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| Title | LONG－RUN FISH STOCK AND IMPERFECTLY COMPETITIVE INTERNATIONAL COMMERCIAL <br> FISHING |
| :---: | :--- |
| Sub Title |  |
| Author | OKUGUCHI，Koji |
| Publisher | Keio Economic Society，Keio University |
| Publication year | 1998 |
| Jtitle | Keio economic studies Vol．35，No．1（1998．），p．9－17 |
| JaLC DOI |  |
| Abstract | In this paper fishery economics and recent theory of international trade under imperfect <br> competition are integrated．For this purpose，international commertial fishing in an open－sea is <br> formulated and its dynamic properties are analyzed under the condition of imperfect competition <br> with two trading countries．The fish stock may become extinct or converge to a steady state level． <br> A sufficient condition is derived for the fish stock to converge to the steady state in relation to the <br> parameters of the fish＇s biological growth equation，of the harvesting costs and of the demand <br> functions for the fish in the two countries． |
| Notes | Journal Article |
| Genre | https：／／koara．lib．keio．ac．jp／xoonips／modules／xoonips／detail．php？koara＿id＝AA000260492－19980001－0 <br> Oo9 |
| URL |  |

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# LONG-RUN FISH STOCK AND IMPERFECTLY COMPETITIVE INTERNATIONAL COMMERCIAL FISHING 

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First version received January 1996; final version accepted April 1996

Abstract: In this paper fishery economics and recent theory of international trade under imperfect competition are integrated. For this purpose, international commertial fishing in an open-sea is formulated and its dynamic properties are analyzed under the condition of imperfect competition with two trading countries. The fish stock may become extinct or converge to a steady state level. A sufficient condition is derived for the fish stock to converge to the steady state in relation to the parameters of the fish's biological growth equation, of the harvesting costs and of the demand functions for the fish in the two countries.

## 1. Introduction

Since the seminal work of Smith (1969) on commercial fishing as distinct from recreational fishing, some economists have conducted dynamic analysis of commercial fishing, taking into account the biological growth law of the fish stock. Leung and Wang (1976) and Wang and Cheng (1978) have analyzed commrecial fishing of a single species; Clark (1976), Solow (1976), May et al. (1979), Okuguchi (1984) and most recently Strobele and Wacker (1995) have dealt with a more complex situation allowing for multispecies with prey-predator interaction. With the exception of Leung and Wang, and Wang and Cheng, in all of these works imperfect competition has been ruled out in the market for the harvested fish; thus the price of the fish has been taken to be constant. Besides, international trade for the harvested fish has never been considerd in all of the above contributions. However, international trade of identical or similar products is of the central issue in the recent theory of international trade under imperfect competition.
In this paper we will formulate and analyze international commercial fishing under imperfect competition, where two countries harvest fish of a single species in an open-access sea. The fish each country harvests is assumed to be sold not

[^0]only in its own country but also in the other country. Each country's harvesting cost is assumed to be proportional to the square of its harvest and inversely proportional to the total level of the fish stock, whose intertemporal movement in the absence of fishing is assumed to be governed by the biological law. We will find that the long-run dynamics of the change in the fish stock will be classified into four possible cases, depending on the relationship between the line for $f(X)$ and the curve for $g(X)$ defined in the next section.

## 2. INTERNATIONAL DUOPOLY MODEL

Let there be two countries which are engaged in commercial fishing in an open-access sea and in selling the harvested fish in the two countries, the home and foreign markets. If $X$ is the fish stock, its rate of change in the absence of fishing is governed by the biological growth law.

$$
\begin{equation*}
d X / d t=X(\alpha-\beta X) \tag{1}
\end{equation*}
$$

where $\alpha$ and $\beta$ are positive constants. If the fish stock is small, there will be abundant foods for the fish, and its stock will grow at the intrinsic growth rate $\alpha$. As the fish stock becomes larger, there will be competition among the fish for the limited foods, forcing the growth rate to decrease. Rewriting (1), we have

$$
d X / d t=\alpha X(1-K X), \quad K \equiv \beta / \alpha .
$$

This is the logistic equation originally due to P.F.Verhulst. The constants $\alpha$ and $K$ are called the intrinsic growth rate and carrying capacity, respectively. If fishing is absent, the fish stock converges to $K$.

Let $x_{i j}$ be the amount of fish harvested by country $i$ and sold in country $j$, $i, j=1,2$. The inverse demand functions for fish in the two countries are given by

$$
\begin{align*}
& p_{1}=a_{1}-b_{1}\left(x_{11}+x_{21}\right),  \tag{2.1}\\
& p_{2}=a_{2}-b_{2}\left(x_{12}+x_{22}\right), \tag{2.2}
\end{align*}
$$

where $a_{1}$ and $b_{1}$ are positive constants for $i=1,2 ; p_{1}$ and $p_{2}$ are the prices of the fish in country 1 and country 2 , respectively. If the fish is not evenly distributed in a open-access sea and if, in addition, the fish stock is given, the unit cost of harvesting the fish will increase as the harvesting rate increases and be proportional to it. Hence under the same condition, the total harvesting cost will be proportional to the square of the harvest rate. If, on the other hand, the harvesting rate is given, the harvesting will be easier and less costly as the fish stock increases. Hence given the harvest rate, the harvesting cost is inversely proportional to the fish stock. We assume therefore that each country's harvesting cost is proportional to the square of its harvest rate and inversely proportional to the fish stock. If $c_{i}$ is country $i$ 's opportunity cost for fishing, country 1 and country 2 's profits, $\pi_{1}$ and $\pi_{2}$, are given by

$$
\begin{align*}
& \pi_{1}=p_{1} x_{11}+p_{2} x_{12}-\gamma_{1}\left(x_{11}+x_{12}\right)^{2} / X-c_{1} .  \tag{3.1}\\
& \pi_{2}=p_{1} x_{21}+p_{2} x_{22}-\gamma_{2}\left(x_{21}+x_{22}\right)^{2} / X-c_{2}, \tag{3.2}
\end{align*}
$$

where $\gamma_{i}$ and $c_{i}$ are positive constants, $i=1,2$. We assume that the two countries behave as Cournot duoposists.

Let country $i$ 's total harvest and total supply to country $i$ be

$$
X_{i} \equiv x_{i 1}+x_{i 2}, \quad i=1,2
$$

and

$$
Y_{i} \equiv x_{1 i}+x_{2 i}, \quad i=1,2
$$

respectively. Given $X$, the first order conditions for countries 1 and 2's profit maximization are given by (4.1) and (4.2), and (4.3) and (4.4), respectively.

$$
\begin{align*}
& \partial \pi_{1} / \partial x_{11}=a_{1}-b_{1} x_{11}-b_{1} Y_{1}-2 \gamma_{1} X_{1} / X=0,  \tag{4.1}\\
& \partial \pi_{1} / \partial x_{12}=a_{2}-b_{2} x_{12}-b_{2} Y_{2}-2 \gamma_{1} X_{1} / X=0,  \tag{4.2}\\
& \partial \pi_{2} / \partial x_{21}=a_{1}-b_{1} x_{21}-b_{1} Y_{1}-2 \gamma_{2} X_{2} / X=0,  \tag{4.3}\\
& \partial \pi_{2} / \partial x_{22}=a_{2}-b_{2} x_{22}-b_{2} Y_{2}-2 \gamma_{2} X_{2} / X=0 . \tag{4.4}
\end{align*}
$$

From (4.1) and (4.3), and (4.2) and (4.4), respectively, we have

$$
\begin{align*}
& Y_{1}=2\left\{a_{1}-\left(\gamma_{1} X_{1}+\gamma_{2} X_{2}\right) / X\right\} / 3 b_{1} .  \tag{5.1}\\
& Y_{2}=2\left\{a_{2}-\left(\gamma_{1} X_{1}+\gamma_{2} X_{2}\right) / X\right\} / 3 b_{2} . \tag{5.2}
\end{align*}
$$

Substituting (5.1) and (5.2) into (4.1) and (4.2) and rearranging, we get

$$
\begin{align*}
& x_{11}=a_{1} / b_{1}-2\left\{a_{1}-\left(\gamma_{1} X_{1}+\gamma_{2} X_{2}\right) / X\right\} / 3 b_{1}-2 \gamma_{1} X_{1} / b_{1} X,  \tag{6.1}\\
& x_{12}=a_{2} / b_{2}-2\left\{a_{2}-\left(\gamma_{1} X_{1}+\gamma_{2} X_{2}\right) / X\right\} / 3 b_{2}-2 \gamma_{1} X_{1} / b_{2} X . \tag{6.2}
\end{align*}
$$

Similarly, (4.3), (4.4), (5.1) and (5.2) lead to

$$
\begin{align*}
& x_{21}=a_{1} / b_{1}-2\left\{a_{1}-\left(\gamma_{1} X_{1}+\gamma_{2} X_{2}\right) / X\right\} / 3 b_{1}-2 \gamma_{2} X_{2} / b_{1} X,  \tag{6.3}\\
& x_{22}=a_{2} / b_{2}-2\left\{a_{2}-\left(\gamma_{1} X_{1}+\gamma_{2} X_{2}\right) / X\right\} / 3 b_{2}-2 \gamma_{2} X_{2} / b_{2} X . \tag{6.4}
\end{align*}
$$

Adding (6.1) and (6.2), and (6.3) and (6.4), respectively, we have

$$
\begin{align*}
& X_{1}=\left(a_{1} b_{2}+a_{2} b_{1}\right) / 3 b_{1} b_{2}-4 \gamma_{1}\left(b_{1}+b_{2}\right) X_{1} / 3 b_{1} b_{2} X+2 \gamma_{2}\left(b_{1}+b_{2}\right) X_{2} / 3 b_{1} b_{2} X,  \tag{7.1}\\
& X_{2}=\left(a_{1} b_{2}+a_{2} b_{1}\right) / 3 b_{1} b_{2}-4 \gamma_{2}\left(b_{1}+b_{2}\right) X_{2} / 3 b_{1} b_{2} X+2 \gamma_{1}\left(b_{1}+b_{2}\right) X_{1} / 3 b_{1} b_{2} X, \tag{7.2}
\end{align*}
$$

Rewrite (7.1) and (7.2) as

$$
\begin{equation*}
\left\{1+4 \gamma_{1}\left(b_{1}+b_{2}\right) X_{1} / 3 b_{1} b_{2} X\right\} X_{1}-2 \gamma_{2}\left(b_{1}+b_{2}\right) X_{2} / 3 b_{1} b_{2} X=\left(a_{1} b_{2}+a_{2} b_{1}\right) / 3 b_{1} b_{2} . \tag{8.1}
\end{equation*}
$$

$$
\begin{equation*}
-2 \gamma_{1}\left(b_{1}+b_{2}\right) X_{1} / 3 b_{1} b_{2} X+\left\{1+4 \gamma_{1}\left(b_{1}+b_{2}\right) X_{2} / 3 b_{1} b_{2} X\right\} X_{2}=\left(a_{1} b_{2}+a_{2} b_{1}\right) / 3 b_{1} b_{2} . \tag{8.2}
\end{equation*}
$$

Solving this system of equations with respect to $X_{1}$ and $X_{2}$, we have

$$
\begin{align*}
& X_{1}=\left(a_{1} b_{2}+a_{2} b_{1}\right)\left\{b_{1} b_{2} X+2 \gamma_{2}\left(b_{1}+b_{2}\right)\right\} X /\left(A X^{2}+B X+C\right),  \tag{9.1}\\
& X_{2}=\left(a_{1} b_{2}+a_{2} b_{1}\right)\left\{b_{1} b_{2} X+2 \gamma_{1}\left(b_{1}+b_{2}\right)\right\} X /\left(A X^{2}+B X+C\right), \tag{9.2}
\end{align*}
$$

Hence the total harvest by two countries is

$$
\begin{equation*}
X_{1}+X_{2}=(D X+E) X /\left(A X^{2}+B X+C\right), \tag{10}
\end{equation*}
$$

where

$$
\begin{gather*}
A=3 b_{1}^{2} b_{2}^{2}>0  \tag{11.1}\\
B=4 b_{1} b_{2}\left(b_{1}+b_{2}\right)\left(\gamma_{1}+\gamma_{2}\right)>0  \tag{11.2}\\
C=4 \gamma_{1} \gamma_{2}\left(b_{1}+b_{2}\right)^{2}>0  \tag{11.3}\\
D=2 b_{1} b_{2}\left(a_{1} b_{2}+a_{2} b_{1}\right)>0  \tag{11.4}\\
E=2\left(b_{1}+b_{2}\right)\left(a_{1} b_{2}+a_{2} b_{1}\right)\left(\gamma_{1}+\gamma_{2}\right)>0 \tag{11.5}
\end{gather*}
$$

Hence the differential equation governing the change of the fish stock in the presence of commercial fishing is

$$
\begin{equation*}
d X / d t=X(f(X)-g(X)), \tag{12}
\end{equation*}
$$

where

$$
\begin{gather*}
f(X) \equiv \alpha-\beta X  \tag{13}\\
g(X) \equiv(D X+E) /\left(A X^{2}+B X+C\right) \tag{14}
\end{gather*}
$$

In the light of (11) and

$$
C D-B E=-8 b_{1} b_{2}\left(b_{1}+b_{2}\right)^{2}\left(a_{1} b_{2}+a_{2} b_{1}\right)\left(\gamma_{1}^{2}+\gamma_{1} \gamma_{2}+\gamma_{2}^{2}\right)<0,
$$

we have

$$
\begin{equation*}
g^{\prime}(X)=\left(-A D X^{2}-2 A E X+C D-B E\right) /\left(A X^{2}+B X+C\right)^{2}<0, \quad \text { for } \quad X \geq 0 \tag{15}
\end{equation*}
$$

Furthermore we have
$g^{\prime \prime}(X)=\frac{-2\left\{-A^{2} D X^{3}-3 A^{2} E X^{2}+3 A(C D-B E) X+B(C D-D E)+A C E\right\}}{\left(A X^{2}+B X+C\right)^{4}}>0$,
since

$$
\begin{aligned}
& B(C D-D E)+A C E \\
& \quad=-8 b_{1}^{2} b_{2}^{2}\left(b_{1}+b_{2}\right)^{3}\left(a_{1} b_{2}+a_{2} b_{1}\right)\left(\gamma_{1}+\gamma_{2}\right)\left(4 \gamma_{1}^{2}+4 \gamma_{2}^{2}+\gamma_{1} \gamma_{2}\right)<0 .
\end{aligned}
$$

The nonextinct steady state or the bionomic equilibrium for the fish stock satisfies

$$
\begin{equation*}
f(X)=g(X) . \tag{17}
\end{equation*}
$$

We have to consider four possibilities:
Case 1. Equation (17) has no real solution.
Case 2. It has only one real solution and $E / C>\alpha$.
Case 3. It has only one real solution and $E / C<\alpha$.
Case 4. It has two distinct real solutions.
These four cases are illustrated in Figs. 1-4.
First, consider Fig. 1. In this case the harvest rate of the fish stock is always greater than its biological growth rate, resulting in extinction of the fish stock. In Fig. 2, the harvest rate is greater than the biological growth rate regardless of the initial stock level unless it equals $X^{*}$. Hence the fish stock converges to the steady state level $X^{*}$ if the initial stock is larger than $X^{*}$ but becomes extinct if the initial stock is less than $X^{*}$. In Fig. 3, the harvest rate is greater (less) than the growth rate for $X>X^{*}\left(X<X^{*}\right)$. Hence the fish stock converges to the steady state level regardless of the initial stock level. In Fig. 4, if the initial stock is larger than the larger steady state stock $X^{* *}$, the fish stock decreases and converges to $X^{* *}$; if the initial stock is less than the smaller steady state stock $X^{*}$, the fish stock decreases and become extinct in the long-run. If, however, the initial stock lies between $X^{*}$ and $X^{* *}$, the biological growth rate is greater than the harvest rate, thus the fish stock increases and converges to the larger steady state level $X^{* *}$.

Let us now clarify some policy implications of the above findings. Consider again the case of Fig. 3, where the unique steady state stock $X^{*}$ is stable. In this case, the following inequality holds,

$$
\begin{equation*}
F \equiv E / C \equiv\left(a_{1} b_{2}+a_{2} b_{1}\right)\left(\gamma_{1}+\gamma_{2}\right) / 2 \gamma_{1} \gamma_{2}\left(b_{1}+b_{2}\right)<\alpha . \tag{18}
\end{equation*}
$$

A simple calculation yields:

$$
\begin{array}{lll}
\partial F / \partial b_{1}>0 & \text { if } & a_{2}>a_{1}, \\
\partial F / \partial b_{2}>0 & \text { if } & a_{1}>a_{2}, \\
\partial F / \partial \gamma_{1}<0, & & \\
\partial F / \partial \gamma_{2}<0 . & &
\end{array}
$$

Hence ceteris paribus, an increase in $b_{1}$ in the case of $a_{2}>a_{1}$, an increase in $b_{2}$ in the case of $a_{1}>a_{2}$ and a decrease either in $\gamma_{1}$ or $\gamma_{2}$ are more likely to lead to violation of (17), hence either the case 4,2 or 1 is more likely to occur. The parameter $\alpha$ is biologically determined and almost beyond any governmental control. Furthermore, any government will find it difficult to change the values


Fig. 1. Case 1


Fig. 2. Case 2


Fig. 3. Case 3


Fig. 4. Case 4
of the demand parameters, $a_{1}, a_{2}, b_{1}$ and $b_{2}$. If any government reduces $R \& D$ subsidies for its fishing firm, $\gamma_{1}$ or $\gamma_{2}$ will increase and (17) is more likely to be satisfied.
Consider next the situation where all parameter values are given and not changeable. In the case of Fig. 4, if the initial fish stock is less than $X^{*}$ and fishing is allowed the fish becomes extinct. In this case, if the two countries cooperate to prohibit fishing until the fish stock level becomes $X^{*}$ and if, in addition, fishing is allowed only then, the fish stock remains at the smaller steady level of $X^{*}$. However, if the two countries cooperate to allow fishing only if the fish stock level becomes larger than $X^{*}$, the fish stock will converges to the larger steady state level $X^{* *}$ in the long-run. In the case of Fig. 2, if the initial fish stock is less than $X^{*}$, the fish becomes extinct in the presence of fishing. Therefore, the two countries must cooperate to allow fishing only when the fish stock reaches the level $X^{*}$ or a higher level. In this way the steady state level is achievable in the long-run. Finally, in the case of Fig. 1, there is no possibility of attaining the steady state by prohibiting fishing. The only possible way for attaining the steady state is to change $\gamma_{1}$ or $\gamma_{2}$ by the government's decrease in $R$ and $D$ subsidies to its fishing firm.

## 3. CONCLUSION

In the literature on commercial fishing, the market for the harvested fish has been assumed to be under perfect competition or monopoly, and international trade for the harvested fish has been entirely neglected. The recent fishery dispute between Ireland and Spain as well as that between the EU and Canada has highlighted the importance of conflicts of interests among countries engaged in commercial fishing in an open-access common fishing area. In this paper we have modelled international duopoly in commercial fishing of a single species. We have shown that the fish stock becomes extinct or approaches to the steady state in the long run, depending on the initial level of the fish stock as well as on the relationship between the line for $f(X)$ and the downward-sloping curve for $g(X)$. We have found, among other things, that the fish stock converges to the unique nonextinct steady state level if (17) holds. We have clarified how changes in the values of the biological, demand and technical parameters will affect the non-extinction condition for the fish stock. Any governmental policy intervention is almost ineffective in changing the values of the biological and demand parameters. However, it may be able to induce a change in the value of the technical parameter for the fishing firm, thus enhancing a possibility for realizing the steady state fish stock level in the long-run. Furthermore, if the initial fish stock level is low, the governments in two countries are in a position to cooperate to prohibit fishing until the fish stock grows to a certain level, thus ensuring the steady state fish stock in the long-run.

In this paper we have analyzed international duopoly in commercial fishing.

Our analysis will be easily adapted to deal with oligopolistic fishery in a single country as in Okuguchi (1996).

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    Acknowledgement. We are grateful to a referee of this journal for very helpful comments.

