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ENDOGENOUS LABOR SUPPLY AND CAPACITY UTILIZATION RATE IN REAL BUSINESS CYCLE MODELS: A RECONSIDERATION OF TIME SEPARABLE PREFERENCES

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Abstract: In Real Business Cycle models, time-separable preferences fail to explain procyclicalities of major economic variables observed in many countries. Performance of time-dependent preferences is no better under diminishing returns technology, because it relies on the Intertemporal Substitution of leisure that has been rejected by empirical studies. The comparative static analysis tells that allowing capacity utilization rate to be determined endogenously makes Real Business Cycle models exhibit the procyclicalities even if preferences are time-separable.

1. INTRODUCTION

Business fluctuations in many countries are featured by persistent shocks and procyclicalities where output, consumption, investment, employment (manhour labor force) and labor productivity move in the same direction.[1] Real Business Cycle (RBC) models are a focus of attention in the recent studies of business cycles. So far, however, none of these is satisfactory in generating the procyclicalities of employment and labor productivity. They regard the economy always in the competitive equilibrium, and fluctuations caused by exogenous shocks to technology. On this view, variations in employment crucially depend on the intertemporal substitution (IS) of consumption and leisure. It is for this reason that much attention has been paid to the problem of consumers' preferences in RBCs. Time-separable utility function, frequently used for simplicity, does not generate the comovements as is shown by Barro and King [5] (BK).

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[1] Long and Plosser [21] and Yoshikawa [33] studies persistent production shocks in the United States and Japan, respectively. Kydland and Prescott [19] explains the comovements for the United States.

The recent RBC models begin to employ the time-dependent utility function. It may generate the observed fluctuations in employment, but not in labor productivity as long as labor is the only short-run variable production factor. In addition, not a few empirical studies on the labor market are unfavorable to the IS hypothesis.[2] This paper examines by comparative static analysis the relationships between consumers' preferences and procyclicalities, suggesting that introduction of variations in capacity utilization rate largely removes difficulties with RBCs in generating the observed comovements.

Following BK, the present paper attempts to rehabilitate the RBC theory, by examining effects of technology shocks, first with time-separable utility function, and then with time-dependent utility function. Main conclusions of this paper are as follows: (i) With minor extensions, BK's result is confirmed that the time-separable utility function does not explain the comovements. (ii) Allowing capacity utilization rate to be determined endogenously, the competitive equilibrium model exhibits the comovements even with time-separable utility function. Hercowitz [14] examined the effect of such an endogenous capacity utilization rate, with utility function having no income effect on leisure and production lags. The present paper differs from it in supposing no production lags and positive income effect on leisure. (iii) If the current leisure is substitute for future consumption and leisure, time-dependent preferences may generate the comovements of employment, as in Lucas and Rapping [23].

The paper is organized in the following way. The general form of the problem and assumptions are presented in section 2. Section 3 carries out comparative static analysis, first with time-separable utility function. Procyclical movements of employment and labor productivity are hardly explained under the assumptions of superior-good leisure and diminishing returns technology. Then, variable capacity utilization rate is introduced to show that it generates the desired dynamics. Some evidence on the Japanese economy concerning the variability of capacity utilization rate is also shown. Section 4 examines time-dependent utility function, followed by section 5 that concludes the paper with some comments on the problem concerning employment.

2. THE GENERAL FORM OF THE PROBLEM

This paper takes the RBCs' approach and examines the dynamics of the economy as the solution to a central planner's problem. The general form of his problem is given by (1).

[2] Altonji [2] and Mankiw, Rotemberg and Summers [27] reject the hypothesis on the US labor market. Kennan [18] finds strong persistency in employment of OECD countries to cast a doubt on RBC equilibrium approaches.

$$\max_{\{c_t\}\{l_t\}\{h_t\}\{k_{k+1}\}} W=\sum_{t=0}^{\infty} \beta^t U(c_t, z_t)$$

subject to $k_{t+1}=G(c_t, l_t, h_t, k_t; x_t)$

$$\text{where} \quad z_t=\sum_{i=0}^{p} \alpha_i l_{t-i}, \alpha_0=1$$

$$k_0 \text{ and } l_{-i} \ (i=0, 1, \cdots, p) \text{ are given .} \tag{1}$$

Decision variables are c_t, l_t, h_t and k_{t+1}, while k_t, l_{t-i} $(i=1, \cdots, n)$ and x_t are state variables in period t. c, l, h, k and x are called consumption, leisure, capacity utilization rate, capital stock and technology shift parameter, respectively. The instantaneous utility depends on a linear combination of current and past leisure as in Ryder and Heal [30] and Kydland and Prescott [19], and is called time-separable when $p=0$. h is treated separately from labor as a production factor since it is not included in the utility function.[3]

The following assumptions are made throughout the paper.

(A1) Differentiability and Concavity:
The instantaneous objective function $U(\cdot)$ is concave and twice continuously differentiable. $G(\cdot)$ is also twice continuously differentiable in l, h and k. It is concave for $-l$ and k, and concave-convex for h.

(A2) Boundary:
$U(\cdot)$ and $G(\cdot)$ satisfy the following Inada conditions.

$$\lim_{i\to\infty} U_i=0 \qquad \lim_{i\to 0} U_i=\infty$$
$$\lim_{s\to\infty} G_s=0 \qquad \lim_{s\to 0} G_s=\infty$$
$$\text{where} \quad i=c, z, \quad s=n, h, k \quad \text{and} \quad n=-l$$

(A3) Discount factor:
β satisfies $\beta\in(0, 1)$.

(A4) Time-separable Hicks-neutral technology shock:
The technology shift parameter x enters production function multiplicatively and

[3] The corresponding consumers' problem is

$$\max_{\{c_t\}\{l_t\}\{k_{k+1}\}} \tilde{W}=\sum_{t=0}^{\infty} \beta^t U(c_t, z_t)$$

subject to $k_{t+1}=\tilde{G}(c_t, l_t; w_t, R_t, k_t)$

$$\text{where} \quad z_t=\sum_{i=0}^{p} \alpha_i l_{t-i}, \alpha_0=1$$

$$k_0 \text{ and } l_{-i} \ (i=0, 1, \cdots, p) \text{ are given} \tag{2}$$

where w and R are factor incomes of labor and capital. The number of decision variables for consumers is different from that of central planner due to the presence of endogenous capacity utilization h.

its exogenous change, namely technology shocks, *only concerns with variables having the same time subscript*. This temporary innovation has no serial correlation, increasing productivities in the Hicksian neutral way. The objective utility function is free from any shocks.

(A5) Superior goods:
All goods included in the utility function are superior so that wealth effect on demand is positive for all of these goods.

Under (A1)–(A3), the unique inner solution is guaranteed (Lucas = Stokey [24]). With (A4), shocks to preferences are excluded from the present analysis.[4] We are to perform analyses in a deterministic way so that expectations are not included in (1). (A5), obtained from cross-sectional empirical studies, is used in almost all models of endogenous labor supply. In addition to the assumption of diminishing returns technology (A1), it plays a crucial role.

The first-order conditions for the central planner are given below where λ is the current-value Lagrange multiplier for the feasibility constraint.

$$U_{it}(c_t, z_t) - \lambda_t G_{it}(c_t, n_t, h_t, k_t; x_t) = 0$$

$$-\lambda_t + G_{kt+1}(c_t, n_t, k_t; x_t)\beta\lambda_{t+1} = 0$$

$$G_{ht}(c_t, n_t, h_t, k_t; x_t) = 0$$

$$G(c_t, n_t, h_t, k_t; x_t) - k_{t+1} = 0 \quad \forall t$$

$$\text{where} \quad i = c, l, \quad \text{and} \quad n = -l.$$

As we deal with short run effects of shocks observed or anticipated at the beginning of period t, we concentrate on the interactions of variables in the adjacent periods t and $t+1$, holding future variables constant.[5]

Differentiating the above equations totally while holding k_t and k_{t+j} $(j = 2, 3, \cdots)$ constant gives the following dynamics.

$$\begin{bmatrix} A & B \\ B' & V \end{bmatrix} \begin{bmatrix} dY_t \\ dY_{t+1} \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} dX_t \\ dX_{t+1} \end{bmatrix} \tag{3}$$

dY_{t+j} $(j = 0, 1)$ is a vector of decision variables having the time subscript $t+j$. (The stock variable k_{t+1} chosen at period t is included in dY_{t+1}.) Matrix A is $n \times n$ square where n is the number of decision variables plus the number of constraints having the time-subscript t. It is the static Hessian matrix for the central planner. Matrix V is $(n+s) \times (n+s)$ square, being composed of matrix A

[4] This assumption does not rule out labor-augmenting (or Harrod-neutral) permanent technological changes, which the stylized facts of economic growth suggest. (Phelps [29] and Swan [32]). We can take care of such permanent technological changes, having all the affected variables either divided by some trend level index of technology, or detrended linearly as in Kydland, Prescott, and Rebello [16].

[5] To be precise, a change in x_{t+1} affects the value of k_{t+2}. The logic of the proofs and conclusions, however, remain the same even if we allow for the effects of changes in x_{t+1} on the value of k_{t+2}.

and a border. s is the number of stock variables that are chosen in period t and become state variables in period $t+1$. Matrix B, $n \times (n+s)$ rectangular, determines the intertemporal dynamics of the system. Its non zero elements pertain to the agents' access to the capital market as well as time-dependency of preferences. dX_{t+j} $(j=0, 1)$ is a vector of exogenous shocks having the time subscript $t+j$. Assumption (A4) of the unique source of shocks requires that dX_{t+j} filled with the same elements. X_1 and X_2 are shock-transmitting matrices.

Comparative statics of current and expected future shocks are[6]

$$\begin{bmatrix} \dfrac{dY_t}{dX_t} \\ \dfrac{dY_{t+1}}{dX_t} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Q_{11}X_1 \\ Q_{21}X_1 \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} \dfrac{dY_t}{dX_{t+1}} \\ \dfrac{dY_{t+1}}{dX_{t+1}} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} 0 \\ X_2 \end{bmatrix} = \begin{bmatrix} Q_{12}X_2 \\ Q_{22}X_2 \end{bmatrix}, \tag{5}$$

$$\text{where} \quad Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ B' & V \end{bmatrix}^{-1}$$

$$\begin{aligned} \text{and} \quad Q_{11} &= A^{-1} + A^{-1}B(V - B'A^{-1}B)^{-1}B'A^{-1} \\ Q_{12} &= -A^{-1}B(V - B'A^{-1}B)^{-1} \\ Q_{21} &= -(V - B'A^{-1}B)^{-1}B'A^{-1} \\ Q_{22} &= (V - B'A^{-1}B)^{-1}. \end{aligned}$$

Thus,

$$\begin{aligned} \left[\frac{dY_t}{dX_{t+1}} \right] &= -A^{-1}BQ_{22}X_2 \\ &= -A^{-1}B \left[\frac{dY_{t+1}}{dX_{t+1}} \right], \end{aligned} \tag{6}$$

$$\begin{aligned} \left[\frac{dY_t}{dX_t} \right] &= A^{-1}X_1 - A^{-1}BQ_{21}X_1 \\ &= A^{-1}X_1 - A^{-1}B \left[\frac{dY_{t+1}}{dX_t} \right]. \end{aligned} \tag{7}$$

Responses of endogenous variables depend on time structure of three factors: i.e. preferences, feasibility constraint and exogenous shocks. Similar to the Slutsky's equation for consumer's behavior, the effect of a current shock in (7) is decomposed

[6] See Magnus and Neudecker [25] p. 11 for matrix calculations.

into two parts; the first term indicates intra-temporal substitutions, the total wealth as well as future leisure held constant, and the second term, the intertemporal substitutions (IS). The latter is further decomposed into the net wealth effect and the tradeoff with future leisure. If matrix B equals zero and agents have no means to save under time-separable utility, the total effect is only intra-temporal.

3. TIME-SEPARABLE PREFERENCES

3.1. *Investment with Constant Utilization and Depreciations*

When agents have access to investment opportunity with time-independent preferences, the central planner's problem is

$$\max_{\{c_t\}\{l_t\}\{k_{t+1}\}} \quad W = \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

$$\text{subject to} \quad k_{t+1} = F(n_t, k_t; x_t) + (1-\delta)k_t - c_t, \quad n_t = -l_t$$

$$k_0 \text{ is given}. \tag{8}$$

The future shock affects the current decision variables only through the capital market because of the special structure of matrix B shown in appendix, which means that the shock is transmitted as changes in the wealth for consumers. Since the intertemporal effect is equivalent to the net wealth effect, we obtain the following proposition.

PROPOSITION 1 (Barro and King [5]). *In problem* (8) *under* (A1)–(A5), *consumption, investment, employment, output and labor productivity do not move all in the same direction in response to the anticipated future technology shock such that*

$$\operatorname{sgn}\left(\frac{dc_t}{dx_{t+1}}\right) \neq \operatorname{sgn}\left(\frac{dk_{t+1}}{dx_{t+1}}\right), \qquad \operatorname{sgn}\left(\frac{dc_t}{dx_{t+1}}\right) \neq \operatorname{sgn}\left(\frac{dn_t}{dx_{t+1}}\right),$$
$$\operatorname{sgn}\left(\frac{dc_t}{dx_{t+1}}\right) \neq \operatorname{sgn}\left(\frac{dF_t}{dx_{t+1}}\right), \qquad \operatorname{sgn}\left(\frac{dc_t}{dx_{t+1}}\right) = \operatorname{sgn}\left(\frac{dF_{nt}}{dx_{t+1}}\right) \tag{9}$$

where F_n is the marginal labor productivity (MPL).

PROOF. Given in appendix.

When consumption and leisure are superior, they must move in the same direction. The current investment, on the other hand, moves in the opposite because a future increase in income stimulates current consumption and discourages present savings. These are inconsistent with the comovements observed.

The foregoing analysis gives us some different implications obtained in BK where an increase in initial wealth is shown to decrease output. As the future production shocks increase the anticipated lifetime wealth of consumers as well as the anticipated gains from investment on the part of producers, the latter effects on the investment demand should be integrated here by carrying out directly the comparative statics of the future production shocks. In BK's notation of Eq. (15) p. 830,

$$Y_t = C^d(B_t, r_t, \alpha_{t+1}) + I^d(B_t, r_t, \alpha_{t+1}) = Y^s(B_t, r_t, \alpha_{t+1})$$

where B, r and α are lifetime wealth, interest rate and the technology level, respectively. Similar to BK's method, the effects of anticipated shocks can be decomposed into three parts: the direct effects of technology shocks, the indirect effects through changes in the interest rate, and the indirect effects via changes in the lifetime wealth.

$$\frac{dY_t}{d\alpha_{t+1}} = \left(\frac{\partial C_t^d}{\partial \alpha_{t+1}} + \frac{\partial I_t^d}{\partial \alpha_{t+1}}\right) + \left(\frac{\partial C_t^d}{\partial r_t} + \frac{\partial I_t^d}{\partial r_t}\right)\frac{dr_t}{d\alpha_{t+1}} + \left(\frac{\partial C_t^d}{\partial B_t} + \frac{\partial I_t^d}{\partial B_t}\right)\frac{dB_t}{d\alpha_{t+1}}.$$

The above equation is different from BK in the sense that it considers the effects of future technology shock on consumption and investment as well as the effects of lifetime wealth on investment. The latter two effects shift the investment demand schedule upward. By inserting the following cross-conditions obtained from the analysis of consumer behavior in BK's Eq. (4) on p. 820

$$\frac{\partial C_t^d}{\partial B_t}\frac{\partial Y_t^s}{\partial r_t} = \frac{\partial C_t^d}{\partial r_t}\frac{\partial Y_t^s}{\partial B_t}, \qquad \frac{\partial C_t^d}{\partial r_t}\frac{\partial Y_t^s}{\partial \alpha_{t+1}} = \frac{\partial C_t^d}{\partial \alpha_{t+1}}\frac{\partial Y_t^s}{\partial r_t},$$

the following inequality may hold if the first term in the square bracket is sufficiently large.

$$\frac{dY_t}{d\alpha_{t+1}} = \Delta^{-1}\left[\left(\frac{\partial I_t^d}{\partial B_t}\frac{\partial B_t}{\partial \alpha_{t+1}} + \frac{\partial I_t^d}{\partial \alpha_{t+1}}\right)\frac{\partial Y_t^s}{\partial r_t} - \left(\frac{\partial Y_t^s}{\partial B_t}\frac{\partial B_t}{\partial \alpha_{t+1}} + \frac{\partial Y_t^s}{\partial \alpha_{t+1}}\right)\frac{\partial I_t^d}{\partial r_t}\right] > 0$$

where

$$\Delta = \left(\frac{\partial Y_t^s}{\partial r_t} - \frac{\partial C_t^d}{\partial r_t} - \frac{\partial I_t^d}{\partial r_t}\right) > 0. \tag{10}$$

On the other hand, a current shock generates the procyclicality if substitution effects on leisure dominate income effects, *and if* the exogenous increase in labor productivity overwhelms its endogenous decrease due to diminishing returns technology. According to (7),

$$\begin{bmatrix} dc_t/dx_t \\ dl_t/dx_t \\ d\lambda_t/dx_t \end{bmatrix} = A^{-1}X_1 - (dk_{t+1}/dx_t)A^{-1}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}. \tag{11}$$

The first and second terms represent the intra-temporal and net wealth effects, respectively. As is well known, the employment (negative leisure) increases if the substitution effect dominates. The current investment rises, reflecting agents' behavior to make their consumption path smooth.

$$\operatorname{sgn}\left(\frac{dc_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dn_t}{dx_t}\right), \qquad \operatorname{sgn}\left(\frac{dc_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dF_t}{dx_t}\right),$$
$$\operatorname{sgn}\left(\frac{dc_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dF_{nt}}{dx_t}\right), \qquad \operatorname{sgn}\left(\frac{dc_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dk_{t+1}}{dx_t}\right),$$
$$\text{if substitution effect dominates}\,. \tag{12}$$

This basic model seems to generate the comovements, at first glance. However, a Hicks-neutral shock has a large income effect.[7] Either a rise in labor productivity (real wage) or the IS elasticity must be so large as to let people work more. Empirical studies on labor supply, however, show low elasticities.[8] Fluctuations in real wage are relatively small compared to manhour employment.

As is mentioned in the introduction, many countries experience a strong positive serial correlation in output. From the stand point of RBC models, there should be a strong positive serial correlation in exogenous technology shocks. The argument so far, however, tells us that they offset each other, pressing the current employment and investment to move in the opposite directions.

3.2. *Investment with Endogenous Capacity Utilization and Depreciation Rates*

In recent studies on investment and labor demand, endogenous utilization and depreciation rates have received much attention.[9] The current output depends on the flow of capital service which is the product of utilization rate h $(0 \leq h \leq 1)$ and the physical amount of existing capital. An increase in utilization rate accelerates the depreciations.[10] The central planner's problem in this case is given by

$$\max_{\{c_t\}\{l_t\}\{h_t\}\{k_{t+1}\}} \quad W=\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$
$$\text{subject to} \quad k_{t+1}=F(n_t, h_t, k_t; x_t)+[1-\delta(h_t)]k_t-c_t\,, \quad n_t=-l_t$$
$$k_0 \text{ is given}\,. \tag{13}$$

Although the time structure is essentially the same as in the previous model, the wealth effect of central planner is different from households'. This leads to the following proposition.

PROPOSITION 2. *In problem* (13) *under* (A1)–(A5), *the anticipated future shock generates the procyclicalities of employment, labor productivity, capacity utilization rate and investment. If the complementarity between capital service and labor is sufficiently large, the current consumption also behaves procyclically.*

[7] Note that it simultaneously raises the real wage and dividends paid to households.

[8] See Altonji [2] and Mankiw [26].

[9] Abel [1], Merrick [28], Calvo [6], Hercowitz [14], and Greenwood, Hercowitz and Huffman [11]. Kydland [20] recently extends their original model by introducing capacity utilization rate.

[10] BK denies the effectiveness of variable utilization with constant depreciation rate.

$$\operatorname{sgn}\left(\frac{dn_t}{dx_{t+1}}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_{t+1}}\right), \quad \operatorname{sgn}\left(\frac{dF_t}{dx_{t+1}}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_{t+1}}\right),$$
$$\operatorname{sgn}\left(\frac{dF_{nt}}{dx_{t+1}}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_{t+1}}\right), \quad \operatorname{sgn}\left(\frac{dk_{t+1}}{dx_{t+1}}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_{t+1}}\right).$$
$$\text{if} \quad F_{nk}>0 \tag{14}$$

PROOF. Given in appendix.
The effects of current shocks are

$$\operatorname{sgn}\left(\frac{dc_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_t}\right), \quad \operatorname{sgn}\left(\frac{dn_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_t}\right)$$
$$\text{if substitution effect dominates}.$$
$$\operatorname{sgn}\left(\frac{dF_t}{dx_t}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_t}\right), \quad \operatorname{sgn}\left(\frac{dF_{nt}}{dx_t}\right)=\operatorname{sgn}\left(\frac{dh_t}{dx_t}\right)$$
$$\text{if} \quad F_{nh}+F_{nn}>0\,. \tag{15}$$

The utilization rate has three effects. First, it raises MPL which equals the real wage to stimulate labor supply when $F_{kn}>0$. This condition always holds if capital and labor are complement in production.[11] Secondly, it increases current output. Thirdly, it accelerates depreciation and raises future MPK to promote replacement investment. Some people may say that any other production factor such as materials plays the same role unless it does not affect utility. An increase in the current amount of materials, however, does not affect future MPK.

A rise in future income stimulates current consumption, while the increased future dividend depresses it, stimulating savings. When the former effect is sufficiently large, the current consumption goes up. A current shock increases current consumption, investment, and employment if intra-period substitution effects on leisure dominate income effect. *When shocks are positively serially correlated, current and future shocks reinforce each other to generate greater and persistent comovements.*

The present settings are different from Greenwood, Hercowitz and Huffman [11] where the shock dx_{t+1} directly affects both current marginal product of h_t and future MPK. In addition to it, they employ production lags and the utility function having no wealth effect on demand for leisure. The proposition states that without production lags introduction of variable capacity utilization rate help RBC models generating the procyclicalities even if consumers have time-separable preferences showing no wealth effect on leisure. Figure 1 shows cross correlations of growth rates between major economic variables. They show that the capital

[11] As the capital service is the product of utilization rate and the amount of capital stock, F_{hn} and F_{kn} have the same sign.

service is as strongly procyclical as consumption and employment.[12]

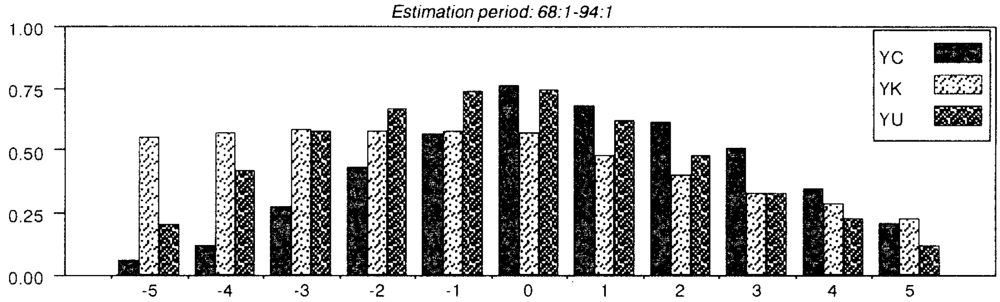

Fig. 1a. Cross correlation between GNP and major economic variables.
Note: YC = cross correlations between GNP and private consumption
YK = c.c. between GNP and private capital stock (on the installation base)
YU = c.c. between GNP and capital service where capital service is the product of capital stock and capacity utilization in the manufacturing industry
Sources: GNP, private capital stock, (The Annual Report on the National Accounts by the Economic Planning Agency), Capacity utilization rate (Industrial Statistics Monthly by the Ministry of International Trade and Industries)

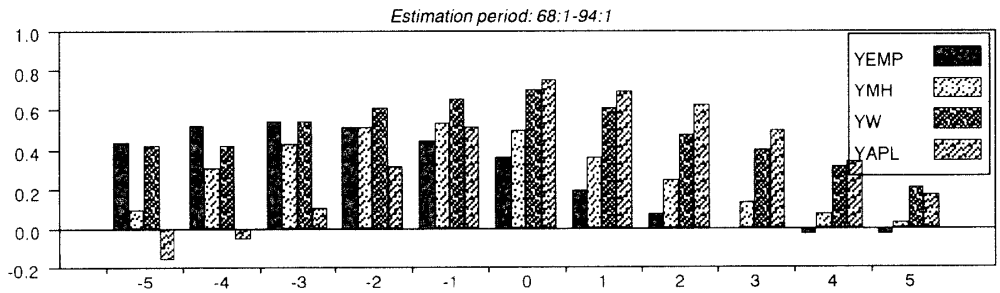

Fig. 1b. Cross correlation between GNP and major economic variables.
Note: YEMP = c.c between GNP and the index of regular employment
YMH = c.c. between GNP and index of manhour labor force (which is the product of the index of regular employment and the index of total hours worked)
YW = c.c. between GNP and the index of total earnings
YAPL = c.c. between GNP and the index of average labor productivity.
Sources: The indices of regular employment, total hours worked, total earnings, and average labor productivity (Monthly Labor Survey by the Ministry of Labor)

[12] As all variables are logarithmed and prefiltered with $(1-L^4)$, any positive correlation indicates that the two variables move in the same direction.

4. TIME-DEPENDENT PREFERENCES

Recent studies on RBC models and labor supply employ time-dependent preferences.[13] When preferences are time-dependent, the intertemporal dynamic matrix B in (3) has nonzero elements besides the one corresponding to investment.

For simplicity, assume $p=1$ in (1). The current utility depends on leisure in the last and present periods. The first order conditions are

$$\begin{aligned} U_{c_t}(c_t, z_t)-\lambda_t&=0 \\ U_{z_t}(c_t, z_t)-\lambda_t F_n(n_t, k_t; x_t)+\beta\alpha_1 U_{z_{t+1}}&=0 \\ F(n_t, k_t; x_t)+(1-\delta)k_t-c_t-k_{t+1}&=0 \\ -\lambda_t+\beta\lambda_{t+1}[F_k(n_{t+1}, k_{t+1}; x_{t+1})+1-\delta]&=0 . \quad \forall t \end{aligned} \tag{16}$$

By assuming $u_{zz_t}=u_{zz_{t+1}}$, matrices A, B, V, and vectors X_1, X_2 and dY_{t+j} $(j=0, 1)$ are

$$A=\begin{bmatrix} U_{cc} & U_{cz} & -1 \\ U_{cz} & (1+\beta\alpha_1^2)U_{zz}+\lambda F_{nn} & -F_n \\ -1 & -F_n & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta\alpha_1 U_{cz} & \beta\alpha_1 U_{zz} & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$V=\left[\begin{array}{c|c} \beta\lambda F_{kk} & \beta\lambda F_{nk} \quad \beta\lambda \quad \beta \\ \hline \begin{matrix} 0 \\ \lambda F_{nk} \\ R \end{matrix} & A \end{array}\right]$$

$$\begin{aligned} X_{1_t}&=(0, \lambda F_{nx}, -F_x)' \\ X_{2_{t+1}}&=(-\lambda F_{kx}, 0, \lambda F_{nx}, -F_x)' \\ dY_t&=(dc_t, dl_t, d\lambda_t)' \\ dY_{t+1}&=(dk_{t+1}, dc_{t+1}, dl_{t+1}, d\lambda_{t+1})' \end{aligned} \tag{17}$$

From (6),

[13] Kydland and Prescott [19], Eichenbaum, Hansen and Singleton [7], and Mankiw, Rotemberg and Summers [27].

$$\begin{bmatrix} dc_t/dx_{t+1} \\ dl_t/dx_{t+1} \\ d\lambda_t/dx_{t+1} \end{bmatrix} = -(dk_{t+1}/dx_{t+1})A^{-1}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$
$$-(dc_{t+1}/dx_{t+1})A^{-1}\begin{bmatrix} 0 \\ \beta\alpha_1 U_{cz} \\ 0 \end{bmatrix}$$
$$-(dl_{t+1}/dx_{t+1})A^{-1}\begin{bmatrix} 0 \\ \beta\alpha_1 U_{cz} \\ 0 \end{bmatrix}. \tag{18}$$

The first term indicates the net wealth effect, while the second and third represent the IS with future consumption and leisure. As mentioned in section 3.2, the wealth effect causes the problem. Assume that $\alpha_1 > 0$, then the current leisure is substitute for future consumption and leisure as in Lucas and Rapping [23]. Since $(dc_{t+1}/dx_{t+1}) > 0$ and $(dl_{t+1}/dx_{t+1}) > 0$, the IS effects have a different sign from the wealth effect. The same holds with a current shock.

Time-dependent preferences appear to be successful in explaining the comovements of employment. Labor productivity, however, move in the opposite direction under diminishing returns technology (A1). In addition to it, Mankiw, Rotemberg and Summers [27] statistically rejects the hypothesis of IS under these preferences for the U.S. data. Their estimates imply that the utility function is convex and that either consumption or leisure is an inferior good.

5. CONCLUDING REMARKS

This paper examined responses of endogenous variables to exogenous technology shocks in RBC models, under some plausible assumptions and various preferences. Under the assumptions of superior-goods leisure and diminishing returns technology, comovements of major macroeconomic variables cannot be generated by anticipated future shocks. If shocks are positively correlated, current shocks let people anticipate that output and productivities also increase in the next period. In this case, we are not sure if the RBC hypothesis is a plausible one to choose. With variable capacity utilization and depreciation rates, the RBC hypothesis and serial correlation of exogenous shocks reinforce each other, even under the assumption of time-separable preferences, to generate the observed patterns of fluctuations.

Previous works on other problems also suggest the need for introducing variable capacity utilization rate. First, Abel [1] reports the good performance of Cobb-Douglas production function estimates when utilization is considered. Secondly, in almost all market economies movements of employment are sluggish (Kennan [18]). One problem with introducing capacity utilization rate is that it is difficult to obtain good data,[14] although it is employed as one of diffusion indices in many countries including Japan.

There are two possibilities to improve performance of RBC models. The one is to introduce some new factors, like capacity utilization rate, that is beyond the control of consumers and affects MPK directly. The other is to introduce intertemporal elements, such as time-dependent preferences and/or production lags. The result of the present analysis suggests that much attention should be paid on the production side.

There remains one disturbing fact about the Japanese economy. As Figure 1b shows, labor productivity and real wage rate do not seem to move together. The RBC models suggest that MPL should always be equal to real wage rate. Introduction of variable capacity utilization may rehabilitate RBC theory, but we still need some other elements to fill the gap between MPL and real wage rate.

APPENDIX

PROOF OF PROPOSITION 1. The first order conditions:

$$\begin{aligned} &U_{ct}(c_t, l_t) - \lambda_t = 0 \\ &U_{lt}(c_t, l_t) - \lambda_t F_n(n_t, k_t; x_t) = 0 \\ &F(n_t, k_t; x_t) + (1-\delta)k_t - c_t - k_{t+1} = 0 \\ &-\lambda_t + \beta\lambda_{t+1}[F_k(n_{t+1}, k_{t+1}; x_{t+1}) + 1 - \delta] = 0 . \quad \forall t \end{aligned} \tag{A1}$$

Matrices A, B and V, and vectors X_1, X_2, dY_{t+j} and dX_{t+j} in (3) are as follows.

$$A = \begin{bmatrix} U_{cc} & U_{cl} & -1 \\ U_{cl} & U_{ll} + \lambda F_{nn} & -F_n \\ -1 & -F_n & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \left[\begin{array}{c|ccc} \beta\lambda F_{kk} & 0 & \beta\lambda F_{nk} & \beta R \\ \hline 0 & & & \\ \lambda F_{nk} & & A & \\ R & & & \end{array}\right]$$

where

[14] Epstein and Denny [8]. As for Japan, data are currently available only for manufacturing industries, published by the Ministry of International Trade and Industries.

$$R = F_k + 1 - \delta$$

$$X_1 = (0, \lambda F_{nx}, -F_x)'$$

$$X_2 = (-\lambda F_{kx}, X_1')'$$

$$dY_t = (dc_t, dl_t, d\lambda_t)'$$

$$dY_{t+1} = (dk_{t+1}, dc_{t+1}, dl_{t+1}, d\lambda_{t+1})'$$

$$dX_t = (dx_t, \cdots, dx_t)'$$

$$dX_{t+1} = (dx_{t+1}, \cdots, dx_{t+1})' . \tag{A2}$$

From (6), as matrix B is almost filled with zero elements,

$$\begin{bmatrix} dc_t/dx_{t+1} \\ dl_t/dx_{t+1} \\ d\lambda_t/dx_{t+1} \end{bmatrix} = -(dk_{t+1}/dx_{t+1})A^{-1} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} . \tag{A3}$$

The product $A^{-1}(0, 0, -1)'$ is precisely the income effects of the central planner.[15]

The element λF_{nn}, by which the central planner's reaction differs from consumers' when $F_n = w$, should be negative under (A1) so that the sign of income effect for the central planner is the same as consumers'. Thus, from (A5),

$$\frac{dc_t/dx_{t+1}}{dk_{t+1}/dx_{t+1}} < 0$$

$$\frac{dl_t/dx_{t+1}}{dk_{t+1}/dx_{t+1}} < 0 .$$

As labor is the unique production factor, employment and output move in the same direction. The former responds against labor productivity under (A1) of concave production function. Q.E.D.

PROOF OF PROPOSITION 2. The first order conditions:

[15] For consumers,

$$\tilde{A} = \begin{bmatrix} U_{cc} & U_{cl} & -1 \\ U_{cl} & U_{ll} & -w \\ -1 & -w & 0 \end{bmatrix}$$

$$\tilde{B} = B$$

$$\tilde{V} = \left[\begin{array}{c|ccc} 0 & 0 & 0 & \beta R \\ \hline 0 & & & \\ 0 & & \tilde{A} & \\ R & & & \end{array} \right] . \tag{A4}$$

$$U_{ct}(c_t, l_t) - \lambda_t = 0$$
$$U_{lt}(c_t, l_t) - \lambda_t F_n(-l_t, h_t, k_t; x_t) = 0$$
$$\lambda_t F_{ht} - \lambda_t \delta_{ht} = 0$$
$$F(-l_t, h_t, k_t; x_t) + [1 - \delta(h_t)]k_t - c_t - k_{t+1} = 0$$
$$-\lambda_t + \beta\lambda_{t+1}[F_k(-l_{t+1}, h_{t+1}, k_{t+1}; x_{t+1}) + 1 - \delta(h_{t+1})] = 0 . \quad \forall t \qquad \text{(A5)}$$

Matrices A, B, V, and vectors X_1, X_2 and dY_{t+1} $(j=0, 1)$ are

$$A = \begin{bmatrix} U_{cc} & U_{cl} & 0 & -1 \\ U_{cl} & U_{ll} + \lambda F_{nn} & -\lambda F_{nh} & -F_n \\ 0 & -\lambda F_{nh} & \lambda(F_{hh} - \delta_{hh}) & 0 \\ -1 & -F_n & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \left[\begin{array}{c|ccc} \beta\lambda F_{kk} & \beta\lambda F_{nk} & \beta\lambda(F_{hk} - \delta_h) & \beta R \\ \hline 0 & & & \\ \lambda F_{nk} & & A & \\ \lambda(F_{hk} - \delta_h) & & & \\ R & & & \end{array}\right]$$

$$X_{1t} = (0, \lambda F_{nx}, -\lambda F_{hx}, -F_x)'$$
$$X_{2t+1} = (-\lambda F_{kx}, 0, \lambda F_{nx}, -\lambda F_{hx}, -F_x)'$$
$$dY_t = (dc_t, dl_t, dh_t, d\lambda_t)'$$
$$dY_{t+1} = (dk_{t+1}, dc_{t+1}, dl_{t+1}, dh_{t+1}, d\lambda_{t+1})' \qquad \text{(A6)}$$

For consumers', matrices $\tilde{A}$, $\tilde{B}$, $\tilde{V}$ are the same as in the previous case.

Since the time structure remains unchanged, we can apply the previous results in (A3) and (11) to this case. What matters is the difference in income effects between the central planner and consumers. By employing Cramer's rule,

$$\frac{dc_t/dx_{t+1}}{dk_t/dx_{t+1}} = \Delta^{-1}[\lambda(F_{hh} - \delta_{hh})(U_{ll} - F_n U_{cl} + \lambda F_{nn}) - \lambda^2 F_{nh}^2] \gtreqless 0$$

$$\frac{dl_t/dx_{t+1}}{dk_t/dx_{t+1}} = \Delta^{-1}[\lambda(F_{hh} - \delta_{hh})(F_n U_{cc} - U_{cl}) < 0$$

$$\frac{dh_t/dx_{t+1}}{dk_t/dx_{t+1}} = \Delta^{-1}\lambda F_{nh}(F_n U_{cc} - U_{cl}) > 0$$

where

$$\Delta = detA$$
$$= -\lambda^2 F_{nh}^2 + \lambda(F_{hh} - \delta_{hh})(2F_n U_{cl} - \lambda F_{nn} - F_n^2 U_{cc}) < 0$$

Clearly, utilization rate, employment and investment move in the same direction. Rises in employment and utilization rate jointly increase the output if the capital stock and labor are complements in production. Thus, they show procyclicalities. The response of consumption is ambiguous. The first and second terms in the bracket have different signs. If F_{nh} is sufficiently large, the second effect dominates and consumption moves in the same direction as output and investment.

Q.E.D.

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