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DYNAMIC EXPORT SUBSIDIES AND OLIGOPOLY

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Abstract: This paper presents an analysis of a dynamic export subsidy game between two countries in a Cournot duopoly with heterogeneous goods which may be substitutes or complements. With linear demand functions for the goods of firms I will show that the steady state equilibrium export subsidies in a dynamic export subsidy game are larger than the equilibrium export subsidies in a static export subsidy game whether the goods are substitutes or complements. And I will show that the welfare of the exporting countries in a dynamic game is lower than their welfare in a static game whether the goods are substitutes or complements.

1. INTRODUCTION

In this paper I consider a steady state equilibrium of a dynamic export subsidy game between two countries in a Cournot duopoly in which firms produce heterogeneous goods. According to studies by Brander and Spencer (1985), de Meza (1986), Eaton and Grossman (1986) and Cooper and Riezman (1989), I consider an international Cournot duopoly with two firms in two countries, one firm in each country. Firms produce heterogeneous goods, which may be substitutes or complements, and export them to the third country. According to Maskin and Tirole (1987) I consider a dynamic game of export subsidies between two countries with alternating moves and an infinite horizon.[1] Firms myopically choose their outputs in each period. On the other hand, the governments of countries provide export subsidies to their firms in each period taking into account the reactions of other countries in the following period. The behavior of firms may be myopic because they must earn short-run profits for their shareholders.

If we assume that firms also play a dynamic game, we must consider a game with four players. In this paper I focus attention to the effects of dynamic behavior of governments. In another paper, Tanaka (1994), I have analyzed equilibrium export subsidies in a dynamic Cournot duopoly when firms play a dynamic game,

[1] Although Maskin and Tirole (1987) considered a dynamic game between two firms not countries, the structure of the games is parallel.

that is, they choose their outputs in each period taking into account the reactions of other firms in the following period, and governments choose their export subsidies once and for all, that is, export subsidies are precommitments. Assuming a linear demand function, I have shown that in a dynamic duopoly the equilibrium export subsidies are smaller, and the outputs of firms are lager than in a static duopoly. In that paper I have considered the implications of the dynamic behavior of firms. On the other hand in the present paper I will consider the implications of the dynamic behavior of governments.

I neglect home consumption of the goods in both producing countries according to the literature on export subsidies under imperfect competition that I referred to above.

With linear demand functions for the goods of firms I will show that the steady state equilibrium export subsidies in a dynamic export subsidy game are larger than the equilibrium export subsidies in a static export subsidy game whether the goods are substitutes or complements. Since firms are myopic, larger export subsidies lead to larger outputs in each period. Therefore, the dynamic behavior of governments increases the outputs of the goods.

In the next section I present a model of dynamic export subsidy game, and consider the steady state equilibrium.

In section 3 I will show the main results of this paper, and argue that when the goods are substitutes (or complements), because the reaction functions of the governments are downward sloping (or upward sloping), the increase in export subsidy by, for example, Country 1 will induce Country 2 to reduce (or increase) its export subsidy in the following period. This reduction (or increase) in export subsidy in Country 2 will increase Country 1's welfare when the goods are substitutes (or complements), and therefore each country has an incentive to choose a higher export subsidy in a dynamic game than in a static game whether the goods are substitutes or complements.

In section 4 I will show that the welfare of the exporting countries in a dynamic game is lower than their welfare in a static game whether the goods are substitutes or complements, and conclude this paper.

2. THE MODEL AND EQUILIBRIUM

Consider an international duopoly with Firm 1 and Firm 2, respectively, in Country 1 and Country 2. Firms produce heterogeneous goods, which may be substitutes or complements, and export them to the third country. The governments of the countries provide export subsidies to their firms. Firms myopically choose their outputs in each period. The governments of the countries choose the export subsidies to their firms in each period taking into account the reactions of the other countries in the following period.

The instantaneous inverse demand functions for the goods of Firm 1 and Firm 2 are represented as follows,

$$p_1 = D - (q_1 + kq_2)$$

and

$$p_2 = D - (q_2 + kq_1)$$

where $D > 0$ and $-1 < k < 1$, $k \neq 0$. p_1 and p_2 are the prices, and q_1 and q_2 are the outputs of the goods of Firm 1 and Firm 2. When $0 < k < 1$ (or $-1 < k < 0$), the goods are substitutes (or complements). In the case where the demand functions are linear, the goods of the firms are *strategic substitutes* (*or strategic complements*) in terms of Bulow, Geanakoplos and Klemperer (1985) if and only if they are substitutes (or complements).

The marginal cost for the firms is c. Each firm chooses its output in each period to maximize its profit given the output of the rival firm and the export subsidies by the governments. Denote the subsidy to Firm i by the government of Country i as s_i, $i = 1, 2$.

The instantaneous profit of Firm i is

$$\pi^i = [d + s_i - (q_i + kq_j)]\, q_i\,, \qquad i = 1, 2,\ j \neq i \tag{1}$$

where

$$d = D - c$$

I assume $d > 0$.

From (1) we obtain the equilibrium outputs of the firms as follows,

$$q_i = \frac{1}{4 - k^2}[(2 - k)d + 2s_i - ks_j]\,, \qquad i = 1, 2\,,\ j \neq i \tag{2}$$

According to Maskin and Tirole (1987) I consider a dynamic export subsidy game between two countries with alternating moves and an infinite horizon. Each government chooses the export subsidy to maximize the present discounted value of welfare, which equals the present discounted value of the profit of its firm net export subsidy. From (2) the instantaneous welfare of Country i is derived as follows

$$\phi^i(s_i, s_j) = \frac{1}{(4 - k^2)^2}[(2 - k)d - (2 - k^2)s_i - ks_j][(2 - k)d + 2s_i - ks_j]\,,$$
$$i = 1, 2\,,\ j \neq i \tag{3}$$

In a static export subsidy game each country chooses its export subsidy given the export subsidy of the other country so that $\partial\phi^i/\partial s_i = 0$ for $i = 1, 2$. The equilibrium export subsidies in a static game are obtained as follows

$$\bar{s}_1 = \bar{s}_2 = \frac{1}{4 + 2k - k^2}k^2 d \tag{4}$$

These are positive whether $k>0$ or $k<0$.

The structure of a dynamic export subsidy game is as follows. Time periods are indexed by $t(=0, 1, 2, \cdots)$. The time between consecutive periods is T. The discount rate for the countries is r, and the discount factor is $\delta(=\exp(-rT))$. The intertemporal welfare of Country i is

$$\Phi^i=\sum_{t=0}^{\infty} \delta^t \phi^i(s_{i,t}, s_{j,t}), \qquad i=1, 2, j \neq i$$

$s_{i,t}$ denotes the export subsidy in Country i in period t.

In an odd numbered period Country 1 chooses its export subsidy, and in an even numbered period Country 2 chooses its export subsidy.[2] The export subsidy of each country is fixed over two periods. We may argue that the economic policies by the governments, especially the trade policies, are determined through the complex political processes, so they can not be changed shortly.

The reaction functions of Country 1 and 2 are represented by

$$s_{1,2m+1}=R_1(s_{2,2m})$$

and

$$s_{2,2m+2}=R_2(s_{1,2m+1})$$

where $m=0, 1, 2, \cdots$.

From (3) we have

$$\frac{\partial^2 \phi^i}{\partial s_i s_j}=-\frac{k^3}{(4-k^2)^2}, \qquad i=1, 2, j \neq i$$

This is negative (or positive) when $0<k<1$ (or $-1<k<0$).

From the dynamic programming there exist valuation functions (V_1, W_1) and (V_2, W_2) such that for any pair of export subsidies $\{s_{1,2m+1}, s_{2,2m}\}$

$$V_1(s_{2,2m})=\max_s [\phi^1(s, s_{2,2m})+\delta W_1(s)] \tag{5}$$

$$R_1(s_{2,2m})=\arg\max_s [\phi^1(s, s_{2,2m})+\delta W_1(s)] \tag{6}$$

$$W_1(s_{1,2m+1})=\phi^1(s_{1,2m+1}, R_2(s_{1,2m+1}))+\delta V_1(R_2(s_{1,2m+1})) \tag{7}$$

and similarly for Country 2.

The first order conditions for welfare maximization for Country 1 and 2 are

$$\frac{\partial \phi^1}{\partial s_{1,2m+1}}+\delta \frac{dW_1}{ds_{1,2m+1}}=0 \tag{8}$$

[2] For details about a dynamic game with alternating moves, see Maskin and Tirole (1987), (1988), Tirole (1988) and Fudenberg and Tirole (1991). In Tirole (1988), pp. 341–343, the capacity choice by firms in a dynamic duopoly has been analyzed.

and

$$\frac{\partial \phi^2}{\partial s_{2,2m+2}} + \delta \frac{dW_2}{ds_{2,2m+2}} = 0 \tag{9}$$

From (5), (6), (7) and the counterparts for Country 2 we obtain

$$\begin{aligned} W_1(s_{1,2m+1}) = {} & \phi^1(s_{1,2m+1}, R_2(s_{1,2m+1})) \\ & + \delta\phi^1(R_1(R_2(s_{1,2m+1})), R_2(s_{1,2m+1})) \\ & + \delta^2 W_1(R_1(R_2(s_{1,2m+1}))) \end{aligned} \tag{10}$$

and

$$\begin{aligned} W_2(s_{2,2m+2}) = {} & \phi^2(s_{2,2m+2}, R_1(s_{2,2m+2})) \\ & + \delta\phi^2(R_2(R_1(s_{2,2m+2})), R_1(s_{2,2m+2})) \\ & + \delta^2 W_2(R_2(R_1(s_{2,2m+2}))) \end{aligned} \tag{11}$$

In an equilibrium the move of each country according to its reaction function maximizes its welfare. Since the welfare functions for the countries are quadratic, and so their partial derivatives are linear, I consider the linear reaction functions:

$$R_1(s_{2,2m}) = a_1 - b_1 s_{2,2m} \tag{12}$$

and

$$R_2(s_{1,2m+1}) = a_2 - b_2 s_{1,2m+1} \tag{13}$$

Differentiating (10) and (11) with respect to $s_{1,2m+1}$ and $s_{2,2m+2}$, and arranging terms, we can show that $b_1 = b_2$. Replacing these by b, we obtain the following equations.[3]

$$\delta^2 k^3 b^4 + 2\delta^2 k^2 b^3 + 2\delta k^2 b^3 + 2\delta k^3 b^2 - 4(1+\delta)(2-k^2)b + k^3 = 0 \tag{14}$$

$$\begin{aligned} & \delta^2 k^3 b^3 a_2 + 2\delta^2 k^2 b^2 a_2 - \delta^2 k^3 b^2 a_1 + 2\delta k^2 b^2 a_2 + \delta k^3 b a_2 + k^3 a_1 \\ & \quad = (1+\delta)k(2-k)(k+2\delta b)bd \end{aligned} \tag{15}$$

and

$$\begin{aligned} & \delta^2 k^3 b^3 a_1 + 2\delta^2 k^2 b^2 a_1 - \delta^2 k^3 b^2 a_2 + 2\delta k^2 b^2 a_1 + \delta k^3 b a_1 + k^3 a_2 \\ & \quad = (1+\delta)k(2-k)(k+2\delta b)bd \end{aligned} \tag{16}$$

From (14) we find

LEMMA 1. *When* $0<k<1$ (*or* $-1<k<0$), *the unique solution of* b *in Eq.* (14), *which yields stable reaction functions, is in the interval* (0, 1/4) (*or* $(-1/4, 0)$).

The proof of this lemma is straightforward. Since $-b$ is the slope of the reaction functions, this lemma implies that when the goods of the firms are (strategic)

[3] These equations are obtained by differentiating (10) and (11) with respect to $s_{1,2m+1}$ and $s_{2,2m+2}$, using (8) and (9), and using (12) and (13).

substitutes (or complements), the reaction functions of the governments are downward (or upward) sloping.

From (15) and (16) we find

$$(a_2-a_1)(\delta^2k^3b^3+\delta^2k^3b^2+2\delta^2k^2b^2+2\delta k^2b^2+\delta k^3b-k^3)=0$$

From Lemma 1 the terms in braces are not zero, so we get $a_1=a_2$. Replacing these by a, from (15) or (16) we obtain

$$a=\frac{1}{A}(1+\delta)k(2-k)(k+2\delta b)bd=\frac{(1+\delta)k(k+2\delta b)}{\delta k^2b+4+2k-k^2}d$$

where

$$A=\delta^2k^3b^3+2\delta^2k^2b^2-\delta^2k^3b^2+2\delta k^2b^2+\delta k^3b+k^3$$

The steady state equilibrium export subsidies (s_1^e, s_2^e) are obtained from the following equations,

$$s_1^e=R_1(s_2^e)=a-bs_2^e=a-b(a-bs_1^e)$$

and

$$s_2^e=R_2(s_1^e)=a-bs_1^e=a-b(a-bs_2^e)$$

Then we have

$$s_1^e=s_2^e=s^e=\frac{a}{1+b}=\frac{k(k+2\delta b)}{\delta k^2b+4+2k-k^2}d \tag{17}$$

From (4) and (17) we find that, as $\delta\to 0$ (or $r\to\infty$), $s_1^e\to\bar{s}_1$ and $s_2^e\to\bar{s}_2$.

Differentiating (14) with respect to δ, and differentiating (17) with respect to δb, we can show

LEMMA 2. *When $0<k<1$ (or $-1<k<0$), δb is increasing (or decreasing) in δ, and s^e is increasing (or decreasing) in δb. Therefore s^e is increasing in δ whether $0<k<1$ or $-1<k<0$.*

The static case is a special case of the dynamic game when $\delta\to 0$. Thus, from this lemma we obtain the following proposition.

PROPOSITION 1. *Whether the goods are substitutes or complements, the steady state equilibrium export subsidies in a dynamic export subsidy game are larger than the equilibrium export subsidies in a static export subsidy game.*

From (2) we know that larger equilibrium export subsidies lead to larger outputs. Therefore whether the goods are substitutes or complements, the dynamic behavior of the governments increases the outputs of the goods.

Differentiating (3) with respect to s_j and using (17) yields

$$\frac{\partial \phi^i}{\partial s_j} = -\frac{k(k^2-4)^2}{(4-k^2)^2(\delta k^2 b + 4 + 2k - k^2)} \tag{18}$$

This is negative (or positive) when $0 < k < 1$ (or $-1 < k < 0$). Thus when the goods are substitutes (or complements), an increase in the export subsidy of one country reduces (or increases) the welfare of the other country.

The outcome in our dynamic export subsidy game is more *competitive* than the outcome in a static game whether the goods of the firms are substitutes or complements, in the sense that the equilibrium export subsidies and firms' outputs are larger than those in a static export subsidy game. A country about to move, say Country 1, takes both the short-run welfare and the reaction it will induce in Country 2 into account. Suppose that Country 1 and 2 are currently at the static equilibrium levels. Then a slight increase in export subsidy above the static equilibrium level by Country 1 will have no effect on its short-run welfare. Consider the case where the goods are substitutes. Because the reaction functions of the governments are downward sloping, the increase in export subsidy by Country 1 will induce Country 2 to reduce its export subsidy in the following period. From (18) this reduction in export subsidy in Country 2 will increase Country 1's welfare. Thus each country has an incentive to choose a higher export subsidy in a dynamic game than in a static game.

Next consider the case where the goods are complements. Because the reaction functions of the governments are upward sloping, the increase in export subsidy by Country 1 will induce Country 2 to increase its export subsidy in the following period. From (18) this increase in export subsidy in Country 2 will also increase Country 1's welfare. Thus each country has an incentive to choose a higher export subsidy in a dynamic game than in a static game also in a case of complements.

3. WELFARE ANALYSIS AND CONCLUSION

In this section I consider the welfare implications of the dynamic behavior of the governments. Since the dynamic equilibrium is symmetric, substituting $s_1 = s_2 = s^e$ into (3) the steady state welfare of the exporting countries in each period is represented as follows,

$$\begin{aligned}\bar{\phi} &= \frac{1}{(4-k^2)^2}[(2-k)d - (2-k^2)s^e - ks^e][(2-k)d + 2s^e - ks^e] \\ &= \frac{1}{(2+k)^2}(d^2 - s^{e2}) \end{aligned} \tag{19}$$

This is unambiguously decreasing in s^e regardless of the sign of k. Therefore, whether the goods are substitutes or complements, the larger the export subsidies are, the lower the welfare of the exporting countries is. Since the equilibrium export subsidies in a dynamic export subsidy game are larger than the equilibrium export subsidies in a static export subsidy game, we obtain the following proposition.

PROPOSITION 2. *The welfare of the exporting countries in a dynamic export subsidy game is lower than the welfare in a static export subsidy game whether the goods are substitutes or complements.*

On the other hand, since the outputs are increased by the dynamic behavior of the governments, the welfare of the importing country in a dynamic export subsidy game is higher than the welfare in a static export subsidy game.

In this paper I have analyzed a dynamic export subsidy game between two countries in a Cournot duopoly with heterogeneous goods. With linear demand functions I have shown that, whether the goods are substitutes or complements, the steady state equilibrium export subsidies in a dynamic export subsidy game are larger than the equilibrium export subsidies in a static export subsidy game, and the welfare of the exporting countries in a dynamic game is lower than their welfare in a static game.

REFERENCES

Brander, A. and B. Spencer (1985), "Export subsidies and international market share rivalry", *Journal of International Economics* 18, 83–100.

Bulow, J., J. Geanakoplos and P. Klemperer (1985), "Multimarket Oligopoly: Strategic substitutes and complements", *Journal of Political Economy* 93, 488–511.

Cooper, R. and R. Riezman (1989), "Uncertainty and the choice of trade policy in oligopolistic industries", *Review of Economic Studies* 56, 129–140.

Eaton, J. and G. M. Grossman (1986), "Optimal trade and industrial policy under oligopoly", *Quarterly Journal of Economics* 101, 383–406.

Fudenberg, D. and J. Tirole (1991), Game theory, Cambridge, The MIT Press.

Maskin, E. and J. Tirole (1987), "A theory of dynamic oligopoly, III: Cournot competition", *European Economic Review* 31, 947–968.

Maskin, E. and J. Tirole (1988), "A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed cost", *Econometrica* 56, 549–570.

de Meza, D. (1986), "Export subsidies and high productivity: cause or effect?", *Canadian Journal of Economics* 19, 347–350.

Tanaka, Y. (1994), "Export subsidies under dynamic duopoly", *European Economic Review* 38, 1139–1151.

Tirole, J. (1988), The theory of industrial organization, Cambridge, The MIT Press.