\partial z} > 0 \) (follows from assumption (b)). Therefore we get that

\[
f(z_i) \frac{\partial \pi_i}{\partial Q_j} \frac{\partial \pi_i}{\partial Q_i} \frac{\partial \pi_i}{\partial Q_j} > 0.
\]

From the assumptions of our model it also follows that,

\[
\int_{z_i}^{z} \left[ \frac{\partial^2 \pi_i}{\partial Q_i \partial Q_j} f(z) \, dz < 0
\]

Therefore the sign of \( \frac{d^2 V_i}{dQ_i dQ_j} \) is uncertain.

It may be noted that the slope of the reaction function \( \phi_i(Q_j) = -\frac{\partial^2 V_i}{\partial Q_i \partial Q_j} \frac{\partial \pi_i}{\partial Q_i} \frac{\partial \pi_i}{\partial Q_j} \). The sign of the slope is uncertain as the denominator is negative (from 2OC) and the numerator can be negative or positive.\(^2\) Below we give an example where the reaction function will be downward sloping if the firm is completely equity financed but it will be upward sloping if it is debt financed with limited liability.

\textbf{An Example.} Consider the following demand function: \( P(Q_1, Q_2, z) = z^2 - Q_1 - Q_2 \), where \( z \in [0, \infty) \) and \( f(z) = e^{-z} \). For simplicity assume that costs are zero. If the firms are completely equity financed then the payoff functions are given by the following:

\[
\int_{0}^{\infty} [z^2 - Q_1 - Q_2] Q_1 e^{-z} \, dz
\]

Then the reaction function of firm \( i \) given by \( R_i(Q_j) = 1/2(2 - Q_j) \). It is obvious

\(^2\) See Campos (1995) and Dastidar and Sengupta (1993) for a similar analysis.
that $R_i(Q_j)$ is strictly decreasing in $Q_j$. However the situation changes dramatically when the firms are debt financed with limited liability and debt is incurred in the first stage and output chosen in the second stage. In the second stage the payoff to firm $i$ is as before

$$V_i = \int_{z_i}^{z_j} \left[ \pi_i(Q_1, Q_2, z) - D_i \right] f(z) dz + D_{mi}$$

where

$$\pi_i(Q_1, Q_2, z) = R_i(Q_1, Q_2, z) - wQ_i - f$$

Let $D_{fi} = 12.822$. Then in our specific example we have

$$V_i = \int_{z_1}^{\infty} \left[ (z^2 - Q_i - Q_j)Q_i - 12.822 \right] e^{-z} dz + D_{mi}$$

where $z_i = (12.822/Q_i + Q_i + Q_j)^{0.5}$ (from (2)).

Let the reaction function of the debt financed firm $i$ be given by $\phi_i(Q_j)$. Now $\phi_i(Q_j)$ will be given implicitly by the solution in $Q_i$ of $\partial V_i/\partial Q_i = 0$ and that is the following in our case.

$$12.822/Q_i - Q_i + 2(12.822/Q_i + Q_i + Q_j)^{0.5} + 2 = 0$$

Note that $\phi_i(0) = 10$. By implicit differentiation and algebraic substitution we get $\phi_i'(0) > 0$. Figure (1) shows both $R_i(Q_j)$ and $\phi_i(Q_j)$.

That is, we have shown that products which are strategic substitutes when firms are completely equity financed may turn into strategic complements when they
are debt financed with limited liability. This has important implications for some Industrial Organisation results (see Bulow et al. (1985a and b)).

In the next section we will derive the reaction function when the states of the world (given by $z$) are finite.

III.

We adhere to the model described in Section II but make the following additional assumption.

(d) For simplicity we assume that $z$ can take only two values $L$ or $H$ (where $L<H$) with probabilities $p$ and $(1-p)$ respectively. It will be clear later that the model can be extended to more than two values of $z$. For the sake of convenience, we will call $L$ the lower state and $H$ the higher state.

Let us define the following,

\[ t_1(Q_2) = \arg \max_{Q_1\geq 0} \left[ p \pi_1(Q_1, Q_2, L) + (1-p) \pi_1(Q_1, Q_2, H) \right] \]

\[ v_1(Q_2) = \arg \max_{Q_1\geq 0} \pi_1(Q_1, Q_2, L) \]

\[ v_H(Q_2) = \arg \max_{Q_1\geq 0} \pi_1(Q_1, Q_2, H) \]

When a firm is completely equity financed it maximises the value of its expected profits given the action (which is the choice of output in a quantity game) of its rival. Therefore it may be noted that $v(Q_2)$ is $1$'s reaction function when completely equity financed. And $t_1(Q_2)$ is $1$'s reaction function when only the lower state prevails and $v_H(Q_2)$ is its reaction function when only the higher state prevails.

The following results may be noted.

**Lemma 1.** $v_H(Q_2) > v(Q_2) > t_1(Q_2), \forall Q_2 \geq 0$

*Proof.* $v(Q_2)$ is the solution in $Q_1$ of the following,

\[ pMR_1(Q_1, Q_2, L) + (1-p)MR_1(Q_1, Q_2, H) = w \]

That is we may write,

\[ pMR_1(v(Q_2), Q_2, L) + (1-p)MR_1(v(Q_2), Q_2, H) = w \tag{3} \]

Similarly for $v_H(Q_2)$ we can write,

\[ MR_1(v_H(Q_2), Q_2, H) = w \tag{4} \]

Since $MR_1(Q_1, Q_2, L) < MR_1(Q_1, Q_2, H)$ [see assumption (b)], it follows from (3) that,

\[ MR_1(v(Q_2), Q_2, L) < w < MR_1(v_H(Q_2), Q_2, H) \tag{5} \]

Suppose on the contrary $v_H(Q_2) \leq v(Q_2)$.

From assumption (a) $R_1(Q_1, Q_2, L)$ is concave $\Rightarrow MR_1(Q_1, Q_2, L)$ is decreasing in $Q_1$. Therefore,
**LEMMA 1.** \(MR_1(\psi(Q_2), Q_2, H) \leq MR_1(\psi_H(Q_2), Q_2, H)\)

But from (4) \(MR_1(\psi_H(Q_2), Q_2, H) = w\). The above implies that \(w \geq MR_1(\psi(Q_2), Q_2, H)\) which contradicts (5). Therefore \(\psi_H(Q_2) > \psi(Q_2)\).

By an argument analogous to the one given above we can show that \(\psi(Q_2) > \psi_L(Q_2), \forall Q_2 \geq 0\).

**Comment.** Lemma 1 implies that if only the higher state prevails then a firm would bring more output to the market as a response to the other firm’s output than in the case of an average expected state which in turn brings forth more output as best response as compared to the case where only the lower state prevails.

**LEMMA 2.** \(\psi(Q_2), \psi_L(Q_2)\) and \(\psi_H(Q_2)\) are strictly decreasing in \(Q_2\).

**Proof.** Trivial.

**Comment.** The above Lemma implies that the reaction functions are strictly downward sloping for equity financed firms.

**LEMMA 3.** \(\pi_1(\psi(Q_2), Q_2, 0), \pi_1(\psi_L(Q_2), Q_2, L)\) and \(\pi_1(\psi_H(Q_2), Q_2, H)\) are strictly decreasing in \(Q_2\).

**Proof.** Trivial.

**Comment.** The above result implies that if the \(i\)th firm sticks to its reaction function then its profits decreases with increases in the \(j\)th firm’s output.

With the above preliminaries out of the way, we now intend to portray clearly a debt financed firm’s reaction function and then investigate the differences in the reaction functions of a debt financed firm vis-a-vis an equity financed one. We will, for convinence’s sake restrict our analysis to firm 1 only. Firm 2, being identical to firm 1, will display similar reaction function properties.

When firm 1 is debt financed it raises debt whose face value is \(D_{f1}\). Then in the second stage of the game (that is the stage where it chooses output) it selects an output level which maximises the following expression,

\[
p \max[\pi_1(Q_1, Q_2, L) - D_{f1}, 0] + (1-p) \max[\pi_1(Q_1, Q_2, H) - D_{f1}, 0]
\]

Let us denote the above expression by \(M\).

Define \(\psi_d(Q_2) = \arg \max_{Q_2 \geq 0} M\).

One may note that \(\psi_d(Q_2)\) is firm 1’s reaction function when it is debt financed with limited liability.

**LEMMA 4.** (i) If \(D_{f1} \leq \pi_1(\psi_L(0), 0, L), \exists Q_2^{\text{min}}(D_{f1}) \) for each \(D_{f1} \) s.t.

\[
\pi_1(\psi_L(Q_2^{\text{min}}(D_{f1}), Q_2^{\text{min}}(D_{f1}), L) = D_{f1}
\]

(ii) \(\pi_1(Q_1, Q_2, L) \leq D_{f1}, \forall Q_2 \geq Q_2^{\text{min}}(D_{f1})\) and \(\forall Q_1 \geq 0\).

**Proof.** (i) Since \(\psi_L(Q_2)\) is strictly decreasing in \(Q_2\) (see Lemma 2) \(\exists \tilde{Q}_2\) s.t.
\[ \psi_L(\tilde{Q}_2) = 0. \]

From lemma 3, \( \pi_1(\psi_L(Q_2), Q_2, L) \) is strictly decreasing in \( Q_2 \). Now \( \pi_1(\psi_L(0), 0, L) \geq D_{f_1} \) (given) and \( \pi_1(\psi_L(\tilde{Q}_2), \tilde{Q}_2, L) = 0 \). Since \( \pi_1(Q_1, Q_2, \theta) \) is continuous in \( Q_2 \), \( \exists Q_2^{\text{min}}(D_{f_1}) \) s.t.

\[ \pi_1(\psi_L(Q_2^{\text{min}}(D_{f_1}), Q_2^{\text{min}}(D_{f_1}), L) = D_{f_1} \]

(ii) Since \( \pi_1(\psi_L(Q_2), Q_2, L) \) is decreasing in \( Q_2 \),

\[ \pi_1(\psi_L(Q_2), Q_2, L) \leq D_{f_1} \forall Q_2 \geq Q_2^{\text{min}}(D_{f_1}) \]

i.e. \( \pi_1(Q_1, Q_2, L) \leq D_{f_1} \forall Q_2 \geq Q_2^{\text{min}}(D_{f_1}) \) because \( \psi_L(Q_2) \) is the best response to \( Q_2 \) and so \( \pi_1(\psi_L(Q_2), Q_2, L) \) is the maximum profit possible given \( Q_2 \).

The above result implies that if the face value of the debt is less than or equal to monopoly profit in the lower state then if 2's output exceeds a certain minimum value then profits of firm 1 will fail to cover debts in the lower state. One may note that if \( D_{f_1} = \pi_1(\psi_L(0), 0, L) \) then \( Q_2^{\text{min}}(D_{f_1}) = 0. \)

**Lemma 5.**

(i) If \( D_{f_1} \leq \pi_1(\psi_L(0), 0, H) \), \( \exists Q_2^{\text{max}}(D_{f_1}) \) for each \( D_{f_1} \) s.t.

\[ \pi_1(\psi_L(Q_2^{\text{max}}(D_{f_1}), Q_2^{\text{max}}(D_{f_1}), H) = D_{f_1} \]

(ii) \( \pi_1(Q_1, Q_2, H) \leq D_{f_1}, \forall Q_2 \geq Q_2^{\text{max}}(D_{f_1}) \) and \( Q_1 \geq 0 \).

**Proof.** The argument is analogous to that of Lemma 4.

**Lemma 6.**

(i) \( Q_2^{\text{min}}(D_{f_1}) \) is decreasing in \( D_{f_1} \) when \( D_{f_1} \in [0, \pi_1(\psi_L(0), 0, L)] \).

(ii) \( Q_2^{\text{max}}(D_{f_1}) \) is decreasing in \( D_{f_1} \) when \( D_{f_1} \in [0, \pi_1(\psi_L(0), 0, H)] \).

**Proof.** (i) For \( D_{f_1} \in [0, \pi_1(\psi_L(0), 0, L)] \) we have (from Lemma 5)

\[ \pi_1(\psi_L(Q_2^{\text{min}}(D_{f_1}), Q_2^{\text{min}}(D_{f_1}), L) = D_{f_1} \]

Now if we increase \( D_{f_1} \) (i.e. the R.H.S. of the above equation), then to maintain the above equality \( Q_2^{\text{min}}(D_{f_1}) \) must come down as \( \pi_1(\psi_L(Q_2), Q_2, L) \) is decreasing in \( Q_2 \). Hence \( Q_2^{\text{min}}(D_{f_1}) \) is decreasing in \( D_{f_1} \).

(ii) Argument analogous to the one given above.

We are now in a position to consider the reaction function of a debt financed firm.

Suppose \( D_{f_1} \leq \pi_1(\psi_L(0), 0, L) \). Then we have \( D_{f_1} \leq \pi_1(\psi_L(0), 0, H) \). Now one may recall that,

\[ \psi_D(Q_2) = \arg_{Q_2 \geq 0} \max \left[ p \max(\pi_1(Q_1, Q_2, L) - D_{f_1}, 0) + (1 - p) \max(\pi_1(Q_1, Q_2, L) - D_{f_1}, 0) \right] \]

Now

\[ Q_2 \leq Q_2^{\text{min}}(D_{f_1}) \Rightarrow \pi_1(\psi_L(Q_2), Q_2, L) \geq D_{f_1} \Rightarrow \pi_1(\psi_L(Q_2), Q_2, H) > D_{f_1} \]

that is for \( Q_2 \leq Q_2^{\text{min}}(D_{f_1}) \), maximum profit net of debt is positive (or rather
non-negative) for all states. Therefore we get the following:

\[ Q_2^{\min}(D_{f1}) < Q_2 \leq Q_2^{\max}(D_{f1}) \Rightarrow \pi_1(Q_1, Q_2, L) < D_{f1}, \quad \forall Q_1, Q_2. \]

The above means that if \( Q_2^{\min}(D_{f1}) < Q_2 \leq Q_2^{\max}(D_{f1}) \) then in the lower state the owners (i.e. the shareholders) of the firm net zero because whatever operating profits are available are given to the bondholders. Therefore the owners will only be interested in maximising profits in the higher state. This follows because the owners are indifferent between small and large losses but prefer more (positive) profits (net of debt) to less.

When \( Q_2 > Q_2^{\max}(D_{f1}) \) then \( \pi_1(Q_1, Q_2, \theta) < D_{f1} \) for \( \theta = L \) or \( H \), that is the owners of the firm net zero in all states as whatever operating profits are available are handed over to the bondholders.

Therefore from the above discussion it is apparent that

\[
\psi_p(Q_2) = \psi(Q_2), \quad \text{when} \quad Q_2 \leq Q_2^{\min}(D_{f1})
\]

\[
= \psi_H(Q_2), \quad \text{when} \quad Q_2^{\min}(D_{f1}) < Q_2 \leq Q_2^{\max}(D_{f1})
\]

\[
= 0, \quad \text{when} \quad Q_2 > Q_2^{\max}(D_{f1})
\]

It may be noted that when \( Q_2 > Q_2^{\max}(D_{f1}) \), owners of the firm net zero in every state, irrespective of whatever it brings in the market. Here we implicitly assume that it will prefer to produce zero quantity, though really it is indifferent to its quantity choices. Figure 2 shows the relevant reaction function when the firm is debt financed with limited liability.

In Figure 2 above the darker lines show firm 1’s reaction function, when it is debt financed with limited liability. As noted earlier its reaction function is \( \psi(Q_2) \)
when it is completely equity financed. However when it is debt financed with limited liability its reaction function is quite different. For $Q_2 \leq Q_2^{\min}(D_{f1})$, it is same as that of the equity financed case. However, for $Q_2^{\min}(D_{f1}) < Q_2 \leq Q_2^{\max}(D_{f1})$, it switches over to the higher state reaction function, i.e. when $Q_2$ lies in that range its reaction function is $\psi_H(Q_2)$. One may note that for the range of $Q_2$ given by $Q_2^{\min}(D_{f1}) < Q_2 < Q_2^{\max}(D_{f1})$, firm 1 displays more aggressive behaviour in the sense that it brings more output in the market as best response to 2's output than it would have had it been completely equity financed. It may also be seen that more is $D_{f1}$, less is $Q_2^{\min}(D_{f1})$, and hence more is $D_{f1}$ more quickly will firm 1 respond aggressively to firm 2's output. Or higher is $D_{f1}$, firm 1's reaction function jumps to coincide with $\psi_H(Q_2)$ for lower values of $Q_2$. It may be noted that firm 2 will have a similar reaction function and it is clear that reaction functions may not intersect; implying thereby the non existence of pure strategy Cournot equilibrium. (see also Campos (1995)).

IV.

We will now consider a model of entry deterrence where the incumbent can finance its capacity installation costs through borrowing or incurring a debt. In a similar set up for the equity financed case Dixit (1980) shows that if $MR_i$ is decreasing in $Q_i$ (which in turn implies that products are strategic substitutes) excess capacity will not appear in equilibrium. Bulow et al. (1985a), however, show that this may not hold true for strategic complements. We will show that an entry deterring incumbent might install a level of capacity, a part of which is not going to be utilised ex post. The important thing to emphasise here is that our analysis is not tethered to any restrictions on the nature of demand functions. In that way it is much more general in its applicability and provides the raison de etre for maintaining excess capacity. Though our example in this Section is based on assumption of finite states of the world the result can be generalised to infinite states of the world as well and this is evident from the example given in Section III, where it was shown strategic complementarities can arise with debt financing. Because of computational simplicity we assume finite states of the world.

We follow closely the structure of Dixit (1980). There are two firms, an incumbent (1) and a potential entrant (2). Output requires labour and capital and is produced under a Leontief technology. Let $r_i$ and $w_i$ denote the capital and labour cost per unit of output for the $i$th firm. Let $f_i$ denote the fixed cost of operating in the industry.

The incumbent firm 1 first chooses to install a capacity level $k$. Capacity can be added later on but is not reducible. The installed capacity thus represents a sunk cost for 1. Therefore the MC for producing upto $k$ level of output is $w_1$, and to produce beyond $k$ the $MC$ is $(w_1 + r_1)$ as capacity has to be added on to make the output possible. Firm 1 also decides to raise a certain amount of debt to finance its capital and fixed cost expenditure.
The potential entrant (2) observes this choice and decides whether to enter the market or not. It will enter only if it expects a positive payoff in the post entry equilibrium. It may be noted that 2 does not have a capacity precommitment but can use debt to finance its entry cost. If entry does take place, the incumbent, acting as a monopolist, selects an output level. With entry the entrant chooses capacity and quantity simultaneously and a Nash equilibrium in output is established immediately.

Let
\[ \pi_1(Q_1, Q_2, z) = R_1(Q_1, Q_2, z) - w_1 Q_1 - \max[(Q_1 - k) r_2, 0] \]
and
\[ \pi_2(Q_1, Q_2, z) = R_2(Q_1, Q_2, z) - (w_2 + r_2) Q_2 \]

Assuming that the debt holders are risk-neutral and debt is fairly priced, the market value of debt, \( D_{mi} \) corresponding to the face value \( D_f \) is given by
\[
D_{mi} = p \min[\max(\pi_i(Q_1, Q_2, L), 0), D_f] \\
+ (1 - p) \min[\max(\pi_i(Q_1, Q_2, H), 0), D_f] 
\]

When firms are completely equity financed for any output pair \((Q_1, Q_2)\), the expected payoffs to the incumbent is given by,
\[ E_1 = p \pi_1(Q_1, Q_2, L) + (1 - p) \pi_1(Q_1, Q_2, H) - r_1 k - f_1 \]
and the payoff to the entrant is given by,
\[ E_2 = p \pi_2(Q_1, Q_2, L) + (1 - p) \pi_2(Q_1, Q_2, H) - f_2 \]

When completely equity financed the entrant will enter if \( E_2 > 0 \) and it gets zero when it does not enter.

Similarly when firms are debt financed their payoffs for each output pair is given by,
\[ T_1 = p \max[\pi_1(Q_1, Q_2, L) - D_{f1}, 0] \\
+ (1 - p) \max[\pi_1(Q_1, Q_2, H) - D_{f1}, 0] - r_1 k - f_1 + D_{m1} \]

The expected payoff to the entrant is given by,
\[ T_2 = p \max[\pi_2(Q_1, Q_2, L) - D_{f2}, 0] \\
+ (1 - p) \max[\pi_2(Q_1, Q_2, H) - D_{f2}, 0] - f_2 + D_{m2} \]

The entrant gets zero if it does not enter. It enters if \( T_2 > 0 \).

The Example
Consider the following scenario. The inverse demand is given by the following:
\[ P = [10 - (Q_1 + Q_2)] z, \]
where the random variable \( z \) can take two values \( L = 20/23 \) and \( H = 3 \) with probabilities 46/49 for \( L \) and 3/49 for \( H \). That is \( z \) has an expected value which is given by \( E = 1 \). The revenue function of the \( i \)th firm is therefore
given by,

\[ R_t(Q_t, Q_2, \theta) = Q_t[10 - (Q_t + Q_2)]z \]

The cost structure of the two firms are symmetric and given as follows: \(w_1 = w_2 = 3\), \(r_1 = r_2 = 1\), and \(f_1 = f_2 = 1\).

**Equilibrium with equity financed firms**

In Figure 3, the line AB denotes the reaction function of the entrant. Note that in the post entry game, the marginal cost of the entrant is 4, while the expected marginal revenue is given by \((10 - 2Q_2 - Q_t)\). (Recall that the expected value of \(\theta\) is 1). AB thus represents the equation \(Q_2 = (6 - Q_t)/2\).

The line CD represents the reaction function of the incumbent under the assumption that \(k = \infty\), that is capital can be added free of charge. The marginal cost of the incumbent in this case is thus 3, representing only the labour costs. CD therefore represents the equation \(Q_1 = (7 - Q_2)/2\). The line EF represents the incumbent’s reaction function when it has no capacity precommitment (that is when its marginal cost is 4 throughout). EF represents the equation \(Q_1 = (6 - Q_2)/2\).

Point T (where \(Q_1 = 2 = Q_2\)) and point V (where \(Q_1 = 8/3\) and \(Q_2 = 5/3\)) are the points of intersection of AB and CD and AB and EF respectively. It may be noted that since the entrant’s reaction function is \(Q_2 = (6 - Q_1)/2\), in the output game the entrant gets a profit of \((6 - Q_1)^2/4\) along its reaction function. One can see
that entrant’s profit is decreasing in $Q_1$. Now at point $V$ we have, $E_2 = 16/9 > 0$. At point $T$ also $E_2 = 3 > 0$.

Therefore following Dixit (1980) we can say that entry always occurs and the following proposition characterises the unique equilibrium outcome for the equity financed firms.

**Proposition 1.** Under equity financing, the following is the unique equilibrium outcome: the incumbent installs a capacity $k^* = 8/3$, entry occurs and the output choices are $Q_1^* = 8/3$ and $Q_2^* = 5/3$. Payoff to firm 1 is $31/9$ and the payoff to firm 2 is $16/9$.

**Equilibrium with Debt Financing**

Let firm 1 incur a debt whose face value is given by $D_{f1} = 6.697$. It will be later clear that this choice of debt is also optimal in a subgame perfect equilibrium. Now from our analysis in the previous section we can say that 1’s reaction function with infinite capacity precommitment (that is when its marginal cost is 3 throughout) is as follows:

$$Q_1 = \frac{7 - Q_2}{2} \quad \text{for } 0 < Q_2 < 1 \quad (DM \text{ in Figure 4})$$

$$Q_1 = \frac{9 - Q_2}{2} \quad \text{for } 1 < Q_2 < 6.018 \quad (SN \text{ in Figure 4})$$

$$Q_1 = 0, \quad \text{for } Q_2 \geq 6.018$$

It should be pointed out that we have approximated 0.99966 by 1.0 to facilitate calculations.

Consider first the case when the entrant is *equity financed*. Figure 4 depicts equilibrium in the output game with an installed capacity of 4 units and $D_{f1} = 6.697$. The line $AB$ as before represents the reaction function for the equity financed entrant while the reaction function of the incumbent is given by the darker broken line $DM \ SN$.

When the entrant is equity financed the equilibrium occurs at $S$ (where $Q_1 = 4, Q_2 = 1$). Here the payoff to the entrant is zero. Therefore it will not enter. And the incumbent will now act as a monopolist and maximise the following expression,

$$p \max [R_1(Q_1, 0, L) - w_1 Q_1 - D_{f1}, 0] + (1 - p) \max [R_1(Q_1, 0, H) - w_1 Q_1 - D_{f1}, 0] - r_1 k - f_1 + D_{m1}$$

Putting the values of $L, H$ etc. it can be seen that the above expression is maximised at $Q_1 = 3.5$. Recall that the incumbent had set up a capacity of 4. Therefore 0.5 units of capacity are left idle as a credible sign of deterring entry. Now it may be noted that $D_{m1} = D_{f1} = 6.697$ [from (6)]. And from (9) we get that the payoff to firm 1 by deterring entry is $T_1 = 7.25$. Had it allowed entry by being equity financed then it would have netted $31/9$, which is less than 7.25. Hence it deters entry by financing expenditure through debt. Also it may be noted that if the incumbent chooses a debt whose face value is less than 6.697, $Q_2^{\text{fin}}$ would go
up. In that case either there is no pure strategy equilibrium or if there is one it occurs at point V where we have shown that the firm gets less payoff than it would have if the face value were 6.697. Choice of debt with a face value more than 6.697 also does not increase payoff. Therefore the choice of debt is optimal.

Here it may be noted that had firm 1 installed a capacity of less than 4 units the equilibrium would have been to the left of point S (see Figure 4). There 2's payoff is greater than zero. Hence a capacity level of at least 4 units is necessary to deter entry. Installing more capacity is useless as it is never going to be used.

Now consider the case when firm 2 is debt financed. 2's reaction function when
it is debt financed is given by (see Section IV).

\[ Q_2 = \begin{cases} 
(6 - Q_1)/2 & \text{for } 0 \leq Q_1 < Q_1^{\text{min}} \\
(26/3 - Q_1)/2 & \text{for } Q_1^{\text{min}} \leq Q_1 < Q_1^{\text{max}} \\
0 & \text{for } Q_1 \geq Q_1^{\text{max}}
\end{cases} \]

Now the question is how much debt will 2 take. That is what is the value of \( D_{f2} \) which in turn will determine the value of \( Q_1^{\text{min}} \) and \( Q_1^{\text{max}} \). One may note that from (10) the payoff to firm 2 is given by the following:

\[ T_2 = p \max[\pi_2(Q_1, Q_2, L) - D_{f2}, 0] + (1 - p) \max[\pi_2(Q_1, Q_2, H) - D_{f2}, 0] - f_2 + D_{m2} \]

Drawing upon our previous analysis we can say that, for \( Q_1^{\text{max}} \geq Q_1 \geq Q_1^{\text{min}} \), by the definition of \( Q_1^{\text{min}} \),

\[ T_2 = (1 - p) \max[\pi_2(Q_1, Q_2, H) - D_{f2}, 0] - f_2 + D_{m2} \]

because

\[ \max[\pi_2(Q_1, Q_2, L) - D_{f2}, 0] = 0 \quad \text{for } Q_1 \geq Q_1^{\text{min}} \]

Now for \( Q_1^{\text{max}} \geq Q_1 \geq Q_1^{\text{min}} \) firm 2’s reaction function is given by \( Q_2 = (26/3 - Q_1)/2 \). This reaction function yields a profit of \( 3(26/3 - Q_1)^2/4 \) in the output game.

Note that from (6) for

\[ Q_1^{\text{max}} \geq Q_1 \geq Q_1^{\text{min}}, \quad D_{m2} = (1 - p) \min[\max(\pi_1(Q_1, Q_2, H), 0), D_{f1}] \]

\[ \Rightarrow \quad \text{for } Q_1^{\text{max}} \geq Q_1 \geq Q_1^{\text{min}}, \quad D_{m2} = (1 - p)D_{f1} \]

Therefore for \( Q_1^{\text{max}} \geq Q_1 \geq Q_1^{\text{min}} \) the payoff to firm 2 is given by, \( (1 - p)(26/3 - Q_1)^2/4 - f_2 \).

Now note that payoff to firm 2 when it is completely equity financed is \( (6 - Q_1)^2/4 - f_2 \) in the output game. Therefore it pays to be debt financed (that is switching over to the higher order reaction function) if the following is true:

For \( Q_1^{\text{max}} \geq Q_1 \geq Q_1^{\text{min}} \).

\[ (1 - p)(26/3 - Q_1)^2/4 - f_2 > (6 - Q_1)^2/4 - f_2 \]

Since \( p = 46/49 \), the above inequality is true for \( Q_1 > 4 \).

Therefore it is optimal for firm 2 to choose a debt level so that \( Q_1^{\text{min}} \) is approximately 4. That is firm 2 incurs a debt whose face value is given by \( D_{f2} = 49/115 \). Then \( Q_1^{\text{max}} = 7.912 \).

In Figure 4 firm 2’s reaction function when it is debt financed is given by the darker broken line AS–UW. It can now be checked that the post entry equilibrium occurs at point S, where \( Q_1 = 4 \) and \( Q_2 = 1 \) (see Figure 4). Here the payoff to firm 2 is zero. Hence it does not enter. As before firm 1 produces 3.5 units of output.
ON SOME IMPLICATIONS OF DEBT FINANCING WITH LIMITED LIABILITY 61

and holds 0.5 units of idle capacity.  

Hence we come to the following result which we state in the form of a proposition.

**Proposition 2.** A debt financed incumbent with limited liability can keep excess capacity as a credible threat to deter entry in equilibrium. This may happen even when the reaction function of the incumbent, when it is completely equity financed, is downward sloping.

Comment. We have seen that firm 1 cannot deter entry when it is completely equity financed. Though by precommitting to capital it becomes aggressive, such aggressiveness is not sufficient to deter entry. However if in addition to capacity precommitment it also precommits to a debt level and becomes more aggressive; that is be a top dog (to use the terminology of Fudenberg and Tirole, 1984), then entry can be optimally deterred as we have seen.

V. TARIFFS AND DEBT FINANCING

We will now provide two examples which shows that imposition of tariff may lead to either an increase in imports or to no change in imports when the foreign firm is debt financed with limited liability. It may be noted that in both the examples imposition of tariff leads to decrease in imports if the foreign firm is completely equity financed. Consider the following scenario. In the domestic market of a certain good two firms are competing. Firm 1 is the foreign firm which is debt financed with limited liability. Firm 2 is the domestic firm which is assumed to be completely equity financed. For simplicity assume that both firms do not compete in any other market. Now, $Q_1$, the amount sold by the foreign firm is also the import of the country in question. $Q_2$ is the amount sold by the domestic firm.

The 1st example (With finite states of the world)

Let the inverse demand curve of the good be given by, $P = z(10 - Q_1 - Q_2)$, where $z$ is a random variable which can take the value $H = 1.5$ with probability $p = 0.5$ and the value $L = 0.75$ with probability $(1 - p) = 0.5$. It may be noted that $z$ has an expected value of 1. Both firms have symmetric cost functions given by $C(Q) = Q$. Let firm 1 incur a face value of debt given by $D_{f1} = 5$.

Firm 2 is completely equity financed. Both firms compete in quantities. First let us consider the case when there are no tariffs imposed on firm 1's outputs.

---

3 How sensitive is our result of excess capacity to the particular specification of the game form? Recall that in this model (as in Dixit, 1980 and Bulow, Geanakoplos and Klemperer, 1985a), after entry, the entrant chooses capacity and output simultaneously. Ware (1984), in his analysis breaks the game into another stage, where after entry but prior to production, the entrant decides on the choice of capacity. This alternative formulation, (as Ware has shown) gives the entrant a way to precommit to a larger output share in the output game, thereby increasing the payoff to the entrant. This is also true in our model.
The reaction functions of the two firms is as follows (see the discussions in the previous sections). Firm 2’s reaction function is \( R_2(Q_1) = \frac{1}{2}(9 - Q_1) \). Firm 1’s reaction function is as follows,

\[
R_1(Q_2) = \begin{cases} 
\frac{1}{2}(9 - Q_2) & \text{for } 0 \leq Q_2 \leq Q_2^{\min} = 3.5026 \\
\frac{1}{3}(14 - 1.5Q_2) & \text{for } 3.5026 < Q_2 < Q_2^{\max} = 5.68 \\
0 & \text{for } Q_2 > 5.68
\end{cases}
\]

Therefore the pre tariff Cournot–Nash equilibrium is given by the following: \( Q_1 \) (pre tariff) = \( Q_2 \) (pre tariff) = 3. That is the domestic country imports 3 units of the commodity.

Now suppose the government of the domestic country decides to impose a per unit tariff \( t = 0.4 \) on the quantity sold by the foreign firm (that is firm 1). Now the reaction functions become

\[
R_2(Q_1) = \frac{1}{2}(9 - Q_1)
\]

and

\[
R_1(Q_2) = \begin{cases} 
\frac{1}{2}(8.6 - Q_2) & \text{for } 0 \leq Q_2 \leq Q_2^{\min} = 2.969 \\
\frac{1}{3}(13 - 1.5Q_2) & \text{for } 2.969 < Q_2 < Q_2^{\max} = 5.415 \\
0 & \text{for } Q_2 > 5.415
\end{cases}
\]

The post tariff equilibrium quantities are as follows:
\( Q_1 \) (post tariff) = 3.044, \( Q_2 \) (post tariff) = 2.977.

Hence the imposition of tariffs increases the import by 0.044 units. The intuitive logic is as follows. In the post tariff situation firm 1 becomes more aggressive for smaller outputs of firm 2 (\( Q_2^{\min} \) goes down) as compared to the pre tariff scenario as 1 makes losses in the lower states due to tariff and so maximises profits in the higher states. Since to counter the tariff it becomes more aggressive, the equilibrium output of firm 1 goes up. Had firm 1 been completely equity financed \( Q_1 \) (post tariff) would have gone down to 2.7333 and \( Q_2 \) (post tariff) would have gone up to 3.1333.

The 2nd example With infinite states of the world

Consider the same story as above. However now let the demand function be \( P = z - Q_1 - Q_2 \), where \( z \in [0, \infty) \) and \( f(z) = e^{-z} \).

Let costs be given by
ON SOME IMPLICATIONS OF DEBT FINANCING WITH LIMITED LIABILITY

\[ C(Q_i) = \frac{1}{2} Q_i \quad \text{for} \quad i = 1, 2 \]

Since the domestic firm 2 is completely equity financed its payoff function is

\[ \pi_2 = \int_0^\infty \left[ z - Q_1 - Q_2 - \frac{1}{2} \right] Q_2 e^{-z} dz \]

The reaction function, \( R_2(Q_1) \), is given by the solution in \( Q_2 \) of \( \partial \pi_2 / \partial Q_2 = 0 \). That is

\[ \int_0^\infty \left[ z - Q_1 - 2Q_2 - \frac{1}{2} \right] e^{-z} dz = 0 \]

Therefore

\[ R_2(Q_1) = \frac{1}{2} \left[ \frac{1}{2} - Q_1 \right] \] (11)

Let the foreign firm (firm 1) incur a debt with face value \( D \). Then before the imposition of tariff the foreign firm’s payoff function is given by

\[ V_1 = \int_{\tilde{z}^i}^\infty \left[ \left( z - Q_1 - Q_2 - \frac{1}{2} \right) Q_1 - D \right] e^{-z} dz \]

where,

\[ \tilde{z}^i = D/Q_1 + Q_1 + Q_2 + \frac{1}{2} \quad \text{(see discussions in Section II).} \]

Firm 1’s reaction function \( R_1(Q_2) \) is given by the solution in \( Q_1 \) of \( \partial V_1 / \partial Q_1 = 0 \). i.e.

\[ \int_{\tilde{z}^i}^\infty \left[ z - 2Q_1 - Q_2 - \frac{1}{2} \right] e^{-z} dz = 0 \]

\[ \Rightarrow e^{-\tilde{z}^i} \left[ \tilde{z}^i - 2Q_1 - Q_2 - \frac{1}{2} \right] = 0 \]

Since \( \tilde{z}^i = D/Q_1 + Q_1 + Q_2 + 1/2 \) and \( e^{-\tilde{z}^i} > 0 \), we get after a bit of manipulation that

\[ R_1(Q_2) = \frac{1}{2} \left[ 1 + (1 + D)^{0.5} \right] \] (12)

Now suppose that the government of country 2 imposes a per unit tariff of “\( t \)” on the output of firm 1 (the foreign firm). Then the foreign firm’s payoff function becomes as follows,
KRISHNENDU GHOSH DASTIDAR and KUNAL SENGUPTA

\[ V_1 = \int_{-\infty}^{\infty} \left( z - Q_1 - Q_2 - \frac{1}{2}t \right) Q_1 - D \right] e^{-z} dz \]

where,

\[ z^i = D/Q_1 + Q_1 + Q_2 + \frac{1}{2} + t \quad \text{(as before)} \]

Now with the same analysis as before firm 1's reaction function after the imposition of tariff remains the same i.e. post tariff

\[ R_1(Q_2) = \frac{1}{2} \left[ 1 + (1 + D)^{0.5} \right] \]

It may be noted that \( R_1(Q_2) \) is not affected by change in \( "t" \). Therefore the equilibrium in the pre tariff and post tariff scenarios are same. Hence the imposition of tariff has no effect on the imports into the domestic country. Also it may be noted that our conclusions remain the same in this case had firm 2 been debt financed with limited liability. This example and the previous one therefore shows that any trade policy must take into account the financial structure of the firms because the outcomes may be radically different. The limited liability nature of debt changes the firm's best response functions and this leads to vastly different outcomes.

REFERENCES


