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INTERNATIONAL INCOME TRANSFERS AND
ENVIRONMENTAL QUALITY

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Abstract: The welfare effects of an international income transfer are examined using a differential game in which environmental quality enters into the utility functions of two countries as a public good. It is shown that if the countries have the same technology of cleaning up pollution (but may differ in any other way), an income transfer has no effect on the steady state or on transitional dynamics. However, if the transfer takes place from a country with a more efficient cleanup technology to the other country, environmental quality and the welfare of not only the donor but also of the recipient country decreases.

1. INTRODUCTION

Environmental problems and their management have received increasing attention from economists, as residuals or wastes from production or consumption become increasingly large and the resulting deterioration of the environmental quality becomes a more serious problem for the welfare of a society. Early studies of the problems, such as Ayres and Kneese (1969), Baumol (1972), and Baumol and Oates (1975), recognized the problem as instances of negative externality or social cost and proposed the incentive-based policies for controlling the externalities. Dynamic dimensions of the problem were studied by Keeler, Spence and Zeckhauser (1972) and d'Arge and Kogiku (1973).

More recent studies by Pethig (1976), Siebert (1979, 1987) and Krutilla (1991) investigated the effects of environmental policies (such as pollution taxes and standards) on the terms and pattern of trade and the environmental quality. In these studies, however, the environmental damage caused by pollution is confined to the country of emission. In the real world, the damage from pollution often goes beyond the geographical boundary of a country. Thus, deterioration of the

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environmental quality caused by pollution in one country may well affect the welfare of another country, and the benefit from the environmental policies and cleanup activities of pollution by one country may be received by another country as well.

In this paper we assume that the environmental quality is a public good and enters into the utility functions of two countries as an argument. We investigate in this model the effects of an international income transfer on the resource allocations and the environmental quality. The adverse effect on the global environmental quality that an income transfer from the North to the South creates is a concern among advanced countries. (See, for example, Environmental Agency of Japan (1994)). As the financial assistance to developing countries help industrialization of these countries, it is concerned that a donor country will suffer from deterioration of the environmental quality caused by pollution as a byproduct.

Neutrality of an income transfer in the private provision of a public good has been pointed out by Warr (1983), and extended by Kemp (1984), Bergstrom and Varian (1985) and Bergstrom, Blume and Varian (1986). They showed that when public goods are provided by private individuals, a redistribution of income among individuals does not change the Nash equilibrium. Yoshioka (1992a) also obtained the neutrality result for a model in which the environmental quality enters the utility functions of two countries as a public good. However, he (1992b) also showed that in such a model the Nash equilibrium is not independent of an income transfer if the technologies of cleaning up pollution are different in two countries. In his paper, however, residuals from industrial output which cause pollution do not accumulate and thus dynamic dimensions of the problem are ignored. In the present paper, it is assumed that residuals accumulate over time, so that the environmental quality continues to deteriorate unless efforts to clean up pollution outweigh the pollution created by the current industrial output. We use a differential game to investigate the nature of the Nash equilibrium for this dynamic model.

It is shown that if the countries have the same technology in the cleanup of pollution (but may differ in any other way), an international income transfer has no effect on the steady state or transitional dynamics. However, if there is a difference in efficiency in the cleanup of pollution, the neutrality result no longer holds; an income transfer from a country with a more efficient technology of the cleanup to a country whose technology is less efficient causes a deterioration in the environmental quality, in spite of an increase in the aggregate resource allocation to the cleanup of pollution, and the utility level of not only the donor but also that of the recipient country deteriorates as a result, both in the long run and in the short run; and the opposite occurs in the case of the reverse income transfer. These results are believed to hold for any utility function and stock dynamics.
II. MODEL

We assume that there are two countries, each producing only consumer goods, which they consume at the time of production, and that there is no foreign trade. The production of consumer goods by each country, however, creates, as a byproduct, pollution, which deteriorates the environmental quality. Each country determines its resource allocation between the production of consumer goods and the cleanup of pollution, the latter prevents deterioration or improves the environmental quality. The utility of each country depends on the amount of consumer goods produced in its own country and the environmental quality, which is considered as a public good.

The utility function of each country is thus given as

\[ u_i = \alpha_i \log y_i + (1 - \alpha_i) \log z, \quad 0 < \alpha_i < 1, \quad i = 1, 2 \]  

(1)

where \( y_i \) is the amount of consumer goods produced by each country, and \( z \) is the level of environmental quality. The production function of consumer goods is given as

\[ y_i = \beta_i r_i, \quad i = 1, 2 \]  

(2)

where \( r_i \) is the resource allocation to the production of consumer goods, and \( \beta_i \) is the coefficient representing the production technology. Pollution, which deteriorates the environmental quality, is assumed to be created, as a byproduct, proportionate to the output of consumer goods by each country. Thus, if \( P \) is the amount of pollution, we have

\[ P = \gamma_1 y_1 + \gamma_2 y_2 \]  

(3)

where \( \gamma_1 \) and \( \gamma_2 \) are the coefficients indicating pollution intensity of the consumer goods production in each country. Each country is assumed to allocate \( s_i \) of its resources to the cleanup of pollution. Thus, if \( C \) is the amount of pollution that is cleaned up, we have

\[ C = \theta_1 s_1 + \theta_2 s_2 \]  

(4)

where \( \theta_1 \) and \( \theta_2 \) represent the cleanup technologies of each country. The level of environmental quality then changes over time according to

\[ \dot{z} = -(P - C) = -(\gamma_1 \beta_1 r_1 + \gamma_2 \beta_2 r_2 - \theta_1 s_1 - \theta_2 s_2), \quad z(0) = z_0 \]  

(5)

The total amount of resources of each country, \( w_i \), which is assumed to be constant, is allocated between the production of consumer goods and the cleanup activities.

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1 Although the utility function is specified in equation (1), the same results are obtained from a general utility function \( f_i(y_i, z), i = 1, 2 \), as long as both \( y_i \) and \( z \) are normal goods. Here, in order to avoid complications of mathematical expressions and make the presentation as simple as possible, we use the utility function specified in (1). See the observation made in page 30 in the last paragraph before the section for the short run analysis.
Each country determines its resource allocation so as to maximize

$$\int_0^\infty e^{-\rho t} u_i(t) dt, \quad i = 1, 2$$

subject to (5), with the initial condition $z(0) = z_0$, which is given. The current value Hamiltonians are

$$(8) \quad H_i = \alpha_i \log y_i + (1 - \alpha_i) \log z - \lambda_i (\gamma_1 \beta_1 r_1 + \gamma_2 \beta_2 r_2 - \theta_1 s_1 - \theta_2 s_2), \quad i = 1, 2$$

The necessary conditions for the Nash equilibrium, assuming an interior solution, are:

$$(9) \quad r_i = \alpha_i / \lambda_i (\gamma_i \beta_i + \theta_i), \quad i = 1, 2$$

$$(10) \quad \dot{s}_i = \rho \lambda_i - (1 - \alpha_i) / z, \quad i = 1, 2$$

$$(11) \quad \lim_{t \to \infty} e^{-\rho t} \lambda_i(t) = 0, \quad i = 1, 2$$

Substitution of (9) together with (6) into (5) yields

$$\dot{z} = \theta_1 w_1 + \theta_2 w_2 - x_1 / \lambda_1 - x_2 / \lambda_2, \quad z(0) = z_0$$

Now we have three differential equations (two equations in (10) and equation (12)) in three unknowns, $\lambda_1, \lambda_2$ and $z$. The motions of $\lambda_1$ and $z$, which satisfy the transversality condition (11), are shown in Figure 1, with $\lambda_2$ being held constant. As shown in Fig. 1, when the environmental quality $z$ is high, its shadow price, $\lambda_1$, for the first country, is low. This implies, from equation (9), that a large amount of resources is allocated to the production of consumer goods (large $r_1$), and a small amount of resources is allocated to the cleanup activities of pollution (small $s_1$.) The consequence is that the pollution made as a byproduct of consumption goods is larger than what is cleaned up, resulting in deterioration of the environmental quality. As the environmental quality deteriorates, its shadow price increases, and hence, more resources are allocated to the cleanup activities and less resources to the production of consumer goods. When the environmental quality deteriorates to its long run steady state value, $z^*$, the resource allocation to the cleanup activities is just enough to clean up pollution created by the current production.

Since the second country does exactly the same, the deterioration or improvement of environmental quality is faster than when only one country is engaged in creating pollution or in its cleanup activities. This can be seen in Fig. 1. Since the $\dot{z} = 0$ line shifts downward or upward as $\lambda_2$ increases or decreases, $z$ approaches $z^*$ faster, from either direction.

What is stated in the above is verified by linearizing the system and solving for the time paths of $\lambda$'s and $z$. (See the Appendix for this derivation.)
\[ \lambda_i(t) = -\frac{1}{(1 - \alpha_i)} z^2 (\rho - \mu) (z(0) - z^*) e^{\mu t} + \lambda_i^* \quad \text{for } i = 1, 2 \]  
\[ z(t) = (z(0) - z^*) e^{\mu t} + z^* \]  
\text{where} 
\[ \lambda_1^* = (1 - \alpha_1) \left( \frac{\alpha_1}{1 - \alpha_1} + \frac{\alpha_2}{1 - \alpha_2} \right) \left( \theta_1 w_1 + \theta_2 w_2 \right) \]  
\[ \lambda_2^* = (1 - \alpha_2) \left( \frac{\alpha_1}{1 - \alpha_1} + \frac{\alpha_2}{1 - \alpha_2} \right) \left( \theta_1 w_1 + \theta_2 w_2 \right) \]  
\[ z^* = \frac{\theta_1 w_1 + \theta_2 w_2}{\rho \left( \frac{\alpha_1}{1 - \alpha_1} + \frac{\alpha_2}{1 - \alpha_2} \right)} \]  
In equations (13) and (14), since \( \mu \) is negative, if the initial value of \( z, z(0) \), is greater than its long run steady state value \( z^* \), \( \lambda \)'s approach their respective steady state values, \( \lambda_1^* \) and \( \lambda_2^* \), from below, while \( z \) approaches its steady state value.
$z^*$ from above. If the initial value of $z$ is smaller than $z^*$, the opposite will happen to the time paths of $\lambda$'s and $z$.

III. EFFECTS OF INTERNATIONAL INCOME TRANSFER

Now we analyze the effects of an international income transfer from one country to the other, on the resource allocations of both countries and the environmental quality, in the long run as well as in the short run. We assume that an income transfer takes place from country 1 to country 2 such that $dw_1 + dw_2 = 0$.

**Long Run Effects**

First we analyze the effects of the income transfer in the long run steady state. By setting $\dot{\lambda}_1 = \dot{\lambda}_2 = z = 0$ in equations (10) and (12), the long run equilibrium values of $\lambda$'s and $z$ are solved, as shown by (15), (16) and (17). The effects of the income transfer in the form: $-dw_1 = dw_2$ on these steady state values are:

\[ \frac{-d\lambda_1^*}{dw_1} + \frac{d\lambda_1^*}{dw_2} = \frac{(1-\alpha_1)\left(\frac{\alpha_1}{1-\alpha_1} + \frac{\alpha_2}{1-\alpha_2}\right)}{\left(\theta_1w_1 + \theta_2w_2\right)^2}(\theta_1 - \theta_2) \quad (20) \]

\[ \frac{-d\lambda_2^*}{dw_1} + \frac{d\lambda_2^*}{dw_2} = \frac{(1-\alpha_2)\left(\frac{\alpha_1}{1-\alpha_1} + \frac{\alpha_2}{1-\alpha_2}\right)}{\left(\theta_1w_1 + \theta_2w_2\right)^2}(\theta_1 - \theta_2) \quad (21) \]

\[ \frac{-dz^*}{dw_1} + \frac{dz^*}{dw_2} = \frac{-(\theta_1 - \theta_2)}{\rho\left(\frac{\alpha_1}{1-\alpha_1} + \frac{\alpha_2}{1-\alpha_2}\right)} \quad (22) \]

where $\Delta < 0$ is defined by (19). Equations (20) through (22) indicate that the effects of an income transfer on the environmental quality and its shadow price in the two countries crucially depend on the relative magnitude of $\theta_1$ and $\theta_2$, the coefficients indicating the technologies of the two countries in their cleanup activities. If the technologies of cleaning up pollution by the two countries are equally efficient ($\theta_1 = \theta_2$), an income transfer from one country to the other does not change the environmental quality or its shadow price in the long run. However, if an income transfer takes place from the country which has a more efficient technology to the country whose technology is less efficient ($\theta_1 > \theta_2$), the environmental quality deteriorates, and its shadow price increases in the steady state. If the transfer is from the country whose technology is inferior to the country which has a superior technology, the environmental quality will improve and the shadow price will decline.

The effects on the resource allocations of these countries to the consumption goods ($r_i$) and the cleanup activities ($s_i$) can be found from equations (6) and (9) as follows:
Thus, if the technologies of the cleanup are the same in the two countries, i.e., \( \theta_1 = \theta_2 \), an international income transfer does not change the resource allocation to the consumer goods of either country. The resource allocated to the cleanup activity decreases in the donor country and increases in the recipient country, by the same amount, equal to the income transferred, thus leaving the level of environmental quality unchanged. Since there is no change in the production of consumer goods in either country and there is no change in the environmental quality, the utility level of either the donor or the recipient country does not change as a result of the income transfer. These results are consistent with Warr (1983), Kemp (1984), Bergstrom and Varian (1985), and Bergstrom, Blume and Varian (1986) which showed neutrality of income transfer in the Nash equilibrium when a public good is provided by the consumers.

However, if the cleanup technology is different in the two countries, neutrality of an income transfer no longer holds. If the cleanup technology is superior in the donor country than in the recipient country, i.e., \( \theta_1 > \theta_2 \), as has been noted previously, the environmental quality deteriorates and its shadow price increases in both countries. This implies, in equations (23) through (26), that since \( \frac{d\theta_i}{d\lambda_i} < 0 \), \( i = 1, 2 \), the following inequalities hold:

\[
- \frac{d\theta_1^*}{dw_1} + \frac{d\theta_1^*}{dw_2} < 0
\]

\[
- \frac{d\theta_2^*}{dw_1} + \frac{d\theta_2^*}{dw_2} < 0
\]

\[
- \frac{d\lambda_1^*}{dw_1} + \frac{d\lambda_1^*}{dw_2} > -1
\]

\[
- \frac{d\lambda_2^*}{dw_1} + \frac{d\lambda_2^*}{dw_2} > 1
\]

Thus, the result is that the resource allocation to the consumer goods decreases in both countries. The resource allocation to the cleanup of pollution increases
in the recipient country by more than the amount of resources transferred, and in the donor country it does not decrease as much as the amount of resources transferred. Thus, if an income transfer takes place from a country which has a more efficient cleanup technology to a country whose technology is less efficient, the aggregate resource allocation to the cleanup of pollution increases, (and yet the environmental quality decreases), and this is done by reducing the resource allocation to the consumer goods in both countries. Since both the environmental quality and the production of consumer goods decrease in both countries, the utility level decreases not only in the donor but also in the recipient country. These results are exactly the same as those obtained by Yoshioka (1992b) in the static setting.

It is to be noted that the result of nonneutrality of an income transfer is obtained only when there is a difference in the cleanup technology. A difference in any other parameter does not matter. This result is due to the way in which the cleanup activity, \( C \) is modeled. Equation (4) can be rewritten as

\[
C = \Omega - \theta_1 r_1 - \theta_2 r_2
\]  

where \( \Omega = \theta_1 w_1 + \theta_2 w_2 \) may be interpreted as an intrinsic rate of cleanup. Given this form for \( C \), and for any utility function and stock dynamics, \( w_i \) enters each country’s maximization problem via \( \Omega \), which is a parameter in their control problems. Thus, it is clear that an income transfer of the type considered here will have no effect on the equilibrium if and only if the transfer does not affect \( \Omega \), i.e., if and only if \( \theta_1 = \theta_2 \). If this equality does not hold, the income transfer does affect \( \Omega \) in an obvious way, and the results are obvious. For example, if \( \theta_1 > \theta_2 \), a transfer from country 1 to country 2 decreases \( \Omega \), to the detriment of both countries.

### Short Run Effects

Next we investigate the effect of the income transfer on the Nash equilibrium off the steady state. Let the income transfer be \(-d_1w = d_2w = d_2w\). We examine the effect of this income transfer on the Nash equilibrium time paths of \( \lambda_1, \lambda_2 \) and \( z \) given by equations (10)'s and (12). Differentiating these equations with respect to \( w \), we obtain:

\[
\frac{d\lambda_1}{dw} = \frac{d(d\lambda_1/dw)}{dt} = \rho \left( \frac{d\lambda_1}{dw} \right) + \left( \frac{1 - \alpha_1}{z^2} \right) \left( \frac{dz}{dw} \right)
\]

\[
\frac{d\lambda_2}{dw} = \frac{d(d\lambda_2/dw)}{dt} = \rho \left( \frac{d\lambda_2}{dw} \right) + \left( \frac{1 - \alpha_2}{z^2} \right) \left( \frac{dz}{dw} \right)
\]

\[
\frac{dz}{dw} = \frac{d(dz/dw)}{dt} = \left( \frac{\alpha_1}{\lambda_1^2} \right) \left( \frac{d\lambda_1}{dw} \right) + \left( \frac{\alpha_2}{\lambda_2^2} \right) \left( \frac{d\lambda_2}{dw} \right) - (\theta_1 - \theta_2)
\]

We have three differential equations in three unknowns, \( d\lambda_1/dw, d\lambda_2/dw \) and
dz/dw. Evaluating the coefficients in equilibrium, we can solve these equations to obtain the time paths of these unknowns in the neighborhood of equilibrium, noting that the effect of the transfer on the environmental quality is initially zero, i.e., dz/dw (0) = 0.

\[
\frac{d\lambda_i}{dw}(t) = \left( \frac{dz^*}{dw} \right) \left[ \frac{1 - \alpha_i}{z^* (\rho - \mu)} \right] e^{\mu t} + \frac{d\lambda_i^*}{dw}, \quad i = 1, 2 \tag{34}
\]

\[
\frac{dz}{dw}(t) = - \left( \frac{dz^*}{dw} \right) e^{\mu t} + \frac{dz^*}{dw} \tag{35}
\]

where \(d\lambda_i^*/dw\), \(i = 1, 2\) and \(dz^*/dw\) are given by equations (20), (21) and (22) respectively, and \(\mu\) is given by (18). Again, the results depend on the cleanup technologies in the two countries. If there is no difference in efficiency in the cleanup between the two countries, i.e., \(\theta_1 = \theta_2\), \(d\lambda_i/dw\) \((t), i = 1, 2,\) and \(dz/dw\) \((t)\) in equations (34) and (35) are always zero, since \(d\lambda_i^*/dw\), \(i = 1, 2,\) and \(dz^*/dw\) are both zero. Thus, in this case an international income transfer has no effect on the environmental quality or the shadow prices (and hence the resource allocations) not only in the steady state but at any other time. However, if an income transfer takes place from a country with a more efficient technology of the cleanup to a country whose technology is less efficient, i.e., \(\theta_1 > \theta_2\), \(d\lambda_i^*/dw\), \(i = 1, 2\) are positive and \(dz^*/dw\) is negative in (34) and (35). Thus, in this case we see, in equations (34) and (35), that \(d\lambda_i/dw\) \((t), i = 1, 2,\) approaches \(d\lambda_i^*/dw\) from below and \(dz/dw\) \((t)\) approaches \(dz^*/dw\) from above. Thus, the effect of the income transfer on the shadow prices is that at first its increase is not as much as in the steady state; however, the increase becomes greater as time passes and it stabilizes at its value in the long run steady state. The environmental quality, on the other hand, does not deteriorate as much at first, but deteriorates more with time and ends up deteriorating as much as the value given by (22) in the long run. In the case of income transfer from a country with a less efficient technology to a country whose technology is more efficient, just the opposite will occur.

IV. CONCLUDING REMARKS

The results of this paper generalize the nonneutrality result of an income transfer obtained by Yoshioka (1992b) in a dynamic setting. Contrary to Warr (1983), Kemp (1984), Bergstrom and Varian (1985) and Bergstrom, Blume and Varian (1985), the Nash equilibrium is not independent of an income transfer if there is a difference in the efficiency among individuals in providing a public good. Thus, an income transfer from an advanced country to a developing country to help clean up pollution might actually deteriorate the environmental quality and end up hurting not only the donor but also the recipient country.

However, the above result is obtained when the environmental quality enters into the utility functions of two countries as a public good. Thus, it is assumed
that the damage from pollution in one country always affects the welfare of the other country just as much. If, however, part of the damage from pollution is confined to the country of emission, an international income transfer may help the recipient country to some extent. Also, this paper deals with the case in which an income transfer takes place in the form of a transfer of resources. If, instead, the transfer is in the form of output of consumer goods, an international income transfer may have different implications on the resource allocations and the environmental quality.

The results obtained in this paper are for the open loop Nash equilibrium, in which both countries determine their respective resource allocations based on the initial level of the environmental quality. The closed loop Nash equilibrium, which is obtained when both countries determine their resource allocations at each moment of time based on the current and the past levels of the environmental quality, may be more efficient and thus the environmental quality may be higher. Although a solution for the closed loop Nash equilibrium is not obtained in this paper because of analytical difficulty, the implications of an international income transfer would probably be the same in such a solution.

REFERENCES

APPENDIX

Derivations of Equations (13) and (14).

Linearizing the system given by equations (10)'s and (12) around the equilibrium point, we have

\[
\begin{bmatrix}
\dot{\lambda}_1 \\
\dot{\lambda}_2 \\
z
\end{bmatrix} =
\begin{bmatrix}
\rho & 0 & (1-a_1)/z^*^2 \\
0 & \rho & (1-a_2)/z^*^2 \\
a_1/z^*^2 & a_2/z^*^2 & 0
\end{bmatrix}
\begin{bmatrix}
\lambda_1 - \lambda_1^* \\
\lambda_2 - \lambda_2^* \\
z - z^*
\end{bmatrix}
\]

(A1)

The characteristic roots of the Jacobian of the above system are:

\[
\rho, \quad \mu_1 = \frac{\rho + (\rho^2 - 4\Delta/\rho)^{1/2}}{2}, \quad \mu_2 = \frac{\rho - (\rho^2 - 4\Delta/\rho)^{1/2}}{2}
\]

where \( \Delta \) is the determinant of the Jacobian and is given by (19). Noting that the characteristic vector associated with \( \rho \) is equal to zero, the solution to the system given by (A1) is

\[
\lambda_1(t) = -\frac{1-a_1}{z^*^2(\rho - \mu_1)} \sigma_1 e^{\mu_1 t} - \frac{1-a_1}{z^*^2(\rho - \mu_2)} \sigma_2 e^{\mu_2 t} + \hat{\lambda}_1^*
\]

(A2)

\[
\lambda_2(t) = -\frac{1-a_2}{z^*^2(\rho - \mu_1)} \sigma_1 e^{\mu_1 t} - \frac{1-a_2}{z^*^2(\rho - \mu_2)} \sigma_2 e^{\mu_2 t} + \hat{\lambda}_2^*
\]

(A3)

\[
z(t) = \sigma_1 e^{\mu_1 t} + \sigma_2 e^{\mu_2 t} + z^*
\]

(A4)

However, if \( \sigma_1 \) is not zero, we have \( \lim_{t \to \infty} e^{-\rho t} \lambda_1(t) = \infty \), violating the transversality condition (11). Thus, \( \sigma_1 = 0 \). Then, from (A4) \( \sigma_2 \) is determined as \( z(0) - z^* \). Now, equations (A2), (A3) and (A4) become equations (13)'s and (14) in the text, with \( \mu = \mu_2 \).