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EFFECTS OF TRADE POLICY FOR INTERNATIONAL DUOPOLY†

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Abstract: The effects of trade policy are investigated for international duopoly with several markets. Deriving the relationship existing between the total outputs of the home and foreign firms, we analyze two cases, in one of which—Krugman's case—the home and foreign firms' marginal costs are both decreasing and in the other, the home firm's marginal cost is increasing but the foreign one is decreasing. Stability conditions are derived for an iterative scheme of computing the equilibrium total output and for the firm's output adjustment system with discrete time scale.

1. INTRODUCTION

Brander [1981] has first proved the possibility of international trade between countries producing identical goods. This pioneering work has led many economists to analyze the effects of strategic trade policy for international oligopoly without product differentiation. Krugman [1984] has shown the export-promoting effect of import protection for international duopoly where both the home and foreign firms are assumed to have decreasing marginal costs. He has derived this result on the basis of the reaction functions for the marginal costs, which are different from the reaction functions for outputs commonly used in analyzing oligopoly without international trade. His analysis, however, was predominantly diagrammatic. In Section 2 of this paper, we will present an alternative proof of Krugman's assertion. In so doing we will derive two trading

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countries' (or firms') reaction functions for the total outputs and make explicit all assumptions which are essential to deriving the assertion.

Our new proof, which is an adaptation of that of Okuguchi [1990] used in establishing the existence of a unique Cournot equilibrium with several regional markets, will be adapted in Section 3 to analyze the effects of trade policy for international duopoly where the home and foreign firms are assumed to have increasing and decreasing marginal costs, respectively. It will be shown that the home country's import protection necessarily leads to its export promotion and that the effects on the foreign country are ambiguous in the sense that its export to the home country may increase, decrease, or remain constant. In Section 4 we will analyze the stability of the Cournot equilibrium for international duopoly taken up in Section 3. Section 5 concludes.

2. DECREASING MARGINAL COSTS AND IMPORT PROTECTION

Let there be two firms, the home and foreign ones which are assumed to be producing identical goods under the condition of decreasing marginal costs. Both firms sell their outputs in n separate markets (countries), which may include the home and foreign ones. If their is no trade barrier,

(1–1) $$R^i = R^i(x_i, x_i^*)$$

and

(1–2) $$R^{*i} = R^{*i}(x_i, x_i^*)$$

are the home and foreign firms' revenues from the i-th market, respectively, where x_i and x_i^* are the home and foreign firms' supplies to the same market. We assume that R^i and R^{*i} satisfy the following conditions.

(2–1) $$R_1^i \geq 0\,, \quad R_{11}^i < 0\,, \quad R_{12}^i < 0\,,$$

(2–2) $$R_2^{*i} \geq 0\,, \quad R_{22}^{*i} < 0\,, \quad R_{21}^{*i} < 0\,,$$

(2–3) $$\Delta_i \equiv R_{11}^i R_{22}^{*i} - R_{12}^i R_{21}^{*i} > 0\,,$$

where $R_1^i \equiv \partial R^i/\partial x_i$, $R_{12}^i \equiv \partial^2 R^i/\partial x_i^* \partial x_i$, etc. for $i = 1, 2, \cdots, n$. If $p_i = p_i(x_i + x_i^*)$ is the inverse demand function in the i-th market, (2–1)–(2–3) hold provided that

(2–4) $$p_i' < 0\,, \quad p_i + x_i p_i' \geq 0\,, \quad p_i + x_i^* p_i' \geq 0\,, \quad p_i' + x_i p_i'' < 0\,, \quad p_i' + x_i^* p_i'' < 0\,.$$

The last two inequalities in (2–4) imply that the home and foreign firms' outputs are strategic substitutes in each market.

If transport costs are assumed away, the home and foreign firms' profits are defined by

(3–1) $$\pi \equiv \sum_i R^i(x_i, x_i^*) - C(X)\,, \quad i = 1, 2, \cdots, n\,,$$

(3–2) $$\pi^* \equiv \sum_i R^{*i}(x_i, x_i^*) - C^*(X^*), \quad i=1,2,\cdots,n,$$

where $X \equiv \sum_i x_i$ and $X^* \equiv \sum_i x_i^*$.

Following Krugman [1984], assume that

(4) $$C'>0, \quad C''<0, \quad C^{*\prime}>0, \quad C^{*\prime\prime}<0.$$

The home (foreign) firm is assumed to form expectations about the foreign (home) firm's supplies a la Cournot. The first-order conditions for profit maximization for the home and foreign firms are

(5–1) $$R_1^i(x_i, x_i^*) - C'(X) = 0, \quad i=1,2,\cdots,n,$$

and

(5–2) $$R_2^{*i}(x_i, x_i^*) - C^{*\prime}(X^*) = 0, \quad i=1,2,\cdots,n,$$

respectively. The second-order conditions are

(6–1) $$R_{11}^i - C''(X) < 0, \quad i=1,2,\cdots,n,$$

(6–2) $$R_{22}^{*i} - C^{*\prime\prime}(X^*) < 0, \quad i=1,2,\cdots,n.$$

Note that the assumptions (2–1), (2–2) and (4) are not sufficient for (6–1) and (6–2). Solving (5–1) and (5–2) with respect to x_i and x_i^*, we get

(7–1) $$x_i = \varphi^i(X, X^*), \quad i=1,2,\cdots,n,$$

where

(7–2) $$\varphi_1^i \equiv \partial\varphi^i/\partial X = C'' R_{22}^{*i}/\Delta_i > 0, \quad \varphi_2^i \equiv \partial\varphi^i/\partial X^* = -C^{*\prime\prime} R_{12}^i/\Delta_i < 0,$$

and

(7–3) $$x_i^* = \varphi^{*i}(X, X^*), \quad i=1,2,\cdots,n,$$

where

(7–4) $$\varphi_1^{*i} \equiv \partial\varphi^{*i}/\partial X = -C'' R_{21}^{*i}/\Delta_i < 0, \quad \varphi_2^{*i} \equiv \partial\varphi^{*i}/\partial X^* = C^{*\prime\prime} R_{11}^i/\Delta_i > 0.$$

The Cournot equilibrium total outputs for the two firms are solutions of

(8–1) $$X = \varphi(X, X^*) \equiv \sum_i \varphi^i(X, X^*)$$

and

(8–2) $$X^* = \varphi^*(X, X^*) \equiv \sum_i \varphi^{*i}(X, X^*).$$

where (7–2) and (7–4) yield

(9–1) $$\varphi_1 > 0, \quad \varphi_2 < 0,$$

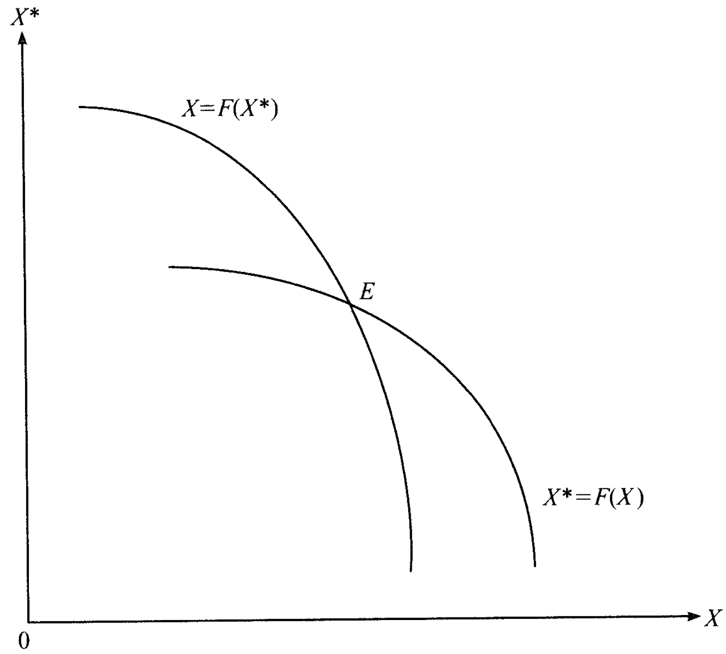

Fig. 1. Cournot equilibrium total output.

(9–2) $$\varphi_1^* < 0\,, \quad \varphi_2^* > 0\,.$$

Making use of (8–1) and (8–2), we can express X as function of X^*, namely

(10–1) $$X = F(X^*)\,, \quad F' < 0\,.$$

Similarly, (8–2) and (9–2) lead to

(10–2) $$X^* = F^*(X)\,, \quad F^{*\prime} < 0\,.$$

We now introduce the following assumptions.

(11–1) $$\varphi_1 < 1\,, \quad \varphi_2^* < 1\,,$$

(11–2) $$(1-\varphi_1)(1-\varphi_2^*) - \varphi_2\varphi_1^* > 0\,.$$

Under (11), we can assert that equations (8–1) and (8–2) or (10–1) and (10–2) have a unique solution in the light of Gale–Nikaido [1965]'s global univalence theorem, as shown by the intersection E of two solid downward-sloping curves in Fig. 1.

We are now in a position to analyze the effects of the home country's imposition of tariff on the foreign firm's export to the home market. Let market 1 be the home market, and let α be a parameter relevant to the home country's tariff. The foreign firm's profits from the home market are now given by

(12) $$\pi^{*1} \equiv R^{*1}(x_1, x_1^*, \alpha) - C^*(X^*)\,,$$

where

(13) $$\partial R^{*1}/\partial\alpha \equiv R_\alpha^{*1} < 0\,, \quad \partial R_2^{*1}/\partial\alpha < 0\,.$$

The first-order conditions for the home firm's profit maximization in all markets and those for the foreign firm in markets other than the home market are given by (5–1) and (5–2) as before. However, the foreign firm's profits from the home market are maximized if

(14) $$R_2^{*1}(x_1, x_1^*, \alpha) - C^{*\prime} = 0$$

is satisfied. Solving (5–1) for $i=1$ and (14), we have

(15–1) $$x_1 \equiv \varphi^1(X, X^*, \alpha), \quad \varphi_1^1 > 0, \quad \varphi_2^1 < 0, \quad \varphi_\alpha^1 > 0,$$

(15–2) $$x_1^* \equiv \varphi^{*1}(X, X^*, \alpha), \quad \varphi_1^{*1} < 0, \quad \varphi_2^{*1} > 0, \quad \varphi_\alpha^{*1} < 0.$$

Equations (7–1) and (7–3) remain valid for all i other than 1. Hence the Cournot equilibrium total outputs are identical to the solution of (16–1) and (16–2) as below.

(16–1) $$X = \varphi^1(X, X^*, \alpha) + \sum_{i \neq 1} \varphi^i(X, X^*) \equiv \varphi(X, X^*, \alpha), \quad \varphi_1 > 0, \quad \varphi_2 < 0, \quad \varphi_\alpha > 0,$$

(16–2) $$X^* = \varphi^{*1}(X, X^*, \alpha) + \sum_{i \neq 1} \varphi^{*i}(X, X^*) \equiv \varphi^*(X, X^*, \alpha), \quad \varphi_1^* < 0, \quad \varphi_2^* > 0, \quad \varphi_\alpha^* < 0.$$

Solving (16–1) and (16–2) with respect to X and X^*, respectively, we get

(17–1) $$X \equiv F(X^*, \alpha), \quad F_{X^*} < 0, \quad F_\alpha > 0,$$

(17–2) $$X^* \equiv F^*(X, \alpha), \quad F_X^* < 0, \quad F_\alpha^* < 0.$$

Assuming (11–1) and (11–2), we can solve (16–1) and (16–2), or (17–1) and (17–2) for an arbitrary given α. Let E_1 in Fig. 2 be the solution for a given α. If α increases, the curve for (17–1) shifts upward, while that for (17–2) shifts downward, as shown by the dotted curves in Fig. 2. Hence E_1 moves to E_2. This means that the home country's total output increases and that for the foreign country decreases, in the event of an increase in the home country's tariff rate. Furthermore we can assert with the help of (7–1), (7–2), (7–3), (7–4), (16–1) and (16–2) that

(18) $$dx_i/d\alpha > 0, \quad dx_i^*/d\alpha < 0, \quad i = 1, 2, \cdots, n.$$

The effects of the home country's subsidy to its export may be similarly analyzed. All we need is to let

(19) $$R^1 \equiv R^1(x, x^*, \beta), \quad \partial R^1/\partial\beta > 0, \quad \partial R_1^1/\partial\beta > 0,$$

where β is the parameter relevant to the subsidy.

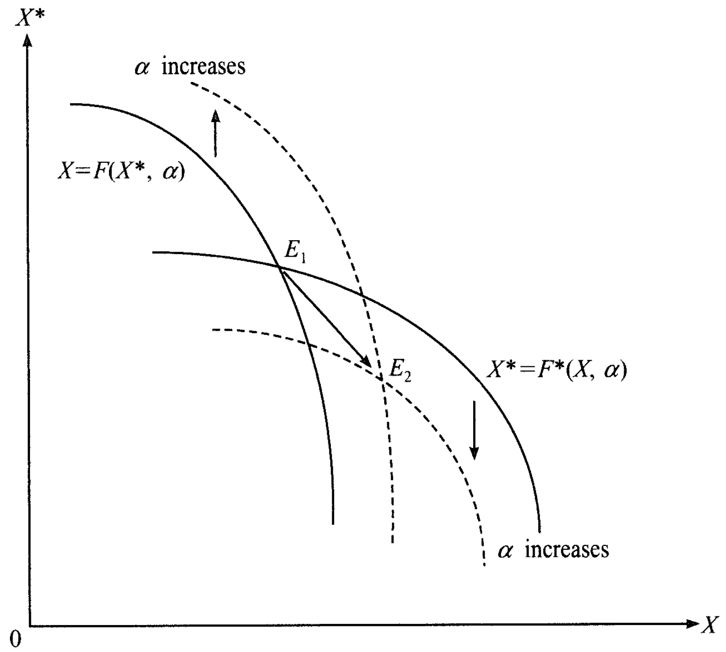

Fig. 2. Effects of trade policy when the marginal costs are decreasing.

3. ASYMMETRIC MARGINAL COSTS

In this section we will analyze the effects of trade policy for international duopoly where the home and foreign firms are assumed to have increasing and decreasing marginal costs, respectively. Hence,

(20) $$C''>0\,, \quad C^{*\prime\prime}<0\,.$$

If α is the parameter relevant to the home country's tariff, similar arguments as in Section 2 yield in the light of (20),

(21–1) $$\varphi_1^i<0\,, \quad \varphi_2^i<0\,, \quad \varphi_\alpha^1>0\,, \quad i=1, 2, \cdots, n\,,$$

(21–2) $$\varphi_1^{*i}>0\,, \quad \varphi_2^{*i}>0\,, \quad \varphi_\alpha^{*1}<0\,, \quad i=1, 2, \cdots, n\,.$$

The Cournot equilibrium total outputs are identical to the solution of

(22–1) $$X=\varphi(X, X^*, \alpha)\,,$$

(22–2) $$X^*=\varphi^*(X, X^*, \alpha)\,,$$

or of

(23–1) $$X=F(X^*, \alpha)\,,$$

(23–2) $$X^*=F^*(X, \alpha)\,,$$

where φ, φ^*, F and F^* are defined as in Section 2. However, in view of (21–1) and (21–2), the partial derivatives of (23–1) and (23–2) have the following signs.

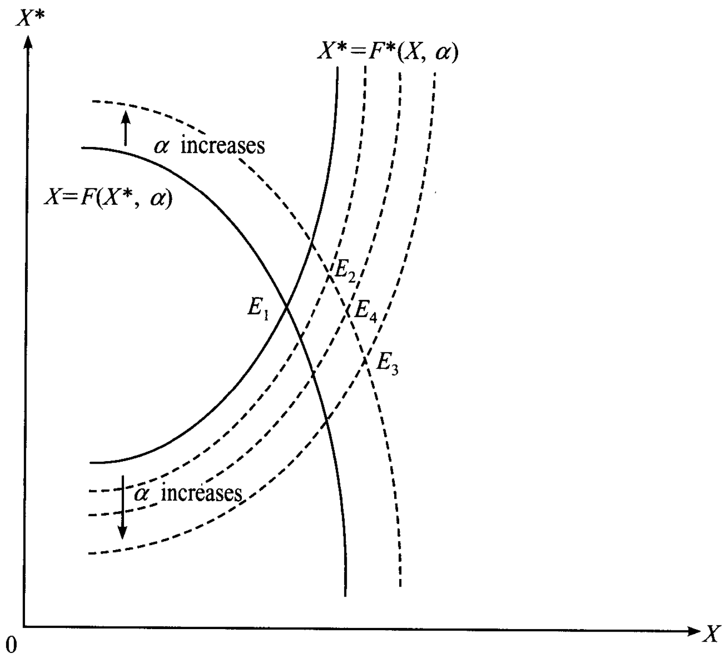

Fig. 3. Asymmetric marginal costs and the effects of trade policy.

(24–1) $$F_{X^*}<0\,, \quad F_\alpha>0\,,$$

(24–2) $$F^*_X>0\,, \quad F^*_\alpha<0\,,$$

where we have to assume

(25) $$1-\varphi^*_2>0\,.$$

Since $\varphi_1<0$, $1-\varphi_1>0$ holds necessarily.

Given α, the equilibrium total outputs are given by the intersection E_1 of the downward-sloping curves for (23–1) and (23–2), respectively, as in Fig. 3. If α increases, the downward-sloping curve shifts upward and the upward-sloping curve shifts downward. The home country's total output increases unambiguously with an increase in α. However, the foreign country's total output may increase, decrease, or remain constant if the new intersection moves to E_2, E_3, or E_4, respectively.

4. STABILITY CONDITIONS

In this section we consider two types of stability conditions. First, let

(26) $$\begin{aligned} X(t+1)&=\varphi(X(t), X^*(t))\,, \\ X^*(t+1)&=\varphi^*(X(t), X^*(t)) \end{aligned}$$

be an *iterative scheme* for computing the Cournot equilibrium total outputs for the home and foreign firms, respectively, where t refers to discrete time.

Adopting the maximum norm for vectors, we can prove that the mapping in (24) is contractive if

$$|\varphi_1|+|\varphi_2|<1\,, \tag{27–1}$$

$$|\varphi_1^*|+|\varphi_2^*|<1 \tag{27–2}$$

hold. Taking into account (7–2), (7–4), (9–1) and (9–2), we rewrite (27–1) and (27–2) as

$$C''\sum_i R_{22}^{*i}/\Delta_i+C^{*\prime\prime}\sum_i R_{12}^i/\Delta_i<1\,, \tag{28–1}$$

$$C^{*\prime\prime}\sum_i R_{11}^i/\Delta_i+C''\sum_i R_{21}^{*i}/\Delta_i<1\,, \tag{28–2}$$

respectively, in the case of $C''<0$ and $C^{*\prime\prime}<0$, and

$$-C''\sum_i R_{22}^{*i}/\Delta_i+C^{*\prime\prime}\sum_i R_{12}^{*}/\Delta_i<1\,, \tag{29–1}$$

$$C^{*\prime\prime}\sum_i R_{11}^i/\Delta_i-C''\sum_i R_{21}^{*i}/\Delta_i<1\,, \tag{29–2}$$

respectively, if $C''>0$ and $C^{*\prime\prime}<0$.

We next consider the following *behavioristic equations* for output adjustments.

$$dx_i/dt=k_i\partial\pi/\partial x_i\,,\quad k_i>0\,,\quad i=1,2,\cdots,n\,, \tag{30–1}$$

$$dx_i^*/dt=k_i^*\partial\pi^*/\partial x_i^*\,,\quad k_i^*>0\,,\quad i=1,2,\cdots,n\,. \tag{30–2}$$

If the Jacobian matrix for the right-hand side of (30–1) and (30–2) is negative dominant diagonal, the Cournot equilibrium is globally stable. The diagonal elements of the Jacobian matrix all being negative by the second-order conditions for profit maximization, the Jacobian matrix is negative dominant diagonal if

$$-(R_{11}^i-C'')>(n-1)|C''|+|R_{12}^i|\,,\quad i=1,2,\cdots,n\,, \tag{31–1}$$

$$-(R_{22}^{*i}-C^{*\prime\prime})>(n-1)|C^{*\prime\prime}|+|R_{21}^{*i}|\,,\quad i=1,2,\cdots,n\,. \tag{31–2}$$

Needless to say, the stability condition consisting of (31–1) and (31–2) is different from that consisting of (28–1) and (28–2) or (29–1) and (29–2).

We now consider what is implied by (31–1) and (31–2) for $n=1$ or 2. By (2–1) and (2–2), $R_{12}^i<0$ and $R_{21}^{*i}<0$. If $n=1$, then (31–1) and (31–2) hold when

$$p'<C''\,, \tag{32–1}$$

$$p'<C^{*\prime\prime} \tag{32–2}$$

hold, respectively. These are the inequalities which we frequently use when we analyze the existence and stability of the Cournot oligopoly equilibrium in an closed economy (see Okuguchi [1976], and Okuguchi and Szidarovszky [1990]). Next let $n=2$. Then, if $C''<0$ and $C^{*\prime\prime}<0$, (31–1) and (31–2) are satisfied provided

(33–1) $$2C'' > p',$$

and

(33–2) $$2C^{*''} > p'$$

hold, respectively. If, on the other hand, $C'' > 0$ and $C^{*''} < 0$, (31–1) holds necessarily. However, (31–2) holds if (33–2) is true.

5. CONCLUSION

In this paper we have analyzed the effects of trade policy for international duopoly without product differentiation and with several markets, which may include the home and foreign ones. In Section 2, we have revisited the problem analyzed by Krugman from an entirely different analytical point of view. The new analytical method, which utilizes the relationships between the total outputs of the home and foreign firms, has enabled us to analyze in Section 3 the effects of trade policy for the case where the home and foreign firms' costs are assumed to be increasing and decreasing, respectively.

In Section 4 we have derived stability conditions for an *iterative scheme* for computing the equilibrium total outputs and for a firms' *behavioristic output adjustment* system with discrete time scale. The stability conditions for the iterative scheme and that for the output adjustment system have been found to be different. We have also considered the economic implications of the stability conditions for the output adjustment system for the case where the number of market is one and two.

Finally, we note that our analytical method using the relationships between the home and foreign firms' total outputs can be easily applied to analyze the remaining case where the home and foreign firms' marginal costs are both increasing and the one where the home firm's marginal cost decreases but the foreign firm's marginal cost increases.

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