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## THE EFFECT OF ENTRY BARRIERS IN A “MIXTURE” MARKET

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*Abstract:* This paper examines the effect of entry barriers in a “mixture” oligopolistic market where a number of private firms maximize their profits and one government firm maximizes its share. The central government is assumed to control the number of private firms in that oligopolistic market. It will be shown that when the share of the government firm is sufficiently large, the number of firms at the free-entry equilibrium is smaller than that of welfare maximizing number of firms; we can obtain what may be called the “insufficient theorem” even under the condition that the outputs are homogenous and strategic substitutes.

### **L13 Oligopoly and Other Imperfect Markets**

**Key-words:** scale-maximizing government firm, insufficient entry, oligopoly, second-best government

#### 1. INTRODUCTION

Numerous attempts have been made to examine if free entry is always desirable for the enhancement of economic welfare. Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) show that in an oligopolistic industry producing homogeneous products, the number of firms at the free-entry equilibrium exceeds the number of welfare maximizing equilibrium. Their results state that the increasing competition, namely free entry, does not contribute to the increase of national welfare in an oligopolistic market. Their conclusion is known as the “excess entry theorem” which provides us with a new perspective in the debates on industrial organization and industrial policy. Moreover, Konishi, Okuno-Fujiwara and Suzumura (1990) show that the “excess entry theorem” is preserved in a general equilibrium setting. Mankiw and Whinston (1986), Suzumura and Kiyono (1987) and Konishi, Okuno-Fujiwara and Suzumura (1990) etc., however,

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neglected the existence of the government firm, which maximizes its share rather than its profit. In the Japanese financial market, for example, we can observe scale-maximizing government firms, together with profit-maximizing private firms. There exist the government financial bank in the banking industry, and post life insurance in the life insurance market. Thus, we would be misled if we do not consider the existence of the government institutions. This paper examines if free entry is desirable for the enhancement of welfare in a “mixture” oligopolistic market where a number of private firms are maximizing their profits and one government firm is maximizing its shares rather than its profit.

Ohyama (1990) and Suzumura (1993) investigate if the message of the “excess entry theorem” is kept intact. However, they do not analyze the strategic interaction between profit maximizing firms and scale maximizing firms as presented in this paper.

In the following sections, we will examine the robustness of the “excess entry theorem” within the framework of Suzumura and Kiyono (1987). It will be shown that, when one firm operates so as to maximize its share, the number of the firms in an industry at free-entry equilibrium may be too small from the view point of economic welfare.

This paper proceeds as follows. In section 2, the basic model is described, and in section 3, we analyze the effect of the existence of a scale-maximizing government firm on competition in a “mixture” oligopolistic market. An example is constructed in section 4, and concluding comments are made in section 5.

## 2. BASIC MODEL

The basic model is a homogeneous product Cournot oligopoly as is used in Mankiw and Whinston (1986), Suzumura and Kiyono (1987) and so on. Consider a “mixture” oligopolistic industry which consists of a number of private firms and one government firm. The private firms are supposed to have the same technology and to maximize their profits; the government firm is assumed to keep its profit at zero level and maximize its scale. And we assume that the government firm is inefficient, that is, its average cost is increasing at the equilibrium. Throughout this paper, the author focus on the symmetric equilibrium about private firms. And the private and government outputs are supposed to be strategic substitutes.

### Notation

- $x_i$ : the output level of the  $i$ -th private firm.
- $y$ : the output level of the government firm.
- $n$ : the number of the private firms.

$$x_{-i} \equiv (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$$

$\pi_i(x_i; x_{-i}, y)$ : the profit function of the  $i$ -th private firm.

$\pi_p(y; x_1, \dots, x_n)$ : the profit function of the government firm.

$Z \equiv \sum x_i + y$ : total outputs.

$P(Z)$ : the inverse demand function.

I assume  $P(Z)$  is twice continuously differentiable and  $P'(Z) < 0$ .

$c_i(x_i)$ : the cost function of the  $i$ -th private firm.

$c_p(y)$ : the cost function of the government firm.

Both cost functions are assumed to be  $U$  shaped.

$AC_i(x_i)$ : the average cost function of the  $i$ -th private firm.

$AC_p(y)$ : the average cost function of the government firm.

$W(x_1, \dots, x_n, y)$ : the net market surplus function.

Now we characterize the equilibrium outputs  $x_i^*(n)$  and  $y^*(n)$ . The profit of the  $i$ -th private firm  $\pi_i(x_i; x_{-i}, y)$  is

$$\pi_i(x_i; x_{-i}, y) = P\left(x_i + \sum_{j \neq i} x_j + y\right) x_i - c_i(x_i) \quad (1)$$

And the profit of the government firm  $\pi_p(y; x_1, \dots, x_n)$  is

$$\pi_p(y; x_1, \dots, x_n) = P\left(\sum_{i=1}^n x_i + y\right) y - c_p(y) \quad (2)$$

Therefore, the first order condition for private firms and the profit-zero constrained scale maximizing condition for government firm are, respectively

$$\frac{\partial \pi_i}{\partial x_i} = P\left(x_i + \sum_{j \neq i} x_j + y\right) - c'_i(x_i) + P'\left(x_i + \sum_{j \neq i} x_j + y\right) x_i = 0 \quad (3)$$

$$\pi_p(y; x_1, \dots, x_n) = P\left(\sum_{i=1}^n x_i + y\right) y - c_p(y) = 0 \quad (4)$$

By using the assumption of symmetric equilibrium concerning private firms,  $x_i^*(n)$  and  $y^*(n)$  are determined to satisfy both (5) and (6).

$$P(nx_i^*(n) + y^*(n)) - c'_i + P'(nx_i^*(n) + y^*(n))x_i^*(n) = 0 \quad (5)$$

$$P(nx_i^*(n) + y^*(n))y^*(n) - c_p(y^*(n)) = 0 \quad (6)$$

In order to obtain the comparative static results for the effects of the change of the number of the private firms on the welfare, totally differentiate (5) and (6). This yields

$$\begin{pmatrix} nP''x + (n+1)P' - c'_i & P''x + P' \\ nP'y & P'y + P' - c'_p \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} P''x^2 + P'x \\ P'xy \end{pmatrix} dn$$

The solution is obtained by matrix inversion:

$$\frac{dx}{dn} = -\frac{1}{\Delta} x(P''x + P')(P - c'_p) \quad (7.1)$$

$$\frac{dy}{dn} = -\frac{1}{\Delta} P'_{xy}(P' - c'_i) \quad (7.2)$$

where

$$\Delta = n(P''_{xx}P')(P - c'_p) + (P' - c'_i)(P - P'y - c'_p)$$

$\Delta > 0$  is assumed for stability of the Cournot equilibrium.

The equilibrium number of private firms  $n_e$  is determined so as to satisfy (8)

$$P(n_e x_i^*(n_e) + y^*(n_e)) x_i^*(n_e) - c_i(x_i^*(n_e)) = 0 \quad (8)$$

### 3. THE EFFECT OF ENTRY BARRIERS IN A "MIXTURE" MARKET

Define the net market surplus function  $W(x_i, y, n)$  by:

$$\begin{aligned} W(x_i, y, n) &\equiv \int^{nx_i + y} P(Z) dZ - (nx_i + y)P(nx_i + y) + n\pi_i(x_i, y) + \pi_p(x_i, y) \\ &= \int^{nx_i + y} P(Z) dZ - nc_i(x_i) - c_p(y) \end{aligned} \quad (9)$$

where  $x_i$  is the output of each private firm,  $y$  is the output of the government firm and  $n$  is the number of the private firms.

Along the line of Suzumura and Kiyono (1987), the author assume the second-best government, which can control the number of private firms but not their competitive behavior.

Differentiating (9) with respect  $n$ , we obtain

$$\begin{aligned} \left. \frac{dW(n)}{dn} \right|_{n=n_e} &= P(n_e x_i^*(n_e) + y^*(n_e)) \left( x_i^*(n_e) + n_e \frac{\partial x_i^*}{\partial n} + \frac{\partial y^*}{\partial n} \right) \\ &\quad - c_i(x_i^*(n_e)) - n_e c'_i \frac{dx_i^*}{dn} - c'_p \frac{dy^*}{dn} \\ &= P(n_e x_i^*(n_e) + y^*(n_e)) x_i^*(n_e) - c_i(x_i^*(n_e)) \\ &\quad + n_e \frac{dx_i^*}{dn} [P(n_e x_i^*(n_e) + y^*(n_e)) - c'_i] \\ &\quad + \frac{dy^*}{dn} [P(n_e x_i^*(n_e) + y^*(n_e)) - c'_p] \end{aligned}$$

The first term is the competition-promotion effect, the second term is the allocational effect of the private firms and the third term is the allocational effect of the government firm. At the free-entry equilibrium, the first term is zero.

$$P - c'_i(x^*(n_e)) = -P' x_i^*(n_e) > 0$$

$$\begin{aligned} P - c'_p(y^*(n_e)) &= P - AC_p(y^*(n_e))y^*(n_e)AC'_p(y^*(n_e)) \\ &= -y^*(n_e)AC'_p(y^*(n_e)) < 0 \end{aligned}$$

And it follows from (7.1), (7.2) and the assumption of strategic substitutes that  $dx_i^*/dn < 0$  and  $dy^*/dn < 0$ . Hence the second term is negative and the third term is positive.

The sign of overall allocation effect depends on which influence dominates.

If the second term dominates, the excess entry theorem holds, and

If the third term dominates, the insufficient entry theorem holds.

Using the comparative static results from (7.1) and (7.2) together with the private firm's first order condition, yields

$$\left. \frac{dW(n)}{dn} \right|_{n=n_e} = \frac{1}{\Delta} x^* y^* AC'_p(y) P' [-P'' x^{*2} - P'(n_e x^* - y^*) - y^* c'_i] \quad (10)$$

Since  $\Delta > 0$ ,  $AC'_p(y^*) > 0$  and  $P' < 0$ , we can assert the following result.

**PROPOSITION.** *When  $-P'' x^{*2} - P'(n_e x^* - y^*) - y^* c'_i < 0$ , welfare decreases by reducing the number of private firms from the free entry equilibrium number.*

This result is in sharp contrast with the "excess entry theorem" which states that the number of firms at the free-entry equilibrium exceeds the number of welfare maximizing number of firms.

**REMARK.** When the government firm is a profit maximizer pure and simple, no matter how low its cost is,  $P - c'_p > 0$  and

$$\left. \frac{dW(n)}{dn} \right|_{n=n_e} = n_e \frac{dx_i^*}{dn} (P(\cdot) - c'_i) + \frac{dy^*}{dn} (P(\cdot) - c'_p) < 0$$

So, "excess entry theorem" always holds.

Especially, when the demand curve is linear and the marginal cost of each private firm is constant, the condition that ensures "insufficient entry theorem" is clear. The following corollary states this condition.

**COROLLARY.** *Suppose that the demand curve is linear and the marginal cost of each private firm is constant.*

*When  $y^* > n_e x^*$ , i.e. the share of the government firm is greater than half, welfare decreases by reducing the number of private firms from the free entry equilibrium number.*

#### 4. AN EXAMPLE

Because the proposition and the corollary stated above depends on the cost function of the private firms and the government firm, it will be useful to provide an example which shows both "excess entry" and "insufficient entry" are possible

in a “mixture” oligopolistic market.

Let

$$P(Z) = a - bZ$$

$$c_i(x_i) = cx_i + K \quad (\text{where } K \text{ is the fixed cost of the } i\text{-th private firm.})$$

then

If  $AC_p\left(\frac{a-c-\sqrt{kb}}{2b}\right) > c + Kb$ , excess entry theorem holds, and

if  $AC_p\left(\frac{a-c-\sqrt{kb}}{2b}\right) < c + Kb$ , insufficient entry theorem holds.

*Proof.* The reaction function of the  $i$ -th private firm, taking  $y$  given, is

$$x_i(y) = \frac{a-c-\sqrt{kb}}{b(n+1)}$$

and the profit of the  $i$ -th private firm, taking  $y$  given, is

$$\pi_i(y) = \frac{(a-c-by)^2}{b(n+1)^2} - K$$

So the equilibrium number of private firms, taking  $y$  given,  $n_e(y)$  is determined to be

$$\frac{a-c-by-\sqrt{Kb}}{Kb}$$

Total output level at free entrance is

$$n_e(y)x + y = \frac{a-c-\sqrt{Kb}}{b}$$

Hence, the profit of the government firm is expressed as

$$\begin{aligned} \pi_p(y) &= P(n_e(y)x + y)y - c_p(y) \\ &= (c + \sqrt{Kb})y - c_p(y) \end{aligned}$$

Because the government firm is a zero-profit constrained scale-maximizer, output level of the government firm is determined to satisfy

$$AC_p(y) = c + \sqrt{Kb}$$

Therefore, we can obtain

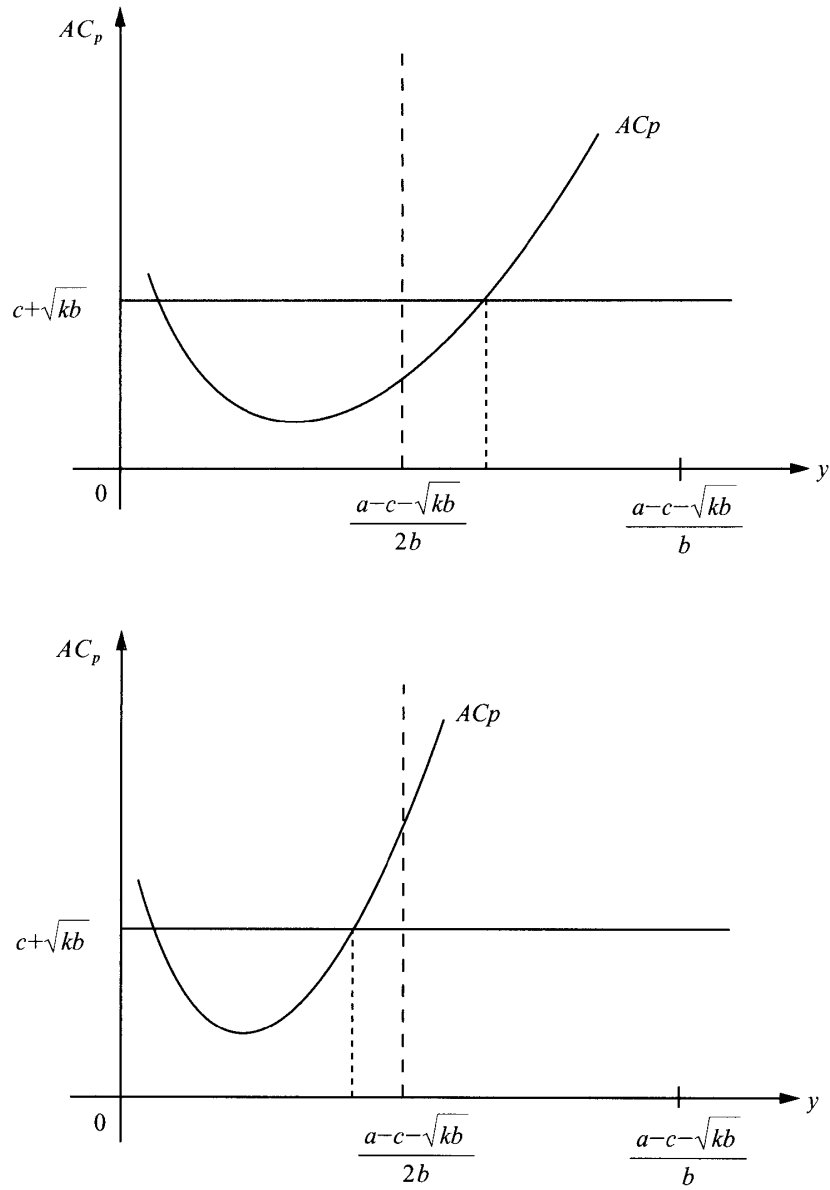


Fig. 1.

$$y \geq n_e x \Leftrightarrow AC_p\left(\frac{a-c-\sqrt{kb}}{2b}\right) \leq c + \sqrt{Kb}$$

The above discussion can be summarized as follows,

If  $AC_p\left(\frac{a-c-\sqrt{kb}}{2b}\right) > c + \sqrt{Kb}$ , excess entry theorem holds, and

if  $AC_p\left(\frac{a-c-\sqrt{kb}}{2b}\right) < c + \sqrt{Kb}$ , insufficient entry theorem holds.



## 6. CONCLUDING REMARKS

This paper examined if free entry is desirable for the enhancement of welfare in a “mixture” oligopolistic market, where a number of private firms are maximizing their profits and one government firm is maximizing its shares rather than its profit. It was shown that, when one firm operates under its scale maximization, the number of the firms in an industry may be too small from the view point of economic welfare, and consequently, more competition among private firms may improve economic welfare.

Many economists believe that increasing competition improves welfare. On the other hand, the government, journalists etc. support the idea that more competition may deteriorate welfare. We conclude that the degree of competition may be excessive or deficient depending on the cost function of scale-maximizing government firm. This is not obtained in Ohyama (1990) nor Suzumura (1993). This paper is an attempt to show that neither belief above can be correct and both belief needs to be reconsidered.

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