Title	FOREIGN AID TIED TO PUBLIC INPUTS
Sub Title	
Author	MICHAEL, Michael S. HATZIPANAYOTOU, Panos
Publisher	Keio Economic Society, Keio University
Publication year	1996
Jtitle	Keio economic studies Vol.33, No.2 (1996.) ,p.35- 45
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19960002-0035

KEIO ECONOMIC STUDIES 33 (2), 35–45 (1996)

FOREIGN AID TIED TO PUBLIC INPUTS

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First version received January 1995; final version accepted March 1996

Abstract: This paper constructs a two-country general equilibrium trade model, where an income transfer finances the production of a public input in the recipient country. Within this framework, the paper examines the effects of the income transfer on the terms of trade and welfare, and shows that the transfer may enrich the donor country and impoverish the recipient country. The paper also demonstrates that a small income transfer raises world welfare if the marginal revenue product of the public input exceeds its unit cost. In this case, a welfare imporvement is possible for both the donor and the recipient countries.

J.E.L. Classification: F35, Foreign Aid; H41, Public Goods.
Key-words: Foreign aid, public inputs, terms of trade improvement, donor enrichment, recipient impoverishment.

1. INTRODUCTION

The debate concerning the welfare and terms of trade effects of an income transfer has a long tradition in the theory of international trade. Three principal views prevail regarding the terms of trade effects of an income transfer. The "orthodox" view holds that the transfer induces a secondary burden on the donor country through lower terms of trade (*e.g.*, Keynes 1929). The "neutral" view argues that no presumption can be made about terms of trade changes due to an income transfer (*e.g.*, Ohlin 1929), and the "anti-orthodox" view suggests that an income transfer is likely to improve the donor country's terms of trade (*e.g.*, Jones 1970 and 1975, Li and Mayer 1990).

Leontief (1936) first provides an example where an income transfer immiserizes the recipient country, and enriches the donor country. Samuelson (1953) demonstrates that in the standard two-country model this "paradoxical" welfare effect can only occur if world commodity markets are Walrasian unstable. Recently, however, numerous authors show that Leontief's result can occur, consistent with

Acknowledgement. The authors gratefully acknowledge the constructive comments by James P. Feehan, Murray C. Kemp, Stephen Miller and an anonymous referee of the Journal. The usual disclaimer applies.

Walrasian stability of world commodity markets, in more general models—for example, Chichilnisky (1980, 1983), Bhagwati *et al.* (1983), Polemarchakis (1983), Yano (1983) and Jones (1984) in a three-agent model; Bhagwati *et al.* (1985) in the presence of endogenous and exogenous distortions; and Kemp and Kojima (1985a, 1985b), and Schweinberger (1990) with foreign aid partially or wholly tied in terms of private goods in the two countries.[1]

In examining the terms of trade and welfare effects of an income transfer, trade theorists assume that the income transfer is either distributed as a lump sum or that it is tied to a certain good(s) in the recipient country. Most nonprivate international aid, however, to less developed countries (LDCs) finances the provision of public consumption goods (*e.g.*, education, health services) or public investment (*e.g.*, infrastructure, government sponsored R & D). To consider these realities, the paper develops a two-country general equilibrium trade model, where the income transfer finances the production of a public input in the recipient country. That is, the income transfer is tied to the public input. Within this framework, the paper examines the terms of trade and welfare effects of the income transfer. The income transfer may lead to paradoxical results, that is, enrich the donor country and impoverish the recipient. The paper also shows that a small income transfer can improve world welfare (*i.e.*, the sum of the two countries welfare). In this case, it is possible for the income transfer to be welfare enriching for both the donor and the recipient countries.

2. THE PUBLIC INPUT ECONOMY

Consider an open economy, home, that produces two traded private goods (*i.e.*, one exported and one imported), and one nontraded public good. Two internationally immobile factors of production, capital and labor, exist in fixed endowments and are used in the production of all three goods. The public good is provided by the government at no cost to the private sector where it is used as an input in the production of the traded goods. Good 1, the numeraire, is exported, and good 2 is imported. Good and factor markets are perfectly competitive, and trade is free.

The production of the i-th traded private good, $i=1, 2$, is given by[2]

$$x_i = f^i(g, v_p^i)\,, \tag{1}$$

where g is the amount of the public input available for the production of private goods, and v_p^i is the vector of private factors used in the production of the i-th traded private good. The function f is assumed to be linearly homogeneous and

[1] Kemp (1984), in a multi-commodity model with many private traded and international public consumption goods, shows that a sufficiently small transfer does not affect welfare in any country.

[2] A complication that may arise with this type of production functions is the possibility of convexities in the production possibilities frontier (p.p.f.), and thus of multiple equilibrium, due to increasing returns to scale from the public input (*e.g.*, see Tawada and Abe 1984). To avoid such complexities, assume that, at least in the neighborhood of equilibrium, the p.p.f. is strictly concave to the origin.

concave in v_p^i, so that the public input is of the "factor-augmenting" variety.[3] The f function is strictly increasing in g.

The public input is produced by the use of capital and labor. Its production function, assumed to be linearly homogeneous and concave in the primary factors, is given by

$$g = g(v^g), \tag{2}$$

where v^g is the vector of factors used in the production of the public input.

Full employment of the primary factors requires that

$$v = v^p + v^g, \tag{3}$$

where v is the vector of fixed primary factor endowments (*i.e.*, $dv = 0$), and v^p is the vector of primary factors used in the production of the traded goods.

The private gross domestic product (GDP) function $\tilde{R}(p, g, v^p)$ gives the maximum value of the traded good production, at the relative price of the imported good p $(= p_2/p_1)$, the level of the public input production g, and the vector of the primary factors v^p available for the production of the public goods. By choice of units $p_1 = 1$. The $\tilde{R}(p, g, v^p)$ function is increasing and convex in p. The derivative of the GDP function with respect to p (*i.e.*, $\tilde{R}_p$) is the supply function of the imported good, with respect to g (*i.e.*, $\tilde{R}_g$) is the marginal revenue product of the public input, and with respect to v^p (*i.e.*, $\tilde{R}_v$) is the marginal revenue product of the primary factors. For the rest of the analysis all subscripts denote partial derivatives.

Equilibrium in the primary factor markets requires that each factor is paid the value of its marginal product at every domestic use. That is,

$$w = \tilde{R}_v(p, g, v^p), \tag{4}$$

where w is the vector of factor rewards.

Let $C^g(w)$ denote the unit cost function of the public input production. The $C^g(w)$ function is concave and linearly homogeneous in factor rewards (*i.e.*, $wC_w^g = C^g$, $C_{ww}^g w = 0$). Using the properties of the $C^g(w)$ function, the demand for factors in the production of the public input is given by

$$v^g = gC_w^g(w). \tag{5}$$

Substituting equation (4) into (5), and then into equation (3) gives

$$v^p + gC_w^g(\tilde{R}_v(p, g, v^p)) = v. \tag{6}$$

Using equation (6) and assuming constant factor endowments, v^p can be

[3] Such are inputs whose benefits, given the production technology, accrue entirely to the primary factors employed by the firms (*e.g.*, education and training workers). Other varieties of public inputs include "firm-augmenting" whose benefits accrue to the firm (*e.g.*, legal or promotional services), and "semi-public" which are collective across industries but congestible within each industry (*e.g.*, a dam that benefits both a farm and an inland fishery). Feehan (1989) provides a detailed account of the properties of such public input varieties.

expressed as a function of p and g, as follows:

$$v^p = v^p(p, g) . \tag{7}$$

Substituting equation (7) into $\tilde{R}(p, g, v^p)$ gives the private GDP function with public input provision as follows[4]

$$R(p, g) = \tilde{R}(p, g, v^p(p, g)) . \tag{8}$$

The Appendix provides a detailed proof of the following properties of the $R(p, g)$ function:[5]

$$\begin{aligned} R_p &= \tilde{R}_p , & R_g &= \tilde{R}_g - C^g(w) , \\ R_{gp} &= \tilde{R}_{gp} - g\tilde{R}_{pv}C^g_{ww}\tilde{R}_{vg} - C^g_p , & R_{pp} &= \tilde{R}_{pp} - g\tilde{R}_{pv}C^g_{ww}\tilde{R}_{vp} \succ 0 . \end{aligned} \tag{9}$$

An increase in the provision of the public input increases the value of traded goods (*i.e.*, $R_g \succ 0$) if the marginal revenue product of the public input exceeds its unit cost of production (*i.e.*, $\tilde{R}_g \succ C^g$).

Let $E(p, u)$ be the minimum expenditure required to achieve a level of utility u, given the relative price p. The expenditure function is increasing and concave in p. Its derivative with respect to p (*i.e.*, E_p) is the compensated demand function for the imported good.

3. EQUILIBRIUM IN THE WORLD ECONOMY

Consider a world of two countries, home and foreign, in which an income transfer occurs. The income transfer (*i.e.*, T) from the foreign (*i.e.*, donor) to the home (*i.e.*, recipient) country, finances the public input in the home country. Equilibrium in the world economy is characterized by the following conditions:

$$E(p, u) = R(p, g) + gC^g(w) , \tag{10}$$

$$B = T - gC^g(w) , \tag{11}$$

$$E^*(p, u^*) = R^*(p) - T , \tag{12}$$

$$Z_p(p, u, g) + Z^*_p(p, u^*) = 0 , \tag{13}$$

where Z_p $(= E_p - R_p)$ is the home country's imports, and Z^*_p $(= E^*_p - R^*_p)$ is the foreign country's exports. Asterisks denote the foreign country's variables.

Equation (10), which is the income expenditure identity of the home country, indicates that expenditure (*i.e.*, $E(p, u)$) must equal the revenue from production of traded goods (*i.e.*, $R(p, g)$) plus revenue from the production of the public input (*i.e.*, $gC^g(w)$). Equation (11), the home government budget constraint, shows that

[4] The term private GDP function with private input provision (*i.e.*, $R(p, g)$) refers to the value of private goods. The country's total GDP function is $R(p, g) + gC^g$.

[5] Abe (1992) discusses the properties of the $R(p, g)$ function. But, in his analysis g is a pure public consumption good.

the cost of the public input is financed through the income transfer from abroad. For simplicity, the foreign country finances the income transfer through lump-sum taxation, and does not produce any public good or input. The foreign country's income-expenditure identity (*i.e.*, equation (12)) requires that expenditure (*i.e.*, $E^*(p, u^*)$) equals revenue form production (*i.e.*, $R^*(p)$) minus the income transfer to the home country. With only two traded goods, Walras's Law implies that world goods markets equilibrium is characterized by equations (5), That is, the world excess demand for good 2 must equal zero.[6]

Equations (10)–(13) contain four endogenous (*i.e.*, u, u^*, p, and g) and one exogenous (*i.e.*, T) variables. Totally differentiating equations (10)–(13), and using the properties of the expenditure and GDP functions with and without public input provision yields the following system of equations:

$$du+(Z_p-gC_p^g)dp-(\tilde{R}_g+gC_g^g)dg=0\,, \tag{14}$$

$$(C^g+gC_g^g)dg+gC_p^g dP=dT\,, \tag{15}$$

$$du^*+Z_p^*dp=-dT\,, \tag{16}$$

$$Z_{pu}du+Z_{pu}^*du^*+Z_{pg}dg+S_{pp}dp=0\,, \tag{17}$$

where by choice of units, $E_u=E_u^*=1$, and $S_{pp}(=Z_{pp}+Z_{pp}^*)$ is the change in the world excess demand for the traded good 2 due to a change in its relative price.

Equation (14) shows that either an improvement in the home country's terms of trade (*i.e.*, $dp<0$), for a given level of the public input, or an increase in the public input (*i.e.*, $dg>0$), at constant terms of trade, has an ambiguous effect on welfare in the recipient country. The improvement in the terms of trade raises welfare if it increases the unit cost of the public input (*i.e.*, $C_p^g<0$). This is because, as the right-hand-side of equation (10) indicates, an increase in the unit cost of the public input, completely financed through foreign aid, increases income, thus expenditure and welfare in the recipient country. For the rest of the analysis, it is assumed that even if $C_p^g>0$, $(Z_p-gC_p^g)$ is positive, which is necessarily true for small enough levels of the public input (*i.e.*, $g\approx 0$). Thus, the improvement in the home country's terms of trade positively affects welfare. On the other hand, at constant terms of trade, the increase in the public input positively affects welfare through its marginal revenue product (*i.e.*, $\tilde{R}_g$), but it has an ambiguous welfare effect through the induced change in its unit cost of production (*i.e.*, $C_g^g>(<)0$). If, at constant terms of trade, the increase in the public input increases its unit cost of production, then it improves the recipient country's welfare. For small levels of the public input (*i.e.*, $g\approx 0$), an increase in its production positively affects

[6] In a world of many countries where a single donor (*e.g.*, DC) and a single recipient (*e.g.*, LDC) are assumed price takers, an income transfer has no effect on the terms of trade (*i.e.*, $(dp/dT)=0$). That is, in such a case, equation (13) no longer holds, since world market equilibrium remains unaffected by the transfer. Consequently, the present framework is more appropriate for the case where the donor DCs as a group make an income transfer to another group of countries, *i.e.*, LDCs. In such a case, the income transfer affects the world market equilibrium, and thus, the terms of trade.

the recipient country's welfare. Since the cost of the public input production in the recipient country is indirectly borne by the donor country, combining equations (15) and (16) gives $du^* = (-Z_p^* - gC_p^g)dp - (C^g + gC_g^g)dg$. Equation (11), however, shows that $(\partial B/\partial g) = -(C^g + gC_g^g)$. Assuming that, at constant commodity prices and at least in the neighborhood of equilibrium, an increase in the public input increases the total cost of producing the public input (*i.e.*, $(dB/dg) < 0$), $(C^g + gC_g^g)$ must be positive. Thus, welfare in the donor country decreases, for given prices, with a small increase in the public input production, and for positive $(-Z_p^* - gC_g^g)$, it increases with an improvement in the country's terms of trade (*i.e.*, $dp > 0$).

4. TERMS OF TRADE EFFECT OF THE TRANSFER

This section considers the effect of the income transfer on the terms of trade, when the transfer finances the public input in the recipient country. Using the system of equations (14) to (17), the effect of the income transfer on the donor country's terms of trade is given by

$$\Delta(dp/dT) = -Z_{pg} - (\tilde{R}_g + gC_g^g)Z_{pu} + (C^g + gC_g^g)Z_{pu}^* \tag{18}$$

where Δ, the determinant of the left-hand-side coefficients matrix of the system of equations (14)–(17), is negative if the world good markets are Walrasian stable.

The first right-hand-side terms of equation (18) captures the effect of the income transfer, and the induced increase in the public input, on p through the effect of g on imports of good 2 in the recipient home country (*i.e.*, Z_{pg}). Note that $Z_{pg} = -R_{pg} = -(\tilde{R}_{pg} - g\tilde{R}_{pv}C_{ww}^g\tilde{R}_{vg} - C_p^g)$ has an ambiguous sign. By assumption, $\tilde{R}_{pg} > 0$; *i.e.*, for given p and v^p, an increase in the public input raises the production of the imported good. An increase in the level of the public input has a direct and an indirect effect on production of the imported good. The direct effect (*i.e.*, $\tilde{R}_{pg}$), in Abe's (1990) terminology, shows the degree of spillover effect of public input on the production of the imported good. The indirect effect (*i.e.*, $R_{pv}v_g^p$) originates from disturbances in factor markets and has two parts. First, the change in the public input influences factor demands and their rewards (*i.e.*, $gC_{ww}\tilde{R}_{vg}$), and second, it induces changes in government demand for factors of production (crowding-out effect) (*i.e.*, $C_w^g = v^g/g$). See also the Appendix. Then, at least for small levels of the public input, when Z_{pg} is negative if an increase in p reduces the unit cost of the public input (*i.e.*, $C_p^g < 0$). Thus, when Z_{pg} is negative, an income transfer that increases the public input in the recipient country increases the domestic production of the imported good, reduces imports, and has a negative effect on the foreign country's terms of trade. The second right-hand-side term of equation (18) (*i.e.*, $(\tilde{R}_g + gC_g^g)Z_{pu}$) measures the effect of higher domestic imports, due to the income transfer, on p. Intuitively, an income transfer that increases the public input in the recipient country by one unit increases income by $(\tilde{R}_g + gC_g^g) = \partial(R + gC^g)/\partial g$. This increase in income increases the consumption

of the imported good by $(\tilde{R}_g + gC_g^g)Z_{pu}$, thus having a positive effect on p. The last right-hand-side term (*i.e.*, $(C^g + gC_g^g)Z_{pu}^*$) reflects the effect of a lower foreign demand for good 2, due to the income transfer, on p. That is, since the cost of the public input in the recipient country is indirectly borne by the foreign country, an income transfer that increases the level of the public input by one unit reduces foreign income by $(C^g + gC_g^g)$, and reduces foreign demand by $(C^g + gC_g^g)Z_{pu}^*$, thus, having a negative effect on the price of the recipient country's imported good.[7]

PROPOSITION 1. *Assume that an income transfer between two countries, finances the production of a public input in the recipient country, and that an increase in the public input reduces net government revenue. The sufficient conditions for this income transfer to improve (worsen) the donor country's terms of trade are* (i) *the marginal propensity to consume good* 2 *is smaller (larger) in the donor than in the recipient country*, (ii) *the marginal revenue product of the public input exceeds (is smaller than) its unit cost, and* (iii) *the increase in the public input increases (reduces) imports of good* 2 *by the recipient country.*

In the present context, the effect of the income transfer on the terms of trade, when the income transfer finances a public input in the recipient country depends not only on the marginal propensities of consumption in the two countries (*i.e.*, Z_{pu} and Z_{pu}^*), but also on (i) the effect of the public input increase on imports of good 2 by the recipient country (*i.e.*, $Z_{pg} \gtrless 0$), and (ii) on the relationship between the marginal revenue product and the unit cost of the public input (*i.e.*, $\tilde{R}_g \gtrless C^g$). Since most LDCs do not provide many public inputs, it is expected that the marginal revenue product of the public input exceeds its unit cost. Thus, the present analysis demonstrates that even if $Z_{pu}^* > Z_{pu}$, it is still possible that $Z_{pu}\tilde{R}_g > Z_{pu}^* C^g$ in which case it is likely for the income transfer to improve the donor country's terms of trade.

5. THE WELFARE EFFECTS OF THE TRANSFER

5.1. *The Welfare Effect in the Recipient Country*

Equation (14) illustrates that domestic welfare is a function of the relative price of good 2, and of the level of the public input. Thus, using equation (14), the welfare effect of the income transfer in the recipient country is given by

$$\begin{aligned}(\mathrm{d}u/\mathrm{d}T) &= (\partial u/\partial p)(\mathrm{d}p/\mathrm{d}T) + (\partial u/\partial g)(\mathrm{d}g/\mathrm{d}p) \\ &= -(Z_p - gC_p^g)(\mathrm{d}p/\mathrm{d}T) + (\tilde{R}_g + gC_g^g)(\mathrm{d}g/\mathrm{d}T)\,. \end{aligned} \tag{19}$$

Equation (19) indicates that a small income transfer (*i.e.*, when initially $g=0$)

[7] The previous analysis assumes that an increase in the public input increases the recipient country's income (*i.e.*, $(\tilde{R}_g + gC_g^g) > 0$), and reduces the donor country's income (*i.e.*, $(C^g + gC_g^g) > 0$). If, however, in the neighborhood of equilibrium an increase in the public input increases the recipient country's cost of producing the public input, then $(C^g + gC_g^g)$ is indeed positive (*i.e.*, no need to assume so), and thus the increase in the public input reduces the donor country's income.

improves the recipient country's welfare if it (i) improves its terms of trade (*i.e.*, $dp/dT \prec 0$), and (ii) increases public input production (*i.e.*, $dg/dT \succ 0$). Using the system of equations (14) to (17), the effect of the income transfer of the public input is given by

$$\Delta(dg/dT) = S_{pp} + (Z^*_{pu} - Z_{pu})(Z_p - gC^g_p) \,. \tag{20}$$

Equation (20) indicates that, at least for small levels of the public input, the condition $Z_{pu} \succ Z^*_{pu}$ guarantees that the income transfer increases the public input in the recipient country (*i.e.*, $(dg/dT) \succ 0$). Note, however, that when initially $g = 0$, $\Delta = C^g[S_{pp} + (Z^*_{pu} - Z_{pu})Z_p]$. Substituting this result into equation (20) shows that when $g = 0$, $(dg/dT) = (1/C^g)$. Thus, a small income transfer unambiguously increases the public input in the recipient country.

Substituting equations (20) and (18) into (19) gives the effect of the income transfer on the recipient country's welfare as follows[8]

$$\Delta(du/dT) = S_{pp}(\tilde{R}_g + gC^g_g) + Z_{pg}(Z_p - gC^g_p) + (Z_p - gC^g_p)Z^*_{pu}(\tilde{R}_g - C^g) \,. \tag{21}$$

Equation (21) indicates that the sufficient conditions for a small income transfer (*i.e.*, when initially $g = 0$) to improve the recipient country's welfare are (i) and increase in the public input reduces the country's imports (*i.e.*, $Z_{pg} \prec 0$), and (ii) the unit cost of the public input exceeds its marginal revenue product (*i.e.*, $\tilde{R}_g \prec C^g$). If either or both conditions do not hold, then it is possible for a small income transfer to be welfare immiserizing for the recipient country.

5.2. *The Welfare Effect in the Donor Country*

Using the same procedure as in the previous section, the effect of the income transfer on the donor country's welfare is given by

$$\Delta(du^*/dT) = -S_{pp}(C^g + gC^g_g) - (-Z^*_p - gC^g_p)Z_{pg} - (Z_p - gC^g_p)Z_{pu}(\tilde{R}_g - C^g) \,. \tag{22}$$

Equation (22) illustrates that a small income transfer impoverishes the donor country if (i) an increase in the public input decreases the recipient country's imports of good 2 (*i.e.*, $Z_{pg} = -R_{pg} \prec 0$), and (ii) the marginal revenue product of the public input is smaller than its unit cost. If either or both conditions do not hold, then it is possible for a small income transfer to be welfare enriching for the donor country.

PROPOSITION 2. *Assume that a small income transfer between two countries finances a public input in the recipient country. This income transfer enriches the recipient and impoverishes the donor country if* (i) *the increase in the public input increases the production of the recipient country's imported good, and* (ii) *the unit cost of the public input exceeds its marginal revenue product. If, however, either or both conditions do not hold, then it is possible for the small income transfer to*

[8] Equation (21) can be directly derived from the system of equations (14) to (17). Here, however, we adopt the present approach for a clearer intuitive explanation.

impoverish the recipient and enrich the donor country.

5.3. The Effect on World Welfare

Using equations (14) to (16) and (20) gives the effect of the income transfer on world welfare as follows:

$$[(du/dT)+(du^*/dT)]=[(\partial u/\partial g)+(\partial u^*/\partial g)](dg/dT)$$
$$=(\tilde{R}_g-C^g)[S_{pp}+(Z_p-gC^g_p)(Z^*_{pu}-Z_{pu})]\Delta^{-1}. \quad (23)$$

Note that $(\partial u/\partial p)=-(\partial u^*/\partial p)$. Equation (23) shows that an income transfer that finances a public input in the recipient country, improves world welfare if (i) the marginal revenue product of the public input exceeds its unit cost of production, and (ii) the income transfer increases the public input. Substituting $g=0$ in equation (23) yields $(du/dT+du^*/dT)=[(\tilde{R}_g-C^g)/C^g]$. Thus, a small income transfer that finances a public input in the recipient country improves world welfare if the marginal revenue product exceeds its unit cost of production.

PROPOSITION 3. *Assume that a small income transfer between two countries finances a public input in the recipient country. This income transfer improves (worsens) world welfare if the marginal revenue product of the public input exceeds (is smaller than) its unit cost. In this case, it is possible for welfare to increase (decrease) in both the donor and the recipient countries.*

Kemp and Kojima (1985b) claim that an income transfer, when tied either in terms of traded private consumption goods or in terms of public consumption or public investment goods, can be welfare immiserizing for the recipient country and welfare improving for the donor. Here, equations (21) and (22) verify this result. In their analysis, however, world welfare remains unchanged since gains by one country are completely offset by losses in the other. In the present framework, equation (23) demonstrates that when the income transfer is used by the recipient country to finance a public input, world welfare can increase or decrease. For example, the present paper shows that a small income transfer increases world welfare if the unit cost of the public input, which is indirectly borne by the donor country, is less than the marginal revenue product of the public input.

6. CONCLUDING REMARKS

International trade theorists, examining the effects of an income transfer on the terms of trade and on welfare, assume that the transfer is either distributed as a lump-sum or tied in terms of a certain commodity in the recipient country. For example, the recipient country may be forced by the donor country to spend a certain amount of the income transfer on the donor's exported good. But, most LDCs use non-private foreign aid to finance public inputs such as the construction of a highway or a dam. With this in mind, we develop a two-country general

equilibrium trade model where an income transfer finances a public input in the recipient country, which then is provided to the private sector at no cost. That is, in the present model, foreign aid is tied to the public input.

Such an income transfer improves (worsens) the donor (recipient) country's terms of trade if (i) the marginal propensity to consume the imported good in the recipient country exceeds the marginal propensity to consume the same good in the donor country, (ii) the marginal revenue product of the public good exceeds its unit cost, and (iii) the increase in the public input increases the recipient country's imports. Under conditions (ii) and/or (iii), a small income transfer can lead to a welfare immiserization of the recipient, and welfare enrichment of the donor country. Under condition (ii) a small income transfer increases world welfare.

Since most LDCs provide little public inputs, it is expected that the marginal revenue product exceeds its unit cost. Thus, a small income transfer than finances a public input in the recipient LDC improves the world welfare. In this case, it is possible for the income transfer to be welfare enriching for both the donor and the recipient country.

To keep the analysis simple, we assume that the donor country does not produce a public input. The results of the paper, however, remain the same if the donor country produces a public input at a fixed level. Qualitatively similar paradoxical results emerge also in the case where the donor (*i.e.*, developed) country produces a public input at a level where the marginal revenue product equals its unit cost using lump-sum taxes to finance its production, while the recipient (*i.e.*, developing) country produces a public input at a fixed level. The paradoxical welfare effects of the transfer do not arise when both countries produce the public input at the level where the marginal revenue product equals its unit cost, using lump-sum taxes to finance its provision. In this case, the transfer is welfare improving for the recipient country, welfare immiserizing for the donor country, and does not affect world welfare.

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APPENDIX

This Appendix proves the properties of the $R(p, g)$ function as stated by equations (9). Note that in the two-traded-good two-factor model, changes in factor endowments do not affect factor rewards (*i.e.*, $\tilde{R}_{vv}=0$).

Differentiating equation (6) yields

$$v_p^p = -gC_{ww}^g \tilde{R}_{vp}\,, \quad \text{and} \quad v_g^p = -gC_{ww}^g \tilde{R}_{vg} - C_w^g\,. \tag{A1}$$

Differentiating equation (8), using equations (A1), and using the properties of the unit cost function (*i.e.*, $C_w^g w = C^g$, and $C_{ww}^g w = 0$) yields

$$R_p = \tilde{R}_p + \tilde{R}_v v_p^p = \tilde{R}_p - wC_{ww}^g \tilde{R}_{vp} g = \tilde{R}_p\,,$$

and

$$R_g = \tilde{R}_g + \tilde{R}_v v_g^p = \tilde{R}_g - w(C_{ww}^g \tilde{R}_{vp} g + C_w^g) = \tilde{R}_g - C^g\,. \tag{A2}$$

Differentiating equations (A2) yields

$$R_{pp} = \tilde{R}_{pp} + \tilde{R}_{pv} v_p^p = \tilde{R}_{pp} - g\tilde{R}_{pv} C_{ww}^g \tilde{R}_{vp} > 0\,,$$

and

$$R_{gg} = \tilde{R}_{gp} + \tilde{R}_{gv} v_p^p - C_p^g = \tilde{R}_{pg} + \tilde{R}_{pv} v_g^p = \tilde{R}_{pg} - g\tilde{R}_{pv} C_{ww}^g \tilde{R}_{vg} - C_p^g > 0\,. \tag{A3}$$