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INTRAINDUSTRY TRADE UNDER VERTICAL PRODUCT DIFFERENTIATION

Luca LAMBERTINI

Dipartimento di Scienze Economiche Università degli Studi di Bologna, Bologna, Italy
and
Institute of Economics University of Copenhagen, Copenhagen, Denmark

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Abstract: The paper focuses on trade between two countries where a vertically differentiated commodity is produced by a single firm in each country, operating initially in autarkic conditions. It is assumed that the two countries have overlapping income distributions, giving thus rise, under certain conditions, to two-way trade, i.e., a proper intraindustry trade. It emerges that while consumers always benefit from trade, especially if two-way trade arises, firms may have conflicting preferences on the choice between (i) autarky and trade, as well as (ii) one-way trade and two-way trade.

JEL classification: F12, L13

Key-words: intraindustry trade, vertical differentiation, income distribution.

1. INTRODUCTION

The role of product differentiation and different consumer preferences across countries have been first advocated as two major factors explaining intraindustry trade between developed countries by Linder (1961). He stressed that a large intraindustry trade occurs between developed countries which have comparable factor endowments, a fact that according to the Heckscher-Ohlin theory we might not expect to observe. Then, a cause of trade other than factor endowment must be identified in the interaction between preferences and income. The role of the former in explaining intraindustry trade has been investigated in several contributions resorting to the Chamberlinian approach (Krugman, 1979; Helpman, 1981; Markusen, 1981). Lancaster (1979, 1980) has introduced this

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issue in the context of the address approach. The relevance of income has been emphasized by Hunter and Markusen (1987). In a different context, Shaked and Sutton (1984) have described the effects of free trade in a two-country setting where vertically differentiated firms compete in prices. In the short run, the exit of low-quality goods leads to an increase in consumer surplus, but a negative effect on social welfare may obtain if the decrease in firms' profit due to the higher intensity of price competition is large enough. In the long run, the increase in global market size favours firms' R & D efforts that ultimately lead to an increase in the quality level. Though, the net effect on welfare is ambiguous. Motta (1992) has extended Shaked and Sutton's analysis to the case of Cournot competition between firms located in countries of different size, showing that (i) if firms compete in the global market with the qualities chosen in autarky, the small country may lose from trade liberalization; and (ii) if firms may optimally set their respective qualities under free trade, then all countries experience a welfare gain. The issue of North-South trade in vertically differentiated products has been focused upon by Flam and Helpman (1987), in a model where two countries are endowed with technologies characterized by different levels of efficiency. They show that the advanced country produces the top quality goods while the other supplies and exports the low quality goods.[1] A dramatic change in the pattern of production and trade may occur if technical progress is faster in the South than in the North.

I focus on the interplay between preferences and income on one side and vertical differentiation on the other side as determinants of intraindustry trade between two countries characterized by different income distributions as well as consumer densities.[2] A partial equilibrium perspective is adopted in order to isolate the linkages between product specification, consumers' preferences and the trade pattern within an industry. I assume that the product variety offered by each firm is first determined under autarky. When trade opens, provided that quality cannot be changed due to a sunk cost, firms can only adjust their respective prices. I adopt the hypothesis of market integration, i.e., firms cannot price-discriminate by charging different prices in the two countries. This may be due to the possibility of arbitrage by consumers or to legal constraints. The alternative cases of one-way and two-way trade are described. The results point to a main consideration, namely that while free trade is generally preferable to autarky from a social standpoint, the choice between one and two-way trade essentially depends on the high-quality firm's preferences, which in this respect appear non-monotone. A proper form of intraindustry trade may benefit the high-quality firm if the income difference between the consumers in the two countries is not too wide, while is not necessarily preferred to simple one-way trade by the firm located in the poor country, that

[1] This is confirmed by the empirical literature available, according to which poor countries specialize in low-quality goods. See Tharakan (1984); Tharakan, Kerstens and Glejser (1994).

[2] The model shares many features with that in Motta, Thisse and Cabrales (1995), where the issue of leadership persistence vs leapfrogging is modelled.

is specialized in the production of a low-quality good, unless her home market is very poor as compared to the foreign one. Neat conclusions can be reached as far as consumer surplus and social welfare are concerned. Under this respect, two-way trade is generally preferred to one-way trade by both countries.

The paper is structured as follows. The model is introduced in section 2. Section 3 describes the autarkic regime. Then, sections 3 and 4 deal with one-way and two-way trade, respectively. The issue of the choice between the two free trade regimes is tackled in section 5. Sections 6 through 8 contain a comparative evaluation of the results. Finally, section 9 provides concluding remarks and suggestions for future research.

2. THE MODEL

Label the two countries as A and B, respectively. In each country, consumers' population is uniformly distributed over the interval $[0, \bar{\theta}_i]$, $i=A, B$, where parameter θ represents each individual's marginal willingness to pay for quality. I assume $\bar{\theta}_A \geq \bar{\theta}_B$, i.e., country A is at least as rich as country B, in that the marginal willingness to pay of the richer consumer living in country A is not lower than that of the richest consumer in country B. If in each country the number of consumers identified by a generic level of θ is s_i, the global dimension of each market is given by $L_i = s_i \bar{\theta}_i$, and I assume that $L_A = nL_B$, with $n>0$, so that country A is larger than country B if $n>1$. Each consumer buys at most one unit of the product if and only if the net surplus he gets from consumption is non negative:

$$U = \theta q - p \geq 0, \tag{1}$$

where q is the quality of the good and p the price at which it is sold.

On the supply side, one firm is active in each country under autarky, offering a good whose production requires a fixed cost which is convex in quality:

$$F = tq^2, \qquad t>0. \tag{2}$$

Variable costs are assumed away. This hypothesis may be given the following justification: quality can be thought of as the result of investments in R & D, whose size is increasing in the quality level of the good being supplied, while it is completely unrelated to the scale of production. It could easily be shown that the introduction of a constant unit variable cost would not modify significantly the results that I am going to derive in the following sections. Consequently, it can be normalised to zero without loss of generality. [3] Finally, I shall assume that fixed costs are sunk, implying that firms choose quality once and for all.

[3] Instead, the assumption of variable costs increasing in quality would radically change the picture.[3] This setting is investigated in Lambertini and Rossini (1994).

3. THE AUTARKY EQUILIBRIUM

Under autarky each firm operates as a monopolist in her own market. Her objective function is

$$\pi_i^m = p_i x_i - t q_i^2 , \qquad i = A, B , \tag{3}$$

where x_i is market demand, defined as follows:

$$x_i = \left(\bar{\theta}_i - \frac{p_i}{q_i} \right) \frac{L_i}{\bar{\theta}_i} , \qquad i = A, B . \tag{4}$$

From the first order conditions for profit maximization w.r.t. quality and price, we have[4]

$$q_i^m = \frac{\bar{\theta}_i L_i}{8t} ; \qquad p_i^m = \frac{\bar{\theta}_i^2 L_i}{16t} , \tag{5}$$

yielding

$$x_i^m = \frac{L_i}{2} ; \qquad \pi_i^m = \frac{\bar{\theta}_i^2 L_i^2}{64t} \tag{6}$$

as the optimal quantity and maximum profit. It appears thus that the monopolist always serves the upper (or richer) half of the market.[5] Besides, all equilibrium magnitudes increase as $\bar{\theta}_i$ and s_i increase. This implies that the monopolist will find it advantageous to improve product quality as the marginal willingness to pay of the richest consumer increases. Analogously, she will increase quality as consumer density increases, provided that the burden of any increase in quality falls upon fixed costs only. These linkages between quality and marginal willingness to pay as well as consumer density entail that the higher quality good is not necessarily being produced in the richer country, unless the following inequality is met:

$$L_A > \frac{L_B \bar{\theta}_B}{\bar{\theta}_A} . \tag{7}$$

Since it appears natural to think that the possibility of serving richer consumers provides an incentive to produce a good of higher quality as compared to a market where consumers are characterized by a lower marginal willingness to pay, in the next section I will specify the conditions under which the above inequality holds.

As for consumer surplus, it is defined as follows:

[4] Second order conditions are met throughout the calculations performed in the paper, although not shown for the sake of brevity.

[5] A social planner aiming at the maximization of social welfare would supply a higher quality as compared to the profit-seeking monopolist. Furthermore, the planner would price at marginal cost in order to serve all consumers. See Appendix A.

$$CS_i^m = \frac{L_i}{\bar{\theta}_i} \int_{p/q}^{\bar{\theta}} (\theta q - p) d\theta \,, \tag{8}$$

while social welfare corresponds to the sum of consumer and producer surplus. Then, straightforward calculations show that consumer surplus and social welfare under autarky amount to

$$CS_i^m = \frac{\bar{\theta}_i^2 L_i^2}{64t}; \qquad SW_i^m = \frac{\bar{\theta}_i^2 L_i^2}{32t} \,. \tag{9}$$

4. FREE TRADE

When trade opens, one has first to define the market demands accruing to the two varieties. Define as h and k the marginal willingness to pay of the consumers indifferent between the two goods and between the low quality good and nothing at all, respectively:

$$h = \frac{(p_A - p_B)}{q_A - q_B}; \qquad k = \frac{p_B}{q_B} \,, \tag{10}$$

where both qualities are fixed at the levels chosen by each firm under autarky. Two alternative settings can emerge. In the first, one-way trade occurs, with the firm located in country B exporting to country A. This situation is depicted in Figure 1. $\bar{\theta}_B$ lies between h and k, so that consumers in country B cannot afford the high quality-good, which is non-traded.

In the second, two-way trade obtains, with both qualities being purchased in both countries, giving rise to a proper intraindustry trade. This situation is described by Figure 2. Here, the marginal willignesses to pay of the consumers indifferent, respectively, between buying either good and between the buying the low-quality good and nothing at all, are both lower than $\bar{\theta}_B$, so that both varieties are traded.

The firm located in the richer country (A) offers a good of higher quality as compared to the firm operating in country B (see below), so that their respective market demands can be indexed as A and B, and are now defined as follows:

$$x_A = (\bar{\theta}_A - h)\frac{L_A}{\bar{\theta}_A}; \qquad x_B = (h - \bar{\theta}_B)\frac{L_A}{\bar{\theta}_A} + (\bar{\theta}_B - k)\left(\frac{L_A}{\bar{\theta}_A} + \frac{L_B}{\bar{\theta}_B}\right) \tag{11}$$

if one-way trade occurs, and

$$x_A = (\bar{\theta}_A - \bar{\theta}_B)\frac{L_A}{\bar{\theta}_A} + (\bar{\theta}_B - h)\left(\frac{L_A}{\bar{\theta}_A} + \frac{L_B}{\bar{\theta}_B}\right); \qquad x_B = (h - k)\left(\frac{L_A}{\bar{\theta}_A} + \frac{L_B}{\bar{\theta}_B}\right), \tag{12}$$

when two-way trade obtains.

As for product quality, the conditions needed for the quality of the variety

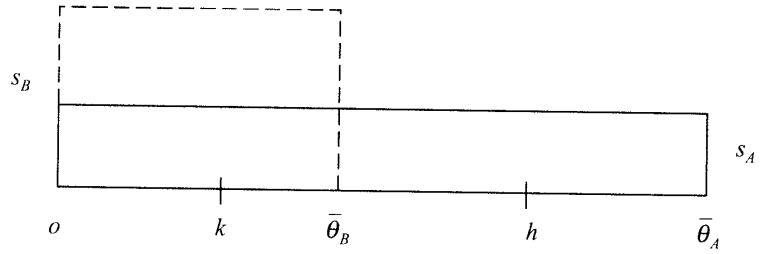

Fig. 1. One-way trade.

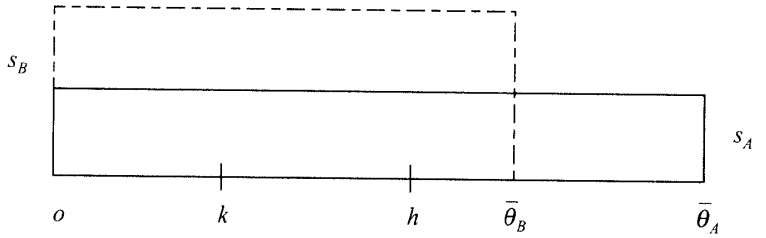

Fig. 2. Two-way trade.

being produced in country A to be higher than that of the variety being produced in country B can be established in the following way. Without loss of generality, set $s_B=1$, and $\bar{\theta}_B=r\bar{\theta}_A$, with $r\in]0, 1]$. Accordingly, from $L_A=nL_B$, $L_A=nr\bar{\theta}_A$ obtains. This allows to reduce significantly the number of parameters involved in the model and ease calculations without prejudicing the validity of the results. Consequently, it can be stated that

$$q_A>q_B \qquad \text{iff} \quad n>r\,. \tag{13}$$

In the remainder of the paper I shall assume that condition (13) holds. Provided that r cannot be greater than one, the above condition implies, as an overall constraint, $r\in]0, 1]$ if $n\geq 1$ i.e., if country A is at least as large as country B, and $r\in]0, n]$ if instead the richer country is smaller than the poorer one.[6]

The two profit functions appear now as follows:

$$\pi_A^d=p_Ax_A-tq_A^2\,; \qquad \pi_B^d=p_Bx_B-tq_B^2\,, \tag{14}$$

where the superscript d stands for *duopoly*, and market demands x_A and x_B are defined as in (11) if one-way trade occurs, or alternatively as in (12) if two-way trade is observed.

4.1. ONE-WAY TRADE

Assume now that trade liberalization leads to a one-way trade from the poor to the rich country, i.e., the low-quality good is exported from the poor country

[6] Notice that these conditions are also sufficient to ensure that under autarky the profit of firm A is at least as large as the profit of firm B as described by expression (6), since $\pi_A^m\geq\pi_B^m$ if $\bar{\theta}_A^2L_A^2\geq\bar{\theta}_B^2L_B^2$, which is true for all $n\geq r$.

(B) to the rich country (A), while the high-quality good produced in country A is non-traded. Market demands are thus given by the expressions in (11), and after the opening of trade, firms simultaneously compete in prices. The first order conditions (FOCs) for profit maximization are:

$$\frac{\partial \pi_A^{1w}}{\partial p_A} = \frac{n(\bar{\theta}_A^3 r^2 - \bar{\theta}_A^3 rn + 16tp_A - 8tp_B)}{\bar{\theta}_A^2(r-n)} = 0\,; \tag{15}$$

$$\frac{\partial \pi_B^{1w}}{\partial p_B} = r\bar{\theta}_A - (1-rn)\frac{16tp_B}{r^2\bar{\theta}_A^2} - (2p_A - p_B)\frac{8tn}{\bar{\theta}_A^2(r-n)} = 0\,, \tag{16}$$

where the superscript $1w$ stands for *one-way trade*. By solving the system (15–16), one gets the following equilibrium prices:

$$p_A^{1w} = \frac{r\bar{\theta}_A^3(n-r)(2n-2r+2n^2r+r^2)}{8t(4n-4r+4n^2r-nr^2)}\,; \tag{17}$$

$$p_B^{1w} = \frac{\bar{\theta}_A^3 r^3(n+2)(n-r)}{8t(4n-4r+4n^2r-nr^2)}\,. \tag{18}$$

Observe that the equation $p_A^{1w} - p_B^{1w} = 0$ has two roots w.r.t. n, i.e., $n_1^p(r) = (r^2 - 2 + \sqrt{4+12r^2+8r^3+r^4})/(4r)$ and $n_2^p(r) = (r^2 - 2 - \sqrt{4+12r^2+8r^3+r^4})/(4r)$, where superscript p indicates that we are concerned with the price sequence. Notice that $r \geq n_1^p(r) \geq n_2^p(r)$ and $p_A^{1w} > p_B^{1w}$, if n lies outside the interval defined by the two roots, for any positive value of r. As a consequence, the condition $n > r$ needed for $q_A > q_B$ suffices to establish that the necessary price difference for the low-quality good to be traded always obtains. The equilibrium quantities for the two goods can be easily calculated:

$$x_A^{1w} = \frac{\bar{\theta}_A nr(2n-2r+2n^2r+r^2)}{4n-4r+4n^2r-nr^2}\,; \tag{19}$$

$$x_B^{1w} = \frac{\bar{\theta}_A r(2+n)(n-r+n^2r)}{4n-4r+4n^2r-nr^2}\,. \tag{20}$$

I can now focus on the distribution of surplus between producers and consumers at equilibrium, in each country. Equilibrium profits are:

$$\begin{aligned}\pi_A^{1w} = &-\bar{\theta}_A^4 nr^2(64n^2r - 16n^3 - 32n^4r - 80nr^2 - 32n^2r^2 + 88n^3r^2 \\ &- 16n^5r^2 + 32r^3 + 64nr^3 - 56n^2r^3 - 32n^3r^3 + 24n^4r^3 - 32r^4 \\ &- 8nr^4 + 32n^2r^4 + n^3r^4 + 8r^5)/(64t(4n-4r+4n^2r-nr^2)^2)\,;\end{aligned} \tag{21}$$

$$\begin{aligned}\pi_B^{1w} = &-\bar{\theta}_A^4 r^4(32nr - 16n^2 - 32n^3 - 8n^4 + 64n^2r + 16n^3r - 32n^4r \\ &- 8n^5r - 16r^2 - 32nr^2 - 16n^2r^2 + 32n^3r^2 + 24n^4r^2 + 8nr^3 \\ &- 8n^3r^3 + n^2r^4)/(64t(4n-4r+4n^2r-nr^2)^2)\,.\end{aligned} \tag{22}$$

Consumer surplus in the two countries is given by:

$$CS_A^{1w} = \bar{\theta}_A^4 nr^2(4n^3 + 4n^2r + 8n^4r - 20nr^2 - 12n^2r^2 + 8n^3r^2 + 4n^5r^2 + 12r^3 + 24nr^3 - 12n^2r^3 - 8n^3r^3 + 5n^4r^3 - 12r^4 - 7nr^4 + 8n^2r^4 + 3r^5)/(16t(4n - 4r + 4n^2r - nr^2)^2); \quad (23)$$

$$CS_B^{1w} = \frac{\bar{\theta}_A^4 r^4(n^2 - 2n + 2r - nr - 4n^2r + nr^2)^2}{16t(4n - 4r + 4n^2r - nr^2)^2} \quad (24)$$

The equilibrium values of social welfare in the two countries, SW_A^{1w} and SW_B^{1w}, can be obtained by summing (22) to (24) and (23) to (25), respectively. The expressions for SW_A^{1w} and SW_B^{1w} are displayed in Appendix B.

4.2. *Two-way trade*

Consider now the setting in which both varieties are traded, i.e., not only the low-quality good produced in country B is exported to country A, but also the high-quality good produced in country A is made available for purchase by consumers living in country B. Demands are now defined as in expression (12) above. As in the case of one-way trade previously treated, after trade liberalization firms noncooperatively and simultaneously set prices. The FOCs w.r.t. prices are:

$$\frac{\partial \pi_A^{2w}}{\partial p_A} = \frac{\bar{\theta}_A^3 r^2(r + nr - n^2 - n) - 8tp_A + 16tp_B + 16tnrp_A - 8tnrp_B}{\bar{\theta}_A^2 r(r - n)} = 0; \quad (25)$$

$$\frac{\partial \pi_B^{2w}}{\partial p_B} = \frac{8t(nr + 1)(2np_B - rp_A)}{\bar{\theta}_A^2 r^2(r - n)} = 0. \quad (26)$$

Superscript $2w$ stands for *two-way trade*. Solving the system (25–26), one gets the Nash equilibrium prices:

$$p_A^{2w} = \frac{\bar{\theta}_A^3 nr^2(n + 1)(n - r)}{4t(4n - r)(nr + 1)}; \quad (27)$$

$$p_B^{2w} = \frac{\bar{\theta}_A^3 r^3(n + 1)(n - r)}{8t(4n - r)(nr + 1)}, \quad (28)$$

with $p_A^{2w} > p_B^{2w}$ if $2n > r$, which is always true. As to the equilibrium quantities for the two firms, they turn out to be the following:

$$x_A^{2w} = \frac{2\bar{\theta}_A nr(n + 1)}{4n - r}; \quad (29)$$

$$x_B^{2w} = \frac{\bar{\theta}_A nr(n + 1)}{4n - r}, \quad (30)$$

so that $x_B^{2w} = x_A^{2w}/2$, i.e., the high quality firm located in country A sells twice as much as the low-quality firm located in country B.

By substituting prices (27–28) into the objective functions and simplifying, the

equilibrium profits under two-way trade obtain:

$$\pi_A^{2w} = \frac{\bar{\theta}_A^4 n^2 r^2 (40nr - 16n^2 + 64n^2 r + 16n^3 r - 33r^2 - 64nr^2 - 24n^2 r^2 - nr^3)}{64t(nr+1)(4n-r)^2}; \quad (31)$$

$$\pi_B^{2w} = \frac{\bar{\theta}_A^4 r^4 (8n^4 + 16n^3 - 8n^2 - 16n^2 r - 24n^3 r - r^2 + 8n^2 r^2 - nr^3)}{64t(nr+1)(4n-r)^2}. \quad (32)$$

Furthermore, consumer surplus in the two countries amounts to

$$\begin{aligned} CS_A^{2w} = \bar{\theta}_A^4 n^2 r^2 (16n^2 - 8nr - 16n^2 r + 16n^3 r + r^2 + 20nr^2 + 8n^2 r^2 \\ - 8n^3 r^2 + 4n^4 r^2 - 4r^3 - 9nr^3 + 6n^2 r^3 + 5n^3 r^3 + 3r^4 \\ + 2nr^4)/(16t(nr+1)^2(4n-r)^2); \end{aligned} \quad (33)$$

$$\begin{aligned} CS_B^{2w} = \bar{\theta}_A^4 n^2 r^3 (4n - 8n^2 + 4n^3 + 5r + 6nr + 9n^2 r - 16n^3 r + 2r^2 + 7nr^2 + 20n^2 r^2 \\ + 16n^3 r^2 - 2r^3 - 4nr^3 - 8n^2 r^3 + nr^4)/(16t(nr+1)^2(4n-r)^2). \end{aligned} \quad (34)$$

Finally, the social welfare levels in the two countries, SW_A^{2w} and SW_B^{2w}, can be obtained by adding (33) to (31) and (34) to (32), respectively. Both magnitudes can be found in Appendix B.

5. ONE OR TWO-WAY TRADE?

Before proceeding to the comparison of the results observed under one and two-way trade with what happens under autarky, it must be firstly established in what parameter range one kind of trade or the other may arise; and it must be also taken into account that the choice between the two alternative trade regimes may well depend upon the performance of the high-quality firm, who can decide whether to export or not to the poor country by comparing the profits she can gain in the two settings. In other terms, the necessary condition for one-way trade to arise is that, in correspondence of the price vector derived under one way-trade, the marginal willingness to pay of the individual who is indifferent between the two varieties lies below $\bar{\theta}_B$. In such a case, there are consumers in both countries that are able to purchase the low-quality good, while the other variety is accessible only to some consumers in the richer country. We will see that this is the case in a particular range of parameters. It can also be shown that two-way trade is possible with no specific restrictions on the relevant parameters, other than those previously introduced. As a result, there exists a range of parameters where both kinds of trade are possible, and which one is going to arise depends on the high-quality firm's preferences. Hence, in the region of parameters where both kinds of trade are feasible, a sufficient condition for one-way trade to obtain is given in terms of the high-quality firm's profit under each alternative regime. This is summarized in the following

CLAIM 1. (a) The necessary condition for one-way trade to arise is that

$r \in]0, 1/2[$ and $n > r > n_2^{1w}(r) > n_1^p(r)$. (b) No restrictions on r and n are required for two-way trade to obtain. (c) The sufficient condition for one-way trade to arise is either (i) $n=1$, $r \in]0, 0.328173[$; or (ii) as r tends to 1/2 from below, $n \in]0, 0.623616[$.

Consider first point (a). In such a case, the following sequence of inequalities must hold:[7]

$$h > \bar{\theta}_B > k\,, \qquad \bar{\theta}_B = r\bar{\theta}_A\,. \tag{35}$$

It can be easily established that $\bar{\theta}_B > k$ for all r and n. As to the first inequality in (35), consider what follows. Solving the equation $h - \bar{\theta}_B = 0$ w.r.t. n yields two roots, $n_1^{1w}(r)$ and $n_2^{1w}(r)$, with (i) $n_2^{1w}(r) > n_1^{1w}(r) \forall r \in]0, 1/2[$, and conversely; (ii) $r > n_2^{1w}(r) > n_1^p(r) \forall r \in]0, 1/2[$; and finally (iii) $h > \bar{\theta}_B$ *iff* $r \in]0, 1/2[$ and $n > n_2^{1w}(r)$. The above condition is met when $n > r > n_2^{1w}(r) > n_1^p(r) \forall r \in]0, 1/2[$, which is acceptable. When instead $r \geq 1/2$, one obtains that $h > \bar{\theta}_B$ *iff* $r > n > n_1^{1w}(r) > n_1^p(r)$, which is not acceptable, provided that $n > r$. Hence, the result stated in point (a) of Claim 1 follows. This amounts to saying that for values of r equal to or exceeding 1/2, the marginal willingness to pay characterizing the consumer indifferent between the two goods falls below $\bar{\theta}_B$, so that two-way trade is the only possible regime. This leads to point (b). It turns out that the inequalities needed for two-way trade to arise, i.e., $\bar{\theta}_B > h > k$, are satisfied for all admissible values of r (the proof is in Appendix C.2).

Hence, in line of principle, if $r \in]0, 1/2[$ and $n > r$, both regimes are possible. This finally leads us to point (c), i.e., to evaluate the preferences of firm A as to the kind of trade. By evaluating the sign of

$$\Delta\pi_A^{21w} = \pi_A^{2w} - \pi_A^{1w}\,, \tag{36}$$

it can be established that (i) if $n=1$, $\Delta\pi_A^{21w} > 0 \forall r \in]0.328173, 1/2[$; and (ii) if $r = 1/4$, $\Delta\pi_A^{21w} > 0 \forall n \in]0.322024, 0.40922[$. Moreover, it can be verified that, as r approaches 1/2, both the lower and the upper bound of such interval increase, and as r tends to 1/2 in the limit, the interval becomes $]0.623616, \infty[$. Conversely, as r approaches 0, the interval obviously tends to disappear since no trade is possible. Thus, when r is very close to 1/2, firm A is better off under two-way trade unless her home market is considerably smaller than country B. If this is not the case, she will decide not to set her price low enough to allow for the high-quality good to be purchased by consumers living in country B, since their number is not sufficient to compensate for the loss due to the decrease in the price charged by firm A and her consequent inability to appropriate a large share of consumer surplus in her domestic market.[8] Otherwise, when r takes intermediate

[7] The equilibrium values of h and k for the case of one-way trade, as well as the roots of the equation $h - \bar{\theta}_B = 0$, are in Appendix C.1. The equilibrium values of h and k under two-way trade are in Appendix C.2.

[8] This is clearly due to the fact that there is no market segmentation, i.e., firms are assumed to be unable to price discriminate between consumers in the two countries. For an analysis of such a setting, though exclusively carried out under two-way trade, see Motta, Thisse and Cabrales (1995).

values in the interval [0, 1/2[, as the ratio between the two countries' consumer mass increases, the preferences of firm A over the kind of trade are non-monotone. This fact can be given the following interpretation. Given r, for very small values of n firm A would prefer not to export because, be the size of foreign demand as considerable as it may, it is insufficient to compensate for the enhanced competition associated with trade liberalization;[9] As n increases, the two countries become comparable in terms of total demand and this, as long as firm A produces the high-quality good, makes exports attractive; finally, n may become so large that the size of foreign demand facing firm A is not worth exporting the high-quality good. If $n=1$, i.e., effects due to relative size are ruled out, one-way trade occurs for $r \in]0, 0.328173[$, i.e., when country B is so poor that firm A has no incentive to export, because no quantity effect is sufficient to compensate for the lower marginal willingness to pay.

Some further considerations are in order. The above analysis has enlightened the existence of a parameter region where the high-quality firm is required to decide over its price policy so as to determine the nature of intraindustry trade on the basis of her profit-maximizing behaviour, provided that the low-quality good is traded anyway. One may wonder whether the choice not to export is available to firm A also outside the intervals established in Claim 1. The answer is negative, for the following reason. When either r or n or both fail to meet the above requirements, firm A is indeed unable to choose between one and two-way trade since only the latter regime is possible, because either the relative wealth or the relative size or both are such that some consumers in country B can afford to buy the high-quality good. In such a situation, the only way firm A can artificially bring about one-way trade is to increase her price well beyond the profit-maximizing level, which is clearly suboptimal.

These results will be taken into account in sections 6–8, where the comparison between autarky, one-way and two-way trade is carried out through numerical simulations (where needed), fixing $n=1$ with (i) $r \in]0, 0328173[$, when comparing one-way trade with autarky; or two way trade; and (ii) $r \in [0.328173, 1[$, when comparing autarky with two-way trade. Finally, the comparative evaluation of all regimes in terms of relative size is worked out over the interval $n \in]r, \infty[$, fixing, respectively, $r = 1/4$ when the comparison involves one-way trade and either autarky or two-way trade, and $r=1$ when comparing two-way trade with autarky. To avoid repetitions as much as possible, in the remainder I shall often refer to each of these intervals as to the admissible or relevant range for the parameter considered in any particular simulation.

[9] The poor performance of firm A when n takes low values appears to be due to the lack of an adequate "home market effect", which is instead working in favour of the low-quality firm (see Helpman and Krugman, 1985).

6. ONE-WAY TRADE VS AUTARKY

In this section, I shall proceed to the comparative evaluation of the equilibrium values of individual and collective surpluses as well as the other relevant magnitudes under autarky and one-way trade. To begin with, consider prices and quantities. The consequences of one-way trade on these magnitudes is summarized by

CLAIM 2. While trade liberalization unambiguously lowers the price of the high-quality good, the price of the low-quality good may change either way. Both firms sell larger quantities after the opening of trade.

Define the following differences:

$$\Delta p_A^{1wm} = p_A^{1w} - p_A^m, \qquad \Delta p_B^{1wm} = p_B^{1w} - p_B^m; \tag{37}$$

$$\Delta x_A^{1wm} = x_A^{1w} - x_A^m, \qquad \Delta x_B^{1wm} = x_B^{1w} - x_B^m. \tag{38}$$

It is easy to verify that Δp_A^{1wm} is always negative in the relevant range, independently of the relative size of the two countries. As it could be expected from the outset, the competition implicit in the opening of trade lowers the price of the high-quality good. A slightly different story must be told about the price of the low-quality good. It turns out that Δp_B^{1wm} is negative for all $n \in]r, r(2-r)/(2(1-2r))[$. If we fix $n=1$, Δp_B^{1wm} is positive for all $r \in]0, 0.328173[$. When size effects are ruled out, the price of the low-quality good is bound to increase after trade liberalization if country B is significantly poorer than country A. As for quantities, a quick exam suffices to conclude that both differences in (38) are positive, increasing and convex in r over the relevant range.

Focus now on profits. Trade exerts opposite effects on the two firms' performances, as stated in

CLAIM 3. The opening of trade decreases the profit of the high-quality firm while it increases that of the low-quality firm.

Again, define

$$\Delta \pi_A^{1wm} = \pi_A^{1w} - \pi_A^m, \qquad \Delta \pi_B^{1wm} = \pi_B^{1w} - \pi_B^m. \tag{39}$$

It appears that $\Delta \pi_A^{1wm}$ is always negative. This result is intuitive and needs no further comments. I can only add that the decrease in firm A's profit after trade liberalization is increasing (in absolute value) and concave in r, i.e., it becomes larger at a decreasing rate as the maximum willingness to pay of the poor country gets closer to that of the rich country. This happens because, as r increases, the varieties offered by the two firms becomes more similar, enhancing thus price competition. As for $\Delta \pi_B^{1wm}$, it is always positive, increasing and convex in r.

The consequences of free trade on consumer surplus and total welfare in the two countries remain to be described.

CLAIM 4. Trade liberalization increases both consumer surplus and social welfare in the rich country. The same generally holds for the poor country as

well, with the exception that when the latter is considerably poorer than the rich country, consumer surplus may be lower than in autarky.

The relevant magnitudes, ΔCS_i^{1wm} and ΔSW_i^{1wm}, $i=A, B$, are defined according to the same criteria adopted above. Trade, if only one-way, increases welfare as compared to autarky over the whole admissible range of r in both countries. Analogously, it increases consumer surplus in the rich country. The same holds for consumer surplus in country B, except when $n=1$ and $r\in]0, 0.328173[$. In such a range, $\Delta CS_B^{1wm}<0$. Notice that, sinceΔSW_B^{1wm} is always positive, any loss suffered by consumers is always more than compensated by the increase in firm B's profit.

7. TWO-WAY TRADE VS AUTARKY

I shall now focus on the setting where both varieties are traded. The procedure and methods I shall adopt here are those established at the end of section 5. Again, I start by treating prices and quantities.

CLAIM 5. Under two-way trade, prices are always lower and quantities are always larger than under autarky.

Define:

$$\Delta p_i^{2wm}=p_i^{2w}-p_i^m\,; \qquad \Delta x_i^{2wm}=x_i^{2w}-x_i^m\,; \qquad i=A, B\,, \tag{40}$$

as the differences between two-way trade and autarky, as far as prices and quantities are concerned. It turns out that $\Delta p_i^{2wm}<0$ and $\Delta x_i^{2wm}>0$ for both countries over the whole admissible range of parameters. This leads one to think that the consequences of two-way trade on producer and consumer surplus, and thus also on social welfare, should be clearcut. Actually, this is not exactly the case, at least as far as firms' profits are concerned. These results are summarized in the following:

CLAIM 6. Two-way trade unambiguously decreases both firms' profits if the two countries have the same size. Otherwise, trade increases the profit of both firms if the rich country is sufficiently larger than the poor one.

CLAIM 7. Two-way trade increases both consumer surplus and social welfare in both countries as compared to autarky.

I take into account firstly the case where $n=1$, i.e., where both countries have the same overall dimension, so that any size effect is ruled out. In such a case, it can be verified that

$$\Delta\pi_i^{2wm}=\pi_i^{2w}-\pi_i^m<0\,; \qquad \Delta CS_i^{2wm}=CS_i^{2w}-CS_i^m>0\,;$$
$$\Delta SW_i^{2wm}=SW_i^{2w}-SW_i^m>0\,, \qquad i=A, B\,, \tag{41}$$

for all admissible values of r. The results displayed in (41) are fully in line with intuition. Things go a slightly different way if $r=1$. In such a case, although one reaches the same conclusions as above as for consumer surplus and social welfare,

two-way trade happens to increase both firms' profits in two distinct parameter ranges:

$$\Delta\pi_A^{2wm} > 0 \qquad \text{iff} \quad n > 17/8\,; \tag{42'}$$

$$\Delta\pi_B^{2wm} > 0 \qquad \text{iff} \quad n > 3.75118\,. \tag{42''}$$

Conditions (42'–42'') imply that the high-quality firm may profit from two-way trade over a wider range of parameters than the low-quality firm. When the upper bound of the marginal willingness to pay is the same in both countries, product differentiation depends solely on n. Hence, if the latter is sufficiently large, both firms may benefit from trade, with a proviso, namely, that providing the high-quality good firm A must enjoy some advantage over firm B, expressed by the wider parameter range in (42').

8. TWO VS ONE-WAY TRADE

The comparison between two-way trade and one-way trade remains to be carried out. Obviously, it shall be limited to the restricted range of parameters where both kinds of trade are possible. It has been established in section 5 that the arising of one type of trade or the other depends upon the relative performance of the high-quality firm in these two settings.

As to prices and quantities' behaviour in the two settings, I can state the following:

CLAIM 8. Both prices are lower under two-way trade than under one-way trade. The quantity sold by the high-quality firm is higher under two-way trade, while that sold by the rival is higher under one-way trade.

Again, define

$$\Delta p_i^{21w} = p_i^{2w} - p_i^{1w}\,; \qquad \Delta x_i^{21w} = x_i^{2w} - x_i^{1w}\,; \qquad i = A, B\,. \tag{43}$$

One quickly checks that $\Delta p_i^{21w} \leq 0$ for both firms, while $\Delta x_A^{21w} > 0$ and $\Delta x_B^{21w} < 0$ over the entire range of parameters. The tougher price competition arising under two-way trade intuitively justifies the first result, while the inequalities concerning quantities are straightforward.

Furthermore, as far as the low-quality firm's performance is concerned, the following holds:

CLAIM 9. The low-quality firm is always better off under one-way trade than under two-way trade.

This obtains by checking that

$$\Delta\pi_B^{21w} = \pi_B^{2w} - \pi_B^{1w} < 0 \tag{44}$$

for all admissible values of r and n. Under two-way trade, the loss due to the competition by the high-quality firm in country B, the low-quality firm's home

market, adds to the loss due to the competitive regime associated with trade, so that firm B always prefers one-way trade.

The opposite preferences obviously characterize consumers living in both countries, since $\Delta CS_i^{21w} = CS_i^{2w} - CS_i^{1w}$ is always positive. This is not surprising, provided that, for given qualities, two-way trade implies a more intense price competition between firms in both countries. As for social welfare, while $\Delta SW_A^{21w} = SW_A^{2w} - SW_A^{1w}$ is always positive, $\Delta SW_B^{21w} = SW_B^{2w} - SW_B^{1w}$ is positive for all admissible values of r if the two countries have the sime size ($n=1$), while, if $r=1/4$, two-way trade is socially preferable to one-way trade only if $n \in [1/4, 0.855862[$. Otherwise, when countries have a comparable size and country B is considerably poorer than country A, the loss suffered by the low-quality firm outweighs the gain in terms of consumer surplus. This recall an analogous result obtained by Motta (1992). I can finally state.

CLAIM 10. Two-way trade is preferable to one-way trade from the consumers' viewpoint. The same applies to social welfare in the rich country, while it holds for the poor country if weither (i) both countries have the same size, or (ii) $r=1/4$ and $n \in [1/4, 0.855862[$.

9. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

I have investigated the issue of free trade in vertically differentiated goods between two countries characterized by different dimensions and income distributions, in a framework suitable to describe North-South trade. Several results have been derived. First, according to the relative size as well as wealth of their respective domestic markets, firms may have conflicting interests as for (i) the opening of trade, be that one or two-way, as against the autarkic *status quo ante*; and (ii) one vs two-way trade. The high-quality firm operating in the rich country may benefit from two-way trade if the poor country is not excessively poor, while the opposite consideration holds for the low-quality firm. The conclusions that the model suggests in terms of consumer surplus and social welfare are rather clearcut. Under both perspectives, trade is generally preferred to autarky and two-way trade is preferred to one-way trade, since the former implies that duopolistic competition extends to both countries, while in the latter the low-quality firms competes against the high-quality firm in the richer market while keeping her monopolistic position at home, though with a lower price than in autarky.

The present analysis can be extended and amended under several respects, and in so doing the robustness of some of the above results could be put into question. First, it has been carried out under a partial equilibrium perspective. Embedding the above results into a general equilibrium framework represents an ambitious and remunerative goal. Second, I have adopted the hypothesis that firms set the quality of their respective products under autarky, so that after trade liberalization they can only adjust prices. In a richer model, this could be considered as the

short run or impact effect of trade liberalization, letting firms reoptimize with respect to quality thereafter.[10] Alternatively, if one prefers to maintain the assumption that quality must be set once and for all due to the existence of sunk costs, then it can be figured that firms choose quality under autarky, anticipating that at some date trade shall open, so that they set quality in order to maximize a discounted flow of profits over a time interval that stretches beyond the time at which liberalization occurs. This would properly embed the analysis in a dynamic perspective. Third, several of the above results may well depend upon the specific assumptions concerning the distributions of the marginal willingness to pay for quality. If both the lower and the upper bound of the distribution are higher in the rich country than in the lower country, a discontinuity may appear in the demand structure produced by the price and quality vectors, in that the supports of the distributions of the marginal willigness to pay characterizing the two countries do not overlap. Such discontinuity might ultimately lead to the isolation of the two markets, and trade would then disappear because the consumers' preferences in the two countries are too heterogeneous to induce any intraindustry trade.[11] Moreover, the introduction of non-uniform distributions could plausibly alter the results in a radical as well as hardly predictable way. To my knowledge, the only contribution available in this particular field is due to Tabuchi and Thisse (1995). They investigate the effects of symmetric triangular consumers' distribution in closed economy described by a duopolistic spatial competition framework, showing that, notwithstanding the *ex ante* symmetry of the model, it yields an asymmetric distribution of firms at the subgame perfect equilibrium. Finally, the general setting presented here opens the way to the analysis of strategic trade policy by the governments of the countries involved. Some instances of the effects exerted by tariffs and quotas are already described by Krishna (1987, 1990) and Lambertini and Rossini (1994), where it is shown that the introduction of a tariff on imports by the rich country may benefit both firms and increase both countries' welfare. The issue of export rivalry on the world market between firms operating in countries characterized by different levels of economic development is tackled by Chang and Kim (1989) and Chang and Chen (1994). These authors establish that the government of the developing country should either introduce a tariff on the input imports or tax its final good exports.

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[10] This is done by Motta, Thisse and Cabrales (1995) by introducing a convex adjustment cost which is completely absent under autarky. However, this is not fully satisfactory, since it seems to imply that firms face two different technologies under the two market regimes.

[11] For an example of such a setting, see Lambertini and Rossini (1994).

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APPENDIX

A. The behaviour of the social planner

A social planner sets both price (or quantity) and quality in order to maximize social welfare, defined as the sum of producer and consumer surpluses:

$$SW=\pi+CS=px-tq^2+s\int_{g}^{\bar{\theta}}(\theta q-p)d\theta\,, \tag{A1}$$

where $g=p/q$. Differentiating (A1) w.r.t. p and q, one gets:

$$p^{sp}=0\,, \qquad q^{sp}=\frac{\bar{\theta}^2 s}{4t}\,. \tag{A2}$$

Substituting and simplifying,

$$SW^{sp}=\frac{\bar{\theta}^4 s^2}{16t}\,, \qquad \pi^{sp}=-\frac{\bar{\theta}^4 s^2}{16t}\,, \qquad CS^{sp}=\frac{\bar{\theta}^4 s^2}{8t}\,, \tag{A3}$$

while the equilibrium quantity amounts to $x^{sp}=\bar{\theta}s$. These results imply that the social planner supplies a quality that is twice as high as that of the profit-seeking monopolist, and sets price equal to marginal cost, serving the whole population of consumers, instead of the richer half, as the profit-maximizing monopolist would do. The divergence between a profit-maximizing and a welfare-maximizing monopolist can also emerge when production involves variable instead of fixed costs. The monopolist's inefficiency under this respect has received wide attention in the existing literature. The main references are Spence (1975), Sheshinski (1976), Mussa and Rosen (1978) and Itoh (1983).

B. Social welfare under free trade

The levels of social welfare in the two countries under one and two-way trade are the following:

i) One-way trade

$$\begin{aligned} SW_A^{1w}=\bar{\theta}_A^4 nr^2(&32n^3-48n^2r+64n^4r-16n^2r^2-56n^3r^2+32n^5r^2\\ &+16r^3+32nr^3+8n^2r^3-4n^4r^3-16r^4-20nr^4-n^3r^4\\ &+4r^5)/(64t(4n-4r+4n^2r-nr^2)^2)\,; \end{aligned} \tag{A4}$$

$$\begin{aligned} SW_B^{1w}=\bar{\theta}_A^4 r^4(&32n^2+16n^3+12n^4-64nr-32n^2r+40n^3r+8n^5r+32r^2\\ &+16nr^2-60n^2r^2+8n^3r^2+40n^4r^2+8nr^3+40n^4r^2+8nr^3-8n^2r^3\\ &-24n^3r^3+3n^2r^4)/(64t(4n-4r+4n^2r-nr^2)^2)\,; \end{aligned} \tag{A5}$$

ii) Two-way trade

$$\begin{aligned} SW_A^{2w}=\bar{\theta}_A^4 n^2r^2(&48n^2+8nr+64n^3r-29r^2+16nr^2+48n^2r^2+32n^3r^2\\ &+32n^4r^2-16r^3-70nr^3-40n^2r^3-4n^3r^3+12r^4+8nr^4\\ &-n^2r^4)/(64t(4n-r)^2(nr+1)^2)\,; \end{aligned} \tag{A6}$$

$$\begin{aligned} SW_B^{2w}=\bar{\theta}_A^4 r^3(&16n^3-32n^4+16n^5+12n^2r+40n^3r+44n^4r-64n^5r\\ &-8n^2r^2-4n^3r^2+96n^4r^2+75n^5r^2-r^3-32n^3r^3-56n^4r^3\\ &-2nr^4+12n^3r^4-n^2r^5)/(64t(4n-r)^2(nr+1)^2)\,. \end{aligned} \tag{A7}$$

C. Marginal willingness to pay of the indifferent consumers under one and two-way trade

C.1: One-way trade. Provided firms do not modify their respective qualities after the opening of trade, the locations of the consumers who are indifferent (i)

between the two varieties, and (ii) between buying the low-quality good or not buying at all, are not invariant with respect to the kind of trade observed, since prices are different under one and two-way trade. The two values of the marginal willingness to pay identifying these consumers are h and k. Under one-way trade, they correspond to:

$$h^{1w}=\frac{\bar{\theta}_A(2n-2r+2n^2r-r^2-nr^2)}{4n-4r+4n^2r-nr^2}; \qquad k^{1w}=\frac{\bar{\theta}_A r(n+2)(n-r)}{4n-4r+4n^2r-nr^2}. \tag{A8}$$

The roots of $h-\bar{\theta}_B=0$ are:

$$n_1^{1w}(r)=\frac{2-4r-r^2+r^3+\sqrt{4-16r+28r^2-44r^3+41r^4-2r^5+r^6}}{4r(2r-1)};$$

$$n_2^{1w}(r)=\frac{2-4r-r^2+r^3-\sqrt{4-16r+28r^2-44r^3+41r^4-2r^5+r^6}}{4r(2r-1)}. \tag{A9}$$

It appears that $n_1(r)-n_2(r)$ exhibits an hyperbolic behaviour, being negative for $r\in[0, 1/2[$, and positive for larger values of r. Then, $h-\bar{\theta}_B>0$ if n lies to the right of the interval which is relevant to the analysis, i.e., $[n_1(r), n_2(r)]$, provided that $r\in[0, 1/2[$.

C.2: Two-way trade. The two indifferent consumers are identified by:

$$h^{2w}=\frac{\bar{\theta}_A r(n+1)(2n-r)}{(4n-r)(nr+1)}; \qquad k^{2w}=\frac{\bar{\theta}_A r(n+1)(n-r)}{(4n-r)(nr+1)}. \tag{A10}$$

Proof of Claim 1(b). It immediately appears that $h^{2w}>k^{2w}\forall r\in]0,n[$. Furthermore, it is quickly verified that

$$\bar{\theta}_B>h^{2w} \qquad \text{iff} \quad r\in]r_1, r_2[, \tag{A11}$$

where

$$r_1=\frac{4n+1-\sqrt{16n^2+9}}{2}, \qquad r_2=\frac{4n+1+\sqrt{16n^2+9}}{2} \tag{A12}$$

While the upper bound of the interval in (a11), i.e., r_2, is always greater than two for all positive values of n, the lower bound, r_1, lies in the interval $[(3-\sqrt{13})/2, (9-\sqrt{73})/2]$, for $n\in[1/2, 2]$. As a consequence, provided $n>r$ and $r\in]0, 1]$ if $n>1$, as far as the analysis carried out in the paper is concerned, the above condition must be considered as satisfied for $r\in r]r_1, 1]$.