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TRADE UNION, MEMBERSHIP DYNAMICS AND CAPITAL ACCUMULATION

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Abstract: This note makes two extensions of the dynamic model of trade union behaviour of Kidd and Oswald (1987). Their major result that the dynamic optimum wage is lower than the static optimum wage may be reversed in both the cases.

1. INTRODUCTION

The static literature on the wage determination in the unionised labour market is substantially large. However, a small subset of the literature considers some dynamic aspects; and this includes the works of Kidd and Oswald (1987), Roberts (1989), Manning (1991), Corneo (1995), Croix and Fagnart (1995), Jones and MacKenna (1994) etc. Kidd and Oswald (1987) present a dynamic model in which the size of the membership of the union changes over time and the monopoly union determines the optimum wage rate solving an infinite horizon dynamic optimization problem.[1] The major result of this model is that the optimum employment (wage) in the dynamic analysis is higher (lower) than the optimum level of employment (wage) obtained in the corresponding static analysis if the future is discounted at a positive and finite rate.[2]

Two points are important to note in the context of their model. First, the equation of membership dynamics is overly simplified. It is based on the assumption that the size of the membership in the current period is a constant fraction of the level of employment in the previous period.[3] Kidd and Oswald (1987) admit that

[1] Jones and Mackenna (1994) also make a similar analysis with a different theory of membership dynamics. These approaches make sense for labour markets in which there is a kind of post-entry closed shop. However, the closed shop arrangements are nowadays very rare.

[2] If the rate of discount is infinitely large, then in Kidd and Oswald (1987), the static optimum and dynamic optimum employments (wages) are same. See the equations (25) and (26) in Kidd and Oswald (1987).

[3] Kidd and Oswald (1987) actually assume the employment-membership ratio to be unity. However, their results remain unaffected if this ratio is constant. See the Section III in Kidd and Oswald (1987).

a more general form of this equation should include the wage-rate as an argument, though they do not use this to derive the properties of the model. One simple way of introducing this is to assume that the membership-employment ratio is positively influenced by the wage rate. Secondly, the trade union in Kidd and Oswald (1987) does not consider the capital accumulation dynamics. The employer's demand for labour is also positively related to the capital stock and this capital stock accumulates over time if the profit is invested. So the membership dynamics is dependent on the capital accumulation dynamics and hence a more appropriate dynamic optimization problem of the trade union should take care of both these two equations of motion.

In section 2 of this note, we reexamine the results of Kidd and Oswald (1987) when only the membership employment ratio is positively related to the wage-rate in the unionised labour market. In the section 3 of the note, we consider the capital-accumulation dynamics in the otherwise identical model of Kidd and Oswald (1987). The importance of these two exercises are easily understood looking at the results. The main result of Kidd and Oswald (1987) is substantially modified. The dynamic optimum employment (wage) in both the cases may appear to be less (greater) than the static optimum employment (wage).

2. MEMBERSHIP DYNAMICS

Here W, n, m, b, r, u and t stand for wage-rate, level of employment, size of the union membership, unemployment benefit, social rate of discount and the level of utility respectively. W, n and m are functions of time, t. But t has been suppressed for notational simplicity.

We consider the following differential equation of membership dynamics:

$$\dot{m} = g \cdot n - m \tag{1}$$

where

$$g = g(w) \qquad \text{with} \quad g'(w) > 0 .$$

In Kidd and Oswald (1987), $g(w) = 1$. A micro-foundation of this equation is given in the Appendix (C).

The monopoly trade union now solves the following dynamic optimization problem.

$$\text{Max} \int_0^\infty \{u(w) \cdot n + (m - n)u(b)\} e^{-rt} dt$$

subject to equation (1) and

$$W = f'(n) . \tag{2}$$

Here the equation (2) is the profit-maximizing condition of the firm.

The appropriate Hamiltonian to be maximized at each t is given by the following:

$$H=[u\{f'(n)\}n+(m-n)u(b)]\mathrm{e}^{-rt}+\lambda(g\{f'(n)\}\cdot n-m) \tag{3}$$

where λ, a function of time, is the co-state variable. m is the state variable and n is the control variable.

From the first-order conditions of optimality we can ob...n the following long-run equilibrium conditions.[4]

$$g(w)(1+\theta)u(b)+(1+r)(\beta(n)-r\cdot u(b))=0 \tag{4}$$

and

$$g\{f'(n)\}\cdot n-m=0\,. \tag{5}$$

Here $\beta(n)$ is same as defined in Kidd and Oswald; and is given by

$$\beta(n)=u(W)+u'(W)\cdot nf'(n)\,.$$

But

$$\theta=\frac{n\cdot g'(W)\cdot W'(n)}{g(W)}<0\,.$$

Note that in Kidd and Oswald (1987), $g(W)\equiv 1$; and hence $g'(W)=0$. This implies that $\theta=0$. So then the equations (4) and (5) are reduced to the followings:

$$(1+r)\cdot\beta(n)-r\cdot u(b)=0 \tag{4A}$$

and

$$n=m\,. \tag{5A}$$

These two are the long-run equilibrium conditions[5] in Kidd and Oswald (1987).

In the conventional static model,

$$\beta(n)=u(b)\,;^{6} \qquad \square$$

and equation (4A) shows that, in the dynamic model of Kidd and Oswald (1987), the long-run equilibrium level of employment is given by

$$\beta(n)=\frac{r}{1+r}\cdot u(b)\,.$$

So if $r<\infty$, $\beta(n)<u(b)$ in the long-run equilibrium. Since $\beta'(n)<0$, it is now clear that the long-run equilibrium level of employment in Kidd and Oswald (1987) is higher than the short-run equilibrium level of employment. So the long-run equilibrium wage is less than the short-run equilibrium wage.

[4] Derivations are similar to those available in Kidd and Oswald (1987) and are shown in Appendix (A). 2nd order conditions are also satisfied because $g\{f'(n)\}\cdot n$ is concave in n. Otherwise the model is similar to Kidd and Oswald (1987).

[5] See the equations (23) and (24) in Kidd and Oswald (1987).

[6] See Oswald (1982); Also equation (26) in Kidd and Oswald (1987).

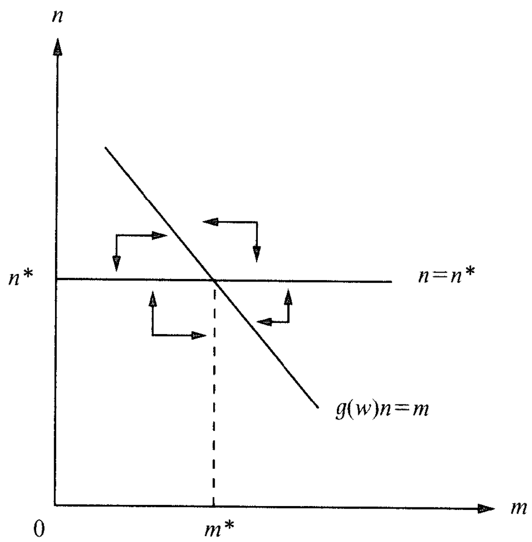

Fig. 1.

But, in the present analysis, $\theta<0$; and we may get a case where $(1+\theta)\leqslant 0$. In that case, equation (4) shows that $\beta(n)\geqslant u(b)$. Hence the long run equilibrium level of employment may not be more than the short run equilibrium level of employment.[7]

Figure 1 sketches the long run equilibrium model. From equation (4), we get

[7] In the Appendix C, it has been shown that

$$g(W)=((u(W)-u(b))/j)\,;$$

and in that case

$$1+\theta=\frac{\beta(n)-u(b)}{u(W)-u(b)}\,.$$

Then using the equation (4), we have.

$$(1+r+(u(b)/j))(\beta(n)-u(b))=0$$

or

$$\beta(n)=u(b)\,.$$

So in this special case, there is no difference between the long run equilibrium level of employment and the short run equilibrium level of employment. But if

$$g(W)=((u(W)-(u(b)+a))/j)$$

then, from equation (4), we have

$$(1+r+(u(b)/j))(\beta(n)-u(b))=(u(b)/j)\cdot a\,.$$

Hence

$$\beta(n)\gtrless u(b) \qquad \text{for} \quad a\gtrless 0\,.$$

Here $a>0$ can be interpreted as the utility from leisure of the unemployed member.

$n=n^*$; and this is shown by a horizontal straight line. But this will be different from the long run equilibrium value of n in Kidd and Oswald (1987) even at the same values of r and b. The equation (5) does not necessarily give us a positively sloped curve. The slope depends on the value of θ because, from equation (5), it can be easily shown that

$$(dn/dm)=\frac{1}{g(W)(1+\theta)};$$

and hence

$$(dn/dm)<0 \quad \text{if} \quad \theta<-1\,.$$

So we get a negatively sloped curve from equation (5) when $\theta<-1$. In the original model of Kidd and Oswald (1987), the relevant equation is (5A) and this shows a 45° line from the origin. The point of intersection (m^*, n^*) is the long run equilibrium point. It can be easily shown that this long run equilibrium (shown in the Figure 1) is stable.

The comparative steady state results with respect to changes in b and r in this model with $\theta<-1$ are completely opposite to those in the original model of Kidd and Oswald (1987) where $g(\cdot)=1$ and $\theta=0$. With a reduction in the value of r and/or b, the $n=n^*$ horizontal straight line shifts upward. So m^* falls in the new long run equilibrium in the present model when $\theta<-1$, i.e., when $\dot{m}=0$ locus slopes negatively. But in Kidd and Oswald (1987), $\dot{m}=0$ locus slopes positively; and hence the new long run equilibrium value of m is increased in this case.

How should one interpret the condition: $\theta<-1$?
Here

$$\theta=\theta_1\cdot\theta_2$$

where,

$$\theta_1=g'(W)\cdot(W/g(W))$$

and

$$\theta_2=W'(n)\cdot(n/W)\,.$$

Here θ_2 is the reciprocal of the wage elasticity of employment and θ_1 is the elasticity of membership flow with respect to the wage rate. Hence $\theta_1\theta_2$ is the elasticity of the flow of membership in the union with respect to the employment in the unionized sector. In Kidd and Oswald (1987), this elasticity is Zero because $g'(W)=0$.

3. CAPITAL ACCUMULATION

In this section, we introduce capital as an input in the production function and assume that capital accumulates over time through investment of the profit. The production function is assumed to satisfy the constant returns to scale property. Otherwise, the model is similar to that of Kidd and Oswald (1987).

Here K stand for capital and $x=(K/n)$ is the capital-labour ratio. Equation (2) is now modified as

$$W=f(x)-f'(x)\cdot x \tag{2A}$$

where $f(\cdot)$ is the intensive production function.

Regarding membership dynamics, we consider the case of Kidd and Oswald (1987); and hence

$$(\dot{m}/m)=(n/m)-1\ ;$$

or,

$$(\dot{m}/m)=(Z/x)-1 \tag{1A}$$

where $Z=K/m$ represents the capital-stock per member.

Here $r=f'(x)$ is the rate of profit and the entire profit is invested. This is a restrictive assumption; and is not necessarily valid when the firm takes its investment-decision solving a dynamic optimization problem. Let ρ be the constant rate at which capital stock depreciates. So the equation of capital accumulation dynamics takes the following form:

$$(\dot{K}/K)=f'(x)-\rho\ ;$$

and using this equation and the equation (1A) we can obtain

$$(\dot{Z}/Z)=f'(x)-(Z/x)+(1-\rho)\ . \tag{6}$$

The union's utility at any point of time is given by

$$((u(W)n+u(b)(m-n))/m)\ ;$$

and we assume that the utility function of the individual, $u(\cdot)$, is linear.[8] So the union's utility function can be written as

$$\{S(x)\cdot Z+b\}$$

where

$$S(x)=(\{f(x)-f'(x)\cdot x-b\}/x)\ . \tag{7}$$

So the trade union solves the following dynamic optimization problem:

$$\text{Max}\int_0^\infty \{S(x)Z+b\}\cdot e^{-rt}dt$$

subject to the equations (2A), (6) and (7). Here Z is the state variable and x is

[8] So the utility of the union is identical to the average income of its members; and is not identical to that considered in the section 2.

the control variable.

Defining the appropriate Hamiltonian and using the relevant first-order conditions of optimality, we obtain the following long-run equilibrium conditions.[9]

$$S'(x)=\frac{S(x)\{f''(x)+(Z/x^2)\}}{\{f'(x)-\rho-(Z/x)+1\}-(r+(Z/x)\}} \tag{8}$$

and

$$f'(x)-\rho-(Z/x)+1=0\,. \tag{9}$$

Using the equations (8) and (9) we can obtain the long run equilibrium values of Z and x. Substituting (9) into (8), we have

$$S'(x)=\frac{-S(x)\cdot\{f''(x)+(Z/x^2)\}}{r+(Z/x)} \tag{10}$$

In the static analysis, $S(x)$ is maximized with respect to x; and the optimum capital intensity is solved from the equation

$$S'(x)=0\,.$$

But, in the present dynamic analysis, equation (10) gives us the optimum capital intensity. Here,

$$S'(x)\gtreqless 0 \quad \text{when} \quad (f''(x)+(Z/x^2))\lesseqgtr 0\,.$$

and $S'(x)<0$ is the case where the conclusion of Kidd and Oswald (1987) is reversed. In Kidd and Oswald (1987), the dynamic optimum wage is less than the static optimum wage. Here $S''(x)<0$.[10] So the long run equilibrium value of x with $S'(x)<0$ should be greater than the value of x satisfying $S'(x)=0$. Equation (2A) shows that W and x are positively related. So if $S'(x)<0$ in the long run equilibrium, the dynamic optimum wage should be greater than the static optimum wage.[11]

Note that $S'(x)<0$ when $(f''(x)+(Z/x^2))>0$; and using equation (9), it can be shown that

$$f''(x)+(z/x^2)=f''(x)+(f'(x)/x)+((1-\rho)/x)$$

By assumption, $\rho<1$; and hence $S'(x)<0$ if $((f'(x)/x)+f''(x))>0$. This condition is valid for a Cobb–Douglas production function.[12]

[9] Derivations are similar to those in Kidd and Oswald (1987). Some remarks on the second-order conditions are made in the Appendix (B).

[10] It is shown in the Appendix (3).

[11] This is true so long the rate of discount, r, is finite. If $r\to\infty$ then the R.H.S. of equation (10) tends to Zero and the dynamic optimum wage tends to be equal to the static optimum wage.

[12] If $f(x)=x^B$ with $0<B<1$, then

$$f''(x)+(f'(x)/x)=B^2\cdot x^{B-2}>0\,.$$

However, capital accumulation does not change the result of Kidd and Oswald (1987) when the union maximizes its total membership utility, given by $\int_0^\infty \{u(W)\cdot n+(m-n)u(b)\}e^{-rt}dt$. It should be noted that we use the expected utility function; and Kidd and Oswald (1987) already pointed out that their main result may be reversed in this case even in the absence of capital accumulation.[13]

4. CONCLUSION

This note shows the cases in which we get results completely opposite to those of Kidd and Oswald (1987). In Kidd and Oswald (1987), the size of the membership of the union in the current period is determined by the level of employment in the previous period and the trade union does not take care of the capital-accumulation dynamics. In this note, we first consider a case where the membership employment ratio varies positively with the wage rate. Secondly, we consider the case where the union also takes care of the capital accumulation dynamics. In both the cases, the dynamic optimum wage rate appears to be more than that obtained in Kidd and Oswald (1987); and under the appropriate sufficient conditions, it may be even more than the static optimum wage rate. So the claim of Kidd and Oswald (1987) that the static monopoly models of trade unions over-state the employment distortions is not necessarily true.

REFERENCES

Corneo, G., (1995), Social Custom, Management opposition and trade union membership; European Economic Review.

Croix, D. and J. F. Fagnart (1995), Underemployment of production factors in a forward looking model; Labour Economics.

Jones, S. R. G. and C. J. KcKenna (1994), A dynamic model of union membership and employment; Economica, 61, 179–189.

[13] If capital accumulation is not allowed then

$$f'(x)-\rho\equiv 0\,;$$

and hence equations (8) and (9) are replaced as follows:

$$S'(x)=\frac{S(x)\{f''(x)+(Z/x^2)\}}{1-(Z/x)-r-(Z/x)} \tag{8A}$$

and

$$(Z/x)=1\,. \tag{9A}$$

But the equation (10) remains same; and hence

$$S'(x)<0 \qquad \text{if} \quad (f''(x)+(Z/x^2))>0\,.$$

However, the equation (9A) shows that $(Z/x^2)=(1/x)$; and so $(f''(x)+(1+x))>0$ is the necessary and sufficient condition for the main result of Kidd and Oswald (1987) to be reversed. Unfortunately this condition is not necessarily satisfied in the case of a Cobb–Douglas production function.

Kidd, D. P. and A. J. Oswald (1987), A dynamic model of trade union behaviour; Economica, 54, 355–365.
Manning, A. (1991), The effects of density on wages and employment: A dynamic monopoly union model; Centre of Economic Performance, DP.
Mcdonald, I. M. and R. M. Solow (1981), Wage bargaining and employment; American Economic Review, 71, 896–908.
Oswald, A. J. (1982), The micro economic theory of the trade union; Economic Journal, 92, 576–595.
Roberts, K., (1989), The theory of union behaviour: Labour hoarding and endogenous hysteresis; STICERD DP.

APPENDIX (A)

The Hamiltonian is written as follows:

$$H=[u\{f'(n)\}\cdot n+(m-n)\cdot u(b)\cdot e^{-rt}+\lambda(g\{f'(n)\}\cdot n-m)\,.$$

The first-order conditions for optimality include

$$(\partial H/\partial n)=0 \tag{A.1}$$

$$\dot{\lambda}=-(\partial H/\partial m) \tag{A.2}$$

and

$$\underset{t\to\infty}{\text{Lim.}}\,\lambda=0\,. \tag{A.3}$$

A sufficient condition for these to describe a maximum is that both the objective function and the differential equation constraint are concave functions in terms of n and m. Kidd and Oswald (1987) have shown that, if $\beta'(n)<0$, then this condition is satisfied where

$$\beta(n)\equiv u(W)+u'(W)\cdot nf''(n)$$

i.e., $\beta(n)$ is the first-derivative of $u\{f'(n)\}\cdot n$ with respect to n.

By solving equations (A.1) and (A.2), we have

$$\{\beta(n)-u(b)\}\cdot e^{-rt}+\lambda\cdot g(W)(1+\theta)=0 \tag{A.4}$$

and

$$\dot{\lambda}=-u(b)\cdot e^{-rt}+\lambda \tag{A.5}$$

where

$$\theta=\frac{n\cdot g'(W)\cdot W'(n)}{g(W)}<0$$

is the product of the elasticity of the employment function and the membership function. We assume it to be constant.

Differentiating equation (A.4) with respect to t we have

$$\beta'(n) \cdot \dot{n} \cdot e^{-rt} - r\{\beta(n) - u(b)\} e^{-rt}$$
$$+ \dot{\lambda} \cdot g(W)(1+\theta) + \lambda(1+\theta) \cdot g'(W) \cdot W'(n) \cdot \dot{n} = 0 . \tag{A.6}$$

Then using equations (A.4) and (A.5), we have

$$\dot{\lambda} = -u(b) \cdot e^{-rt} - \frac{\{\beta(n) - u(b)\} \cdot e^{-rt}}{g(W)(1+\theta)} . \tag{A.7}$$

In the long-run equilibrium, $\dot{n}=0$; and hence using equations (A.6) and (A.7) we get

$$g(W)(1+\theta)u(b) + (1+r)(\beta(n) - u(b)) = 0 . \tag{A.8}$$

Also we have

$$\dot{m} = g(W) \cdot n - m$$

and, in the long-run equilibrium, we have

$$g(W) \cdot n - m = 0$$

or

$$g\{f'(n)\} \cdot n - m = 0 \tag{A.9}$$

These equations (A.8) and (A.9) are the equations (4) and (5) in the section 2 of this paper.

APPENDIX (B)

This Hamiltonian is

$$H = [S(x) \cdot Z + b] \cdot e^{-rt} + \lambda\left[f'(x) - \rho - \frac{Z}{x} + 1\right] \cdot Z ;$$

and the first-order condition of optimality describe a maximum if the objective function is concave and the differential equation constraint is also concave.

Here $\{S(x)Z + b\}$ is linear in Z, i.e., weak concave.

$$S(x) = \frac{f(x) - f'(x) \cdot x - b}{x}$$

$$\therefore \quad S'(x) = -f''(x) - \frac{S(x)}{x^2} ;$$

$$\therefore \quad S''(x) = -f'''(x) - \frac{S'(x)}{x} - \frac{S(x)}{x^2}$$

or,

$$S''(x) = -f'''(x) + \frac{f''(x)}{x} < 0 .$$

Hence $S''(x) < 0$ and $\{S(x) \cdot Z + b\}$ is strictly concave in x.

Suppose that

$$Q = \left(f'(x) - \rho - \frac{Z}{x} + 1 \right) Z$$

Hence,

$$\frac{\partial^2 Q}{\partial Z^2} = -\frac{2}{x} < 0$$

which proves that Q is concave in Z.

$$\frac{dQ}{dx} = f''(x) + \frac{Z}{x^2} ;$$

and

$$\frac{d^2 Q}{dx^2} = f'''(x) - \frac{2Z}{x^3} .$$

So a sufficient condition for concavity is given by

$$f'''(x) < \frac{2Z}{x^3} .$$

APPENDIX (C)

If the worker joins the union, then his probability of getting employment in the unionized labour market is (n/m); and hence $(1-(n/m))$ is the probability of not getting job in the non-unionized labour market. This assumption is, however, restictive because the probability for a member to be employed in a given period is independent of his status as employed or unemployed. As W and b are the wage rates in the unionized and non-unionized labour markets respectively and $U(\cdot)$ is the utility function of the worker, the expected utility of the member worker is $(u(W) \cdot (n/m) + u(b) \cdot (1-(n/m)))$. The representative member worker joins (leaves) the union if

$$(u(W) \cdot (n/m) + u(b) \cdot (1-(n/m))) > (<) u(b) + j$$

where j is the membership fee per worker. So in equilibrium

$$(u(W) \cdot (n/m) + u(b) \cdot (1-(n/m))) = u(b) + j ; \qquad \text{(C.1)}$$

and hence the equilibrium value of m, denoted by m^*, is given by

$$m^* = ((u(W) - u(b))/j) \cdot n$$

As b and j are exogenously given to the system we can express this equation as

$$m^* = g(W) \cdot n\,;$$

where $g(W) = ((u(W) - u(b))/j)$.

As $u(W) > u(b)$, and $u(b) + j$ is the weighted average of $u(W)$ and $u(b)$, then $u(W) > u(b) + j$; and this implies that $g(W) > 1$.

Note that this m^* is not automatically attained at a particular point of time because m changes only over time. $(m^*_{t-1} - m_{t-1})$ is the desired change in membership in period $t-1$; and we assume that the actual change is exactly equal to the desired change. (This is a restrictive one and needs another microfoundation which we can not supply at present). Hence

$$m_{t+1} - m_{t-1} = m^*_{t-1} - m_{t-1}\,.$$

If time, t, is a continuous variable, we have

$$\dot{m} = m^* - m$$

or

$$\dot{m} = g(W) \cdot n - m$$

which is our equation (1) of the model presented in the section 2.