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PRICE UNCERTAINTY AND CREDIT-PRODUCT INTERLINKAGE—A NOTE

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Abstract: In the existing theoretical literature on credit-product interlinkage, the trader (interlocker) is not allowed to offer a Fixed-Rental Contract (FRC) where the trader leases-in land from the farmer in exchange of a fixed rent for self-cultivation. But if the trader is allowed to offer an FRC, then the existence and optimality of an interlinked credit-product contract cannot be established. This note purports to provide an explanation for the existence and optimality of an interlinked credit-product contract, even when the trader is allowed to offer an FRC, introducing price uncertainty in the product market.

1. INTRODUCTION

One of the important empirical findings of the village survey report of Rudra[1] (1982) is the existence of output-cum-credit contracts in many villages of India. A private trader often gives production loan to the farmers at subsidized interest rate and inturn purchases at least a part of the farmers' output at a precontracted price which is below the open market price of the product. The trader earns profit by selling the product in the open market. An Interlinked Credit-Product Contract (ICPC) is more efficient than a Non-Interlinked Credit-Product Contract (NICPC) where the farmer takes the loan from a professional moneylender and sells the product in the free market directly. This is because the trader's opportunity cost of funds is less than the interest rate charged by the moneylender. The existing theoretical literature[2] adopts a principal-agent framework and proves the optimality of the ICPC in terms of credit market imperfections. Here the trader is viewed as the principal and the farmer as the agent. The farmer is assumed to possess no bargaining power and hence the trader is able to extract the entire

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[1] Also see Gupta and Dutta (1993), Duvvury (1986) and Nagaraj (1985).

[2] See Gangopadhyay and Sengupta (1987), Bell and Srinivasan (1989), Gangopadhyay (1994) and Chaudhuri and Gupta (1995a).

surplus from the contract pushing the farmer down to his reservation income (utility) level. Besides, the assumptions of the presence of credit market imperfections and the absence of any bargaining power of the farmer, there is also an implicit assumption in the existing literature that the trader unlike the farmer does not know the technique of cultivation. The unique optimality of the ICPC cannot be proved without making this assumption. This is because, if this assumption is relaxed and the trader is allowed to cultivate, then the trader may alternatively lease-in the given land endowment of the farmer for self-cultivation and will be able to extract the same amount of surplus after paying the farmer a fixed cash rent just sufficient to keep the latter at the reservation income (utility) level. This we may call a Fixed Rental Contract (FRC). So if the trader is allowed to offer an FRC, then the ICPC and the FRC would be identical in terms of the surplus, the trader can extract. Hence there is no reason for the ICPC to exist.

The assumption that the trader does not know the technique of cultivation cannot be justified because the traders, in many cases, belong to the families of large farmers. On the other hand, there are enough empirical evidences[3] to support the separate existence of the ICPC. The existing literature fails to explain these two evidences simultaneously. So there should be a theory which can explain the unique optimality of the ICPC even if the trader is allowed to offer an FRC. The present paper purports to provide such a theory in the presence of price uncertainty in the product market.

There are two types of marketing contract involved in an ICPC. First, the trader purchases the product from the farmer at a predetermined price stipulated in the contract and profit is determined by the price in the free market. This we may call a Fixed-Price ICPC. Alternatively, the farmer sells the product in the free market through the trader at an argeed commission per unit of the product sold. This is called a Fixed-Commission ICPC. There are examples[4] of both types of contract in the agricultural sector in India. However, these two types of ICPC are two special cases of the more general risk-sharing ICPC where the trader and the farmer share the burden of risk. In the absence of price uncertainty, these two contracts become identical.

However, in the presence of price uncertainty, these two contracts deserved separated attention from a theoretical point of view. This is because, while the Fixed-Price ICPC places the entire burden of uncertainty to the trader, the Fixed-Commission ICPC throws this burden on the farmer. On the other hand, in the Fixed-Rental Contract (FRC), the burden of uncertainty is solely borne by the trader-cum-cultivator. So the existence and optimality of these contracts depend upon the attitudes towards risk of the two parties to the contract, namely, the trader and the farmer.

The present theoretical analysis derives its importance from the interesting results that are obtained. The farmer is always on the reservation utility level and

[3] See for example, Rudra (1982), Nagaraj (1985), Duvvury (1986), Gupta and Dutta (1993), etc.

[4] See Rudra (1982), Nagaraj (1985), Gupta and Dutta (1993), etc.

the trader derives higher utility from the more productive (from the view point of agricultural productivity) contract. However, which contract will be more productive (and hence will be chosen) depends on the relative risk-aversion of the trader and the farmer. The Fixed-Rental Contract is always equivalent to a Fixed-Price ICPC. However, the Fixed-Commission ICPC is more productive and utility yielding to the trader when the trader is more risk-averse relative to the farmer. Lastly, risk-sharing is optimal when both the farmer and the trader are risk-averse. In this situation, the risk-sharing ICPC is preferable even to an FRC from the trader's point of view.

2. THE MODEL

Let us consider a stylised agrarian economy with N identical farmers, one trader and one moneylender. Each farmer cultivates a plot of land of a given size. The representative farmer in our model, has to borrow funds from the trader or the moneylender.

The trader is assumed to know the technique of cultivation. So he can behave either like a capitalist farmer and offer a Fixed-Rental Contract (FRC) or like a trader-cum-lender (interlocker) and offer an Interlinked Credit-Product Contract (ICPC). We adopt a principal-agent framework with the trader as the principal and the farmer as the agent. The farmer being the agent does not possess any bargaining power.

The free market price, P is a random variable with the probability density function $f(P)$ for $\infty > P \geqslant 0$. Credit is considered as the only input in the production function.[5] If B is the amount of credit application of the farmer, then $Q(B)$, with $Q'(\cdot)>0$ and $Q''(\cdot)<0$, is the production function. Any loan is paid back with interest at the end of the crop-cycle.

Let us denote the farmer and the trader by the subscripts 1 and 2 respectively. We assume that the ith economic agent maximizes his expected utility which is a function of both $E(Y_i)$ and $V(Y_i)$, where $E(Y_i)$ and $V(Y_i)$ represent the expected value and the variance of income of the ith economic agent for $i=1, 2$. We here consider the following algebraic form[6] of the expected utility function:

$$U_i = E(Y_i) + \rho_i \cdot V(Y_i) \tag{1}$$

where, ρ_i is the risk-aversion coefficient of the ith economic agent. Here $\rho_i > (<) 0$ implies that the ith economic agent is risk-lover (averter). He is risk-neutral when $\rho_i = 0$.

[5] This is in the line of Gangopadhyay and Sengupta (1987). Any production function which is well-behaved, may be written as a function of the total expenditure, if input markets are perfectly competitive. Consequently this production function can be interpreted as a general production relationship with many inputs under competitive conditions.

[6] This has been considered for the sake of simplicity. However, such a simplification is well within the spirit of the simplified conceptual framework of the problem. The results of this paper will hold in the case of more general forms of the expected utility function as well.

2.1 The Reservation Utility

The reservation utility of the farmer is derived from an NICPC. In an NICPC, the farmer takes the loan from the professional moneylender at the parametric interest rate, $\bar{r}$, per period and sells his product directly in the free market at the price, P. The farmer's income, Y_1, is a random variable and is given by

$$Y_1 = P \cdot Q(B) - B \cdot (1 + \bar{r}) \tag{2}$$

The farmer maximizes the following expected utility function through a choice of B:

$$U_1 = E(P) \cdot Q(B) - B \cdot (1 + \bar{r}) + \rho_1 \cdot V(P) \cdot (Q(B))^2 \tag{3}$$

His optimal loan is given by

$$B^0 = \arg\max_{B>0} E(P) \cdot Q(B) - B \cdot (1 + \bar{r}) + \rho_1 \cdot V(P) \cdot (Q(B))^2$$

which is the solution to

$$[E(P) + 2 \cdot \rho_1 \cdot V(P) \cdot Q(B)] \cdot Q'(B) = (1 + \bar{r}) \tag{4}$$

The reservation utility of the farmer is then given by

$$U_1^0 = E(P) \cdot Q(B^0) - B^0 \cdot (1 + \bar{r}) + \rho_1 \cdot V(P) \cdot (Q(B^0))^2 \tag{3.1}$$

If the trader wants the farmer to enter into any deals with him, he must ensure him at least U_1^0 level of utility.

2.2 The Fixed-Rental Contract:

In a Fixed-Rental Contract (FRC), the trader behaves like a capitalist farmer and leases-in the given land endowment of the representative farmer (agent 1) in exchange of a fixed cash rent, R^* and $R^* \geqslant U_1^0$. Since agent 1 has no bargaining power, the trader-cultivator can always choose a value of R^* which is just equal to U_1^0. So we have

$$R^* = U_1^0 \tag{5}$$

The income of the trader-cultivator is then given by

$$Y_2 = P \cdot Q(m) - (1 + g) \cdot m - U_1^0 \tag{6}$$

where, m and g denote the amount of credit application and the opportunity interest rate of the trader-cultivator, respectively. He maximizes his expected utility through a choice of m and his expected utility is given by

$$U_2 = E(P) \cdot Q(m) - (1 + g) \cdot m - U_1^0 + \rho_2 \cdot V(P) \cdot (Q(m))^2 \tag{7}$$

The first-order condition of maximization is

$$[E(P) + 2 \cdot \rho_2 \cdot V(P) \cdot Q(m)] \cdot Q'(m) = (1 + g) \tag{8}$$

The optimum level of utility of the trader-cultivator derived from an FRC is

$$U_2^* = E(P) \cdot Q(m^*) - (1+g) \cdot m^* - U_1^0 + \rho_2 \cdot V(P) \cdot (Q(m^*))^2 \tag{9}$$

where m^* is the solution to equation (8).

2.3 The Fixed-Price ICPC

This is a credit-cum-trade contract, with trade precontracted at a fixed price. The farmer takes the production loan from the trader at the interest rate, i, per period and gets a fixed price, P_1, per unit for his product. The terms of the contract, i.e., the values of i and P_1 are set by the trader. In this type of ICPC, the entire burden of price uncertainty falls upon the trader (interlocker).

Since the farmer does not face any uncertainty, his expected utility is given by

$$U_1 = E(Y_1) = P_1 \cdot Q(B) - (1+i) \cdot B \tag{10}$$

and this is maximized with respect to B. The first-order condition for a maximum is given by

$$P_1 \cdot Q'(B) - (1+i) = 0 \tag{11}$$

which yields the optimum borrowing function

$$\bar{B} = \bar{B}(P_1, i) \tag{12}$$

It is easy to check that $(\partial B/\partial P_1) > 0$ and $(\partial B/\partial i) < 0$. The optimum level of utility of the farmer is then

$$\bar{U}_1 = P_1 \cdot Q(\bar{B}) - (1+i) \cdot \bar{B} \tag{13}$$

The farmer will accept a Fixed-Price ICPC iff

$$\bar{U}_1 \geqslant U_1^0 \tag{14}$$

The trader's problem is to select a contract (P_1, i) so as to maximize his own expected utility taking into account the farmer's borrowing function $\bar{B}(P_1, i)$. He also faces the reservation utility constraint of the farmer. Formally, the trader's problem is

$$\underset{(P_1, i)}{\text{Max}}\ U_2 = (E(P) - P_1) \cdot Q(\bar{B}) + (i-g) \cdot \bar{B} + \rho_2 \cdot V(P) \cdot (Q(\bar{B}))^2 \tag{15}$$

subject to $\bar{U}_1 \geqslant U_1^0$.

Since the farmer possesses no bargaining power, the reservation utility constraint will be binding again.[7] So we write

$$\bar{U}_1 = U_1^0 \tag{16}$$

The trader's problem then reduces to

[7] This is quite obvious. However, the mathematical proof of this result is available from the author on request.

$$\underset{(P_1,i)}{\text{Max}}\,(U_2-U_1^0)=(E(P)-P_1)\cdot Q(\bar{B})+(i-g)\cdot\bar{B}+\rho_2\cdot V(P)\cdot(Q(\bar{B}))^2-U_1^0 \quad (17)$$

In this maximization exercise the trader has two instrumental variables, namely, P_1 and i, at his disposal. The trader through the adjustments of his instrumental variables, influences credit application of the farmer such that the farmer demands exactly that amount of credit at which the objective function of the former is maximized. Hence the maximization of $(U_2-U_1^0)$ with respect to P_1 and i is equivalent[8] to the maximization of the same function with respect to $\bar{B}$. Maximizing (17) with respect to $\bar{B}$, we have the following first-order condition:

$$[(E(P)-P_1)+2\cdot\rho_2\cdot V(P)\cdot Q(\bar{B})]\cdot Q'(\bar{B})+(i-g)=0$$

and with the help of (11) this reduces to

$$[E(P)+2\cdot\rho_2\cdot V(P)\cdot Q(\bar{B})]\cdot Q'(\bar{B})=(1+g) \quad (18)$$

Equation (18) determines $\bar{B}$ as a function of the parameters of the model. Given the value of $\bar{B}$, equations (12) and (16) together then determine the equilibrium values of P_1 and i. The trader's optimum level of expected utility derived from a Fixed-Price ICPC is now obtained by using equations (13), (15) and (16) as the following:

$$\bar{U}_2=E(P)\cdot Q(\bar{B})+\rho_2\cdot V(P)\cdot(Q(\bar{B}))^2-(1+g)\cdot\bar{B}-U_1^0 \quad (19)$$

2.4. The Fixed-Commission ICPC

This is another credit-cum-trade contract, with trade precontracted at a fixed absolute discount (per unit of output). The lender (trader) lends his funds at the interest rate, v per period and receives a fixed commission, x, per unit of output sold through him and the entire burden of price uncertainty is borne by the borrower (farmer). The farmer's income and expected utility are given by the following respectively:

$$Y_1=(P-x)\cdot Q(B)-(1+v)\cdot B \quad (20)$$

and

$$U_1=(E(P)-x)\cdot Q(B)-(1+v)\cdot B+\rho_1\cdot V(P)\cdot(Q(B))^2 \quad (21)$$

$U_1(\cdot)$ is maximized with respect to B and the first-order condition is

$$[E(P)-x+2\cdot\rho_1\cdot V(P)\cdot Q(B)]\cdot Q'(B)=(1+v) \quad (22)$$

which yields the optimum borrowing function

$$B^{**}=B^{**}(x,v) \quad (23)$$

One can check that B^{**} is a decreasing function of both x and v. The farmer's

[8] Interested readers may check it or can obtain the proof from the author on request.

optimum level of expected utility is then given by

$$U_1^{**} = (E(P) - x) \cdot Q(B^{**}) - (1 + v) \cdot B^{**} + \rho_1 \cdot V(P) \cdot (Q(B^{**}))^2 \tag{24}$$

In a Fixed-Commission ICPC, the trader does not bear any uncertainty. So the trader's problem is

$$\underset{(x,v)}{\text{Max}}\, U_2 = Y_2 = x \cdot Q(B^{**}) + (v - g) \cdot B^{**} \tag{25}$$

subject to $U_1^{**} \geqslant U_1^0$. The reservation utility constraint will be binding again. So in equilibrium, we have

$$U_1^{**} = U_1^0 \tag{26}$$

Since the maximization of (25) with respect to x and v is equivalent to the maximization of the same function with respect to B^{**}, the trader's problem now reduces to

$$\underset{(B^{**})}{\text{Max}}\, (U_2 - U_1^0) = x \cdot Q(B^{**}) + (v - g) \cdot B^{**} - U_1^0 \tag{27}$$

The first-order condition of the maximization problem is

$$x \cdot Q'(B^{**}) + (v - g) = 0$$

and with the help of equation (22) this becomes

$$[E(P) + 2 \cdot \rho_1 V(P) \cdot Q(B^{**})] \cdot Q'(B^{**}) = (1 + g) \tag{28}$$

The equilibrium values of x and v are found by solving equations (23) and (26) given the value of B^{**} obtained from equation (28). The trader's optimum level of utility is then obtained by using equations (24), (25) and (26) as follows:

$$U_2^{**} = E(P) \cdot Q(B^{**}) + \rho_1 \cdot V(P) \cdot (Q(B^{**}))^2 - (1 + g) \cdot B^{**} - U_1^0 \tag{29}$$

3. THE RESULTS

From equations (5), (16) and (26), we can establish the following proposition:

PROPOSITION 1. *The trader always keeps the farmer at the latter's reservation utility level.*

Let us now suppose that there is no price uncertainty. It implies that $V(P) = 0$. Putting $V(P) = 0$ into equations (4), (8), (18) and (28), we find that $B^0 = m^* = \bar{B} = B^{**}$ if $\bar{r} = g$. However, if $\bar{r} > g$, we have $B^0 < m^* = \bar{B} = B^{**}$. So the equilibrium credit application is the same in all the three contracts and this equal to (greater than) the farmer's credit intensity of cultivation in an NICPC if the trader's opportunity interest rate is equal to (less than) the interest rate charged by the moneylender. Since credit is the only input of production, we can write $Q(B^0) < Q(m^*) = Q(\bar{B}) = Q(B^{**})$ if $\bar{r} > g$. This proves the standard result in the

existing literature[9] that in the absence of price uncertainty, the credit market imperfection is the only explanation of interlinkage between the credit and the product markets. From equations (9), (19) and (29), we note that $U_2^* = \bar{U}_2 = U_2^{**}$ when $V(P) = 0$ and $m^* = \bar{B} = B^{**}$. Hence the trader derives the same level of utility from each of the three contracts. This establishes the following proposition:

PROPOSITION 2. *A Fixed-Rental Contract (FRC) is equivalent to either a Fixed-Price ICPC or a Fixed-Commission ICPC in terms of utility the trader can derive when there is no price uncertainty.*

So in the absence of price uncertainty, if the trader is allowed to offer an FRC, there is no reason for an ICPC to exist. The existing literature fails to analyse this point because of its implicit assumption that the trader does not know the technique of cultivation.

We now consider the case of market price uncertainty. So we have $V(P) > 0$. From equations (8) and (18), we find that $m^* = \bar{B}$. From equations (9) and (19), it now follows that $U_2^* = \bar{U}_2$ when $m^* = \bar{B}$. This leads to the following proposition:

PROPOSITION 3. *Even when there is price uncertainty, a FRC and a Fixed-Price ICPC are equivalent in terms of utility the trader can derive.*

From equations (8) and (28), one can show[10] that

$$Q(m^*) > (=)(<) Q(B^{**}) \quad \text{if} \quad \rho_2 > (=)(<) \rho_1 \tag{30}$$

We now write the following lemma[11] which characterizes the relationship between the relative agricultural productivity of a contract and its optimality:

LEMMA 1. *The trader derives higher level of expected utility from the more productive (from the view point of agricultural productivity) contract.*

With the help of (30) and lemma 1, one can now establish the following proposition:

PROPOSITION 4. *In the presence of price uncertainty, the trader derives higher (lower) level of expected utility from an FRC than what he derives from a Fixed-Commission ICPC, if the trader is less (more) risk-averse relative to the farmer.*

Hence when the trader is more risk-averse relative to the farmer, a Fixed-Commission ICPC is the best among three contracts to the trader even if he is allowed to offer a FRC.

[9] See Gangopadhyay and Sengupta (1987), Gangopadhyay (1994) and Chaudhuri and Gupta (1995b). However, Chaudhuri and Gupta (1995b) shows that even in the absence of credit market imperfections, price uncertainty in the free market may lead to credit-product interlinkage.

[10] The proof of this is available from the author on request.

[11] The proof of lemma 1 has been presented in the Appendix.

4. RISK-SHARING ICPC

So far, we have considered an extreme type of ICPCs where either the trader or the farmer completely bears risk. However, we can think of intermediate risk burden ICPC which may be called a Risk-Sharing ICPC. Here the farmer receives a proportion of the market price denoted by c and a fixed price per unit of sales denoted by s. Here $0 \leqslant c \leqslant 1$ and the risk is shared by the farmer and the trader when $0 < c < 1$. When $c = 1$ and $s < 0$, then we have the Fixed-Commission ICPC. But in the case of a Fixed-Price ICPC, $c = 0$ and $s > 0$.

The farmer's problem is

$$\underset{B}{\text{Max}}\,[c \cdot E(P) + s] \cdot Q(B) - B(1+t) + \rho_1 \cdot c^2 \cdot V(P) \cdot (Q(B))^2 .$$

Here t is the interest rate on the loan. The first-order condition determines the farmer's optimum level of credit, $\widetilde{B}$,

$$[(c \cdot E(P) + s) + 2 \cdot \rho_1 \cdot c^2 \cdot V(P) \cdot Q(\widetilde{B})] \cdot Q'(\widetilde{B}) = (1+t) \tag{31}$$

The farmer's utility under this contract is given by

$$\widetilde{U}_1 = [c \cdot E(P) + s] \cdot Q(\widetilde{B}) - \widetilde{B} \cdot (1+t) + \rho_1 \cdot c^2 \cdot V(P) \cdot (Q(\widetilde{B}))^2 \tag{32}$$

Now the lender's problem is

$$\underset{(c,s,t)}{\text{Max}}\, U_2 = [(1-c)E(P) - s] \cdot Q(\widetilde{B}) + \widetilde{B} \cdot (t-g) + \rho_2 \cdot (1-c)^2 \cdot V(P) \cdot (Q(\widetilde{B}))^2$$

subject to $\widetilde{U}_1 \geqslant U_1^0$.

The Lagrangian expression is

$$T = U_2 + \beta \cdot (\widetilde{U}_1 - U_1^0) \tag{33}$$

where β is the Lagrangian multiplier and $\beta \geqslant 0$.

Since the agent (the farmer) does not possess any bargaining power, the principal (the trader) will once again be able to keep the former at his reservation utility level. The trader has three instrumental variables, namely c, s and t. However, the trader can keep the farmer at the reservation utility level using only s and t□.[12] When T is maximized with respect to s, t and β, from the first-order conditions, one can prove[13] that in equilibrium

$$\widetilde{U}_1 = U_1^0 \tag{34}$$

$$\beta = 1 \tag{35}$$

Maximization of $T(\cdot)$ with respect to s and t is equivalent to maximization of the same function with respect to $\widetilde{B}$. So maximizing $T(\cdot)$ with respect to $\widetilde{B}$ and

[12] Interested readers may check it or can obtain it from the author on request.

[13] The proof is available from the author on request.

using equation (31), we have the following first-order condition:

$$[E(P)+2\cdot V(P)\cdot Q(\tilde{B})\cdot(c^2\cdot\rho_1+(1-c)^2\cdot\rho_2)]\cdot Q'(\tilde{B})=(1+g) \tag{36}$$

Equation (36) determines $\tilde{B}$ as a function of c and other parameters of the system. Given the value of $\tilde{B}$, equations (31) and (34) together then determine the equilibrium values of s and t as functions of c.

Now risk-sharing is optimal to the trader if there exists an interior value of c satisfying the following conditions:

$$(\partial T/\partial c)=0$$

$$(\partial^2 T/\partial c^2)<0\,.$$

Applying the envelope theorem from equation (33) one can write

$$\begin{aligned}(\partial T/\partial c)&=-E(P)\cdot Q(\tilde{B})-2\cdot\rho_2(1-c)\cdot V(P)\cdot(Q(\tilde{B}))^2\\&\quad+\beta\cdot[E(P)\cdot Q(\tilde{B})+2\rho_1\cdot c\cdot V(P)\cdot(Q(\tilde{B}))^2]\\&=2\cdot V(P)\cdot(Q(\tilde{B}))^2\cdot(c\rho_1-(1-c)\rho_2)\end{aligned} \tag{37}$$

(since $\beta=1$). If there exists an interior solution for c, we must have

$$(\partial T/\partial c)=2\cdot V(P)(Q(\tilde{B}))^2\cdot(c\rho_1-(1-c)\rho_2)=0$$

or

$$c=(\rho_2/(\rho_1+\rho_2)) \tag{38}$$

and

$$(\partial^2 T/\partial c^2)=2\cdot V(P)\cdot(Q(\tilde{B}))^2\cdot(\rho_1+\rho_2)<0$$

or

$$(\rho_1+\rho_2)<0\,.$$

From (38) it follows that $0<c<1$ in situations where ρ_1 and ρ_2 are of the same sign. But $(\partial^2 T/\partial c^2)$ cannot be negative if ρ_1 and ρ_2 are both positive. So only when ρ_1 and ρ_2 are both negative the interior solution for c is optimal. This establishes the following proposition.

PROPOSITION 5. *It is optimal to share the risk when both the farmer and the trader are risk-averse.*

So when both the economic agents are risk-averse, the risksharing ICPC is preferable to an FRC from the trader's point of view.

From (37), it follows that $(\partial T/\partial c)>(<)0$ if $\rho_1\geqslant(\leqslant)0$, $\rho_2\leqslant(\geqslant)0$ and if either ρ_1 or ρ_2 is non-zero. This means that the optimal value of c must be equal to unity (zero) in that case. So when the trader is a risk-lover (or risk-neutral) but the farmer is not so, the optimum value of c is zero and we have a Fixed-Price ICPC (or an FRC; see proposition 3). However, a Fixed-Commission ICPC is the

optimal policy of the trader when he is risk-averse but the farmer is not so.

However, when both the economic agents are risk-lovers, $(\partial^2 T/\partial c^2)>0$. So risk-sharing will not be economically viable in this case. Thus the Fixed-Price ICPC/the FRC (Fixed-Commission ICPC) is the optimal policy of the trader when $\rho_2>(<)\rho_1$.[14]

Now putting $c=(\rho_2/(\rho_1+\rho_2))$ into equation (36), we can easily prove[15] that

$$Q(\tilde{B})>Q(B^{**}), \qquad Q(m^*)=Q(\bar{B}).$$

So when both the trader and the farmer are risk-averters the risk-sharing ICPC is the most productive contract from the point of view of agricultural productivity.

5. CONCLUDING REMARKS

In this note, we have presented a theory of the existence and optimality of the credit-product interlinkage introducing price uncertainty. A limitation of the existing literature on interlinkage is its implicit assumption that the interlocker (trader) does not know the technique of cultivation and so he is not allowed to offer a Fixed-Rental Contract. If this assumption is relaxed, then the unique optimality of an Interlinked Credit-Product Contract cannot be established. This paper is devoted to explain the optimality of the ICPC even when a Fixed-Rental Contract is allowed, introducing price uncertainty. The introduction of price uncertainty makes the Fixed-Price ICPC analytically different from the Fixed-Commission ICPC because these two different contracts shift the burden of risk-bearing on the two different economic agents: on the trader in the case of a Fixed-Price ICPC and on the farmer in the case of a Fixed-Commission ICPC. On the other hand, in a Fixed-Rental Contract (FRC), the entire burden of price uncertainty is borne by the trader. The Fixed-Price ICPC is found to be equivalent to an FRC in terms of utility the trader can derive. However, the trader derives higher level of utility from a Fixed-Commission ICPC than what he derives from an FRC, if he is more risk-averse relative to the farmer.

Lastly, we have considered the case of a risk-sharing ICPC. Risk-sharing is optimal when both the economic agents are risk-averters and in this situation the risk-sharing ICPC is preferable to an FRC from the trader's point of view.

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[14] The proof of this is left for the readers.

[15] Interested readers may check it.

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APPENDIX

PROOF OF LEMMA 1.

From (9) and (29), one can write

$$\begin{aligned}(U_2^{**}-U_2^{*}) &= E(P)\cdot Q(B^{**})-E(P)\cdot Q(m^{*})+\rho_1\cdot V(P)\cdot(Q(B^{**}))^2\\ &\quad -\rho_2\cdot V(P)\cdot(Q(m^{*}))^2-(1+g)\cdot B^{**}+(1+g)\cdot m^{*}\cdot\\ &= (E(P)\cdot Q(B^{**})/2)+((1+g)\cdot Q(B^{**})/2\cdot Q'(B^{**}))-(1+g)\cdot B^{**}\\ &\quad -(E(P)\cdot Q(m^{*})/2)-((1+g)\cdot Q(m^{*})/2\cdot Q'(m^{*}))+(1+g)\cdot m^{*}\,.\end{aligned}$$

[This is because from equations (8) and (28), we can write

$$(E(P)\cdot Q(m^{*})/2)+\rho_2\cdot V(P)\cdot(Q(m^{*}))^2=((1+g)\cdot Q(m^{*})/2\cdot Q'(m^{*}))\,,\quad \text{and,}$$

$$(E(P)\cdot Q(B^{**})/2)+\rho_1\cdot V(P)\cdot(Q(B^{**}))^2=((1+g)\cdot Q(B^{**})/2\cdot Q'(B^{**}))]$$

So,

$$\begin{aligned}(U_2^{**}-U_2^{*}) &= (E(P)/2)\cdot[Q(B^{**})-Q(m^{*})]\\ &\quad +((1+g)/2)\cdot\left[\frac{(Q(B^{**})-2\cdot B^{**}\cdot Q'(B^{**}))}{Q'(B^{**})}\right.\\ &\quad \left.-(Q(m^{*})-2\cdot m^{*}\cdot Q'(m^{*}))/Q'(m^{*})\right]\end{aligned}\tag{A.1}$$

We know that $B^{**}\gtreqless m^{*}$ implies that $Q(B^{**})\gtreqless Q(m^{*})$. Now to prove that $B^{**}\gtreqless m^{*}\Rightarrow U_2^{**}\gtreqless U_2^{*}$, we have to show that $B^{**}\gtreqless m^{*}\Rightarrow$

$$[(Q(B^{**})-2\cdot B^{**}\cdot Q'(B^{**}))/Q'(B^{**})]\gtreqless[(Q(m^{*})-2\cdot m^{*}\cdot Q'(m^{*}))/Q'(m^{*})]\,.$$

For this it is necessary to show that $T=[(Q(B)-2\cdot B\cdot Q'(B))/Q'(B)]$ rises as B rises. Note that

$$T=(B\cdot[(Q(B)/B)-2\cdot Q'(B)]/Q'(B))=B\cdot\left[\frac{((AP-MP)-MP)}{MP}\right].$$

As B rises MP falls but $(AP-MP)$ rises when the AP curve of credit is linear or concave. So T rises as credit intensity, B, increases. (However, the relationship between T and B is not so clear when the AP curve of credit is convex). One should note that the algebraic value of $T(B^{**})$ is greater (less) than that of $T(m^*)$ when B^{**} is greater (less) than m^*. So when T is negative, $T(B^{**})$ is less (more) negative than $T(m^*)$ and when T is positive, $T(B^{**})$ is more (less) positive than $T(m^*)$ according to $B^{**}>(<)m^*$. Hence the second term (and also the first term) of the right-hand side of (A.1) is always positive (negative) if $B^{**}>(<)m^*$ since T rises as B rises. So $U_2^{**}>(<)U_2^*$ when $Q(\mathrm{B}^{**})>(<)Q(m^*)$. This completes the proof.