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THE OUTSIDE OPTION, BARGAINING SKILL AND THE NASH BARGAINING SOLUTION

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Abstract: The threat point of the Nash bargaining solution is variously identified with the impasse point or the outside option vector of the two players. It has been argued however, that the outside option either does not affect the outcome at all, or is relegated to the role of a corner solution. We examine a non-cooperative model where bargaining skill is explicitly introduced via a probabilistic move structure. We show that this approach manages to unify the two competing viewpoints about the threat point. Depending on the parameter values either interpretation may be valid. Moreover, an intermediate case exists where both the interpretations hold partially.

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1. INTRODUCTION

In this paper we adopt a non-cooperative framework to address some interpretational ambiguities associated with the Nash bargaining solution.

We begin by identifying some conceptual inadequacies of the cooperative approach to bargaining problems. Firstly, since the cooperative approach abstracts from the actual negotiation procedure, it is difficult to judge the relevance of any given cooperative bargaining solution. One approach is to provide an axiomatic characterisation of the solution concept in question and then discuss the validity of the concerned axioms. But even these axioms are often couched in rather abstract terms, creating interpretational problems. (Perhaps the most well known example of such ambiguity concerns the axiom of Independence of Irrelevant Alternatives used in characterising the Nash bargaining solution.) Secondly, this approach assumes that binding contracts can be written. In reality, however,

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binding contracts are costly to write, in terms of legal expenses, as well as the time taken. Furthermore, the cost of implementing such contracts is often quite high.

At the conceptual level the Nash programme (see Nash (1951)) provides an answer to both these problems. The programme involves the formulation of some explicit non-cooperative game form so that the Nash equilibrium (or some refinement of it) of this game yields the cooperative outcome we are interested in. Clearly, by explicitly modelling the negotiation procedure, this approach provides an institutional structure where the cooperative solution can be expected to hold. Furthermore, since we look for the Nash equilibrium of the game we construct, the outcome is self-enforcing, thus obviating the need for binding contracts.

This paper is in the spirit of the Nash programme in the sense that we use a non-cooperative framework to investigate some ambiguities about the interpretation of the threat point in the Nash bargaining solution. The first interpretation identifies the threat point with the impasse point, i.e. the payoffs that would result if the contestants do not leave the table but continue to bargain even though no agreement is reached. (This definition was also used by Binmore, Rubinstein and Wolinsky (1986).) Under the second interpretation, the threat point is identified with the outside option vector of the two players, where the outside options denote the payoffs that result if the players leave the bargaining table.

In applications of the Nash bargaining solution, especially in wage bargaining, the threat point is often identified with the outside option. Some examples are Grout (1984), Nickell and Wadhwani (1990), Layard, Nickell and Jackman (1991), Bronars and Deere (1991), to mention only a few. Examinations of the non-cooperative foundations of the Nash bargaining solution, however, do not appear to support such an identification. Both Binmore (1985) and Shaked and Sutton (1984a) demonstrate that the outside options of the players do not affect the outcome, if the values of the outside options lie below the perfect equilibrium payoff levels that would prevail in the absence of any outside options. If, however, the value of the outside option of one of the players exceeds this critical level, then the payoff of the concerned player equals the value of the outside option. These two results together form the Outside Option Principle (Shaked and Sutton (1984a)).[1] In fact Binmore, Shaked and Sutton (1989) perform a bargaining experiment where the Outside Option Principle provides a better prediction compared to the Nash bargaining solution.[2]

1 Shaked and Sutton (1984a) states that (pp. 3) "we believe that the standard method of analysing... outside options—whereby they are incorporated in a Nash bargaining framework by means of identifying the outside option as the status quo point—is unfounded."

2 Binmore, Rubinstein and Wolinsky (1986) argue that the threat point ought to be identified with the impasse point (the impasse point refers to outcome that comes about when the players continue to bargain without reaching an agreement) in case of the standard Rubinstein model and with the breakdown point in models with exogenous risks of breakdown.

We consider a model of bilateral bargaining with outside options, where the move structure is probabilistic rather than deterministic.[3] At the start of every period nature selects one of the two players with the right to make the first offer. The selected player then makes an offer which the other player may either accept or reject. If he rejects, then he may either opt out of the game, when the players immediately receive their outside option payoffs, or remain in the game, when the game passes to the next period where nature again selects a player to make the first offer.

The probability of any player being selected as the proposer, depends on which of the players was the proposer in the previous period. These probabilities can be interpreted as arising from the interplay of bargaining skills and social conventions, where bargaining skills refer to the relative ability of the contestants in formulating an offer. The Rubinstein (1982) model with outside options is obtained as a special case when the transition rule is deterministic and always selects the player who was the responder in the previous period.

We begin by proving existence and uniqueness of subgame perfect equilibrium and then go on to characterise the outcome for different parameter values. We find that our analysis provides an unification of the two different interpretations of the threat point.

We show that for high values of the outside options, the outside option vector can be identified with the threat point of the Nash bargaining solution. Whereas for low values of the outside options, the threat point is identified with the impasse vector. For intermediate values of the outside options, both the interpretations hold partially. In this case the threat point of one of the players is identified with his outside option, whereas the threat point of the other player is identified with his impasse payoff. Hence depending on the parameter values, either one of the interpretations may be valid.

Thus for any non-deterministic transition rule there always exists a range of parameter values (where the outside options are large enough) for which the Outside Option Principle is not binding. In this case the outcome leads to the asymmetric Nash bargaining solution, where the probability of any player being selected as the proposer, is interpreted as his bargaining power. Moreover, the parameter zone for which the above result holds has an area that is bounded away from zero, even in the limit as we approach the deterministic model. However, for lower values of the outside options the critique offered by the Outside Option Principle is still valid since the outcome depends on the outside option of at most one of the players. Thus we find that depending on the parameter values the Outside Option Principle may or may not hold.

The introduction of bargaining skills supplements the notion of patience as the only source of bargaining power. In the standard analysis bargaining skills enter the problem only via the bargaining structure. For a nonstationary bargaining

[3] Binmore (1987) also analyses a model with random proposers.

structure, however, solving the game could become complicated, as most techniques rely on the stationary nature of these bargaining games for the solution. This paper parametrizes the problem by introducing bargaining skills in a way that maintains the stationary nature of the game.

Inter alia, this model also provides an interpretation of the weights used in the asymmetric Nash bargaining solution. We show that the weights reflect the relative bargaining skills of the players. For large values of the outside options, the weights depend on the offer probabilities alone. If, however, the outside options are not too large then the weights also depend on the common discount factor of the two players. This interpretation is similar in spirit to the one offered by Binmore, Rubinstein and Wolinsky (1986). They interpret the weights as the relative time taken for a counteroffer by the two players. It can be argued that the relative time taken for a counteroffer reflects another facet of the bargaining skills of the players. Notice, however, that we adopt a probabilistic offer structure, whereas Binmore, Rubinstein and Wolinsky (1986) consider a deterministic one.

Besides, to the best of our knowledge, this is the only model which implements the Nash bargaining solution exactly. All other models implement the Nash solution in the limit, as the time period between offers goes towards zero.[4]

Dalmazzo (1992) provides a different justification for treating the outside option vector as the threat point. He considers a model with decay in the size of the cake.[5] He shows that in the limit, as the time lag between successive offers goes towards zero, the outcome approaches the Nash bargaining solution, where the outside option is taken to be the threat point. In many cases, however, the assumption of a decay in the size of the cake may not be appropriate. Besides, the value of the outside option may also be decreasing for precisely the same reasons that cause a decrease in the cake size. If the rate of decline is high enough (so that the cake always remains larger than the sum of the outside options), the Dalmazzo approach is not applicable.

In the next section we set down the model and establish the main theorem, that for high values of the outside option, it is legitimate to treat the outside option as the threat point. We also characterise the equilibrium for other values of the outside option. Section 3 concludes.

2. THE MODEL

The game involves two players, P1 and P2, bargaining over a cake of size 1. Time is discrete and continues forever. Periods are indexed by $t=0, 1, 2, \cdots$. The

[4] Binmore and Dasgupta (1987) remark (pg. 24) "Finally, it is necessary to comment that none of the non-cooperative bargaining models that have been studied implement the Nash bargaining solution *exactly*. In each case, the implementation is *approximate* (or exact only in the limit)."

[5] Dalmazzo suggests several possible economic reasons to justify the decay; physical decay of production oppurtunities, loss of market due to customers defecting to other firms, increasing amount of interest maturing over time when there is a fixed debt to be repaid etc.

common discount factor of the two players is δ,[6] where $1>\delta>0$. The outside option vector is denoted by (d_1, d_2) where d_i is Pi's payoff if either of the players leave the game.[7] We assume that $d_1, d_2 \geq 0$ and that $d_1+d_2<1$, i.e. mutual gains from agreement are possible.

The move structure in this game is probabilistic rather than deterministic and is governed by a transition rule of the following kind. We define two states. State 1 corresponds to the subgame where nature is about to choose the proposer and P1 was the proposer in the previous period. (See Fig. 1.) Similarly, state 2 corresponds to the subgame where nature is about to choose the proposer and either, $t=0$ or player 2 was the proposer in the previous period. The transition probability from state i to state j, where $i, j=1, 2$ is defined as p_{ij}. To restate things, in state 1, nature selects P1 as the proposer with probability p_{11} and P2 as the proposer with probability p_{12}. In state 2, nature selects P1 with probability p_{21} and P2 with probability p_{22}.

Subsequent to the selection of a proposer, the selected player makes an offer. An offer is a vector of the form $(x, 1-x)$ where x and $1-x$ denote the shares of the first and the second player respectively. An offer of x by the first player corresponds to the vector $(1-x, x)$. Coming from the second player an offer of x corresponds to the vector $(x, 1-x)$. Acceptance or rejection of the offer is

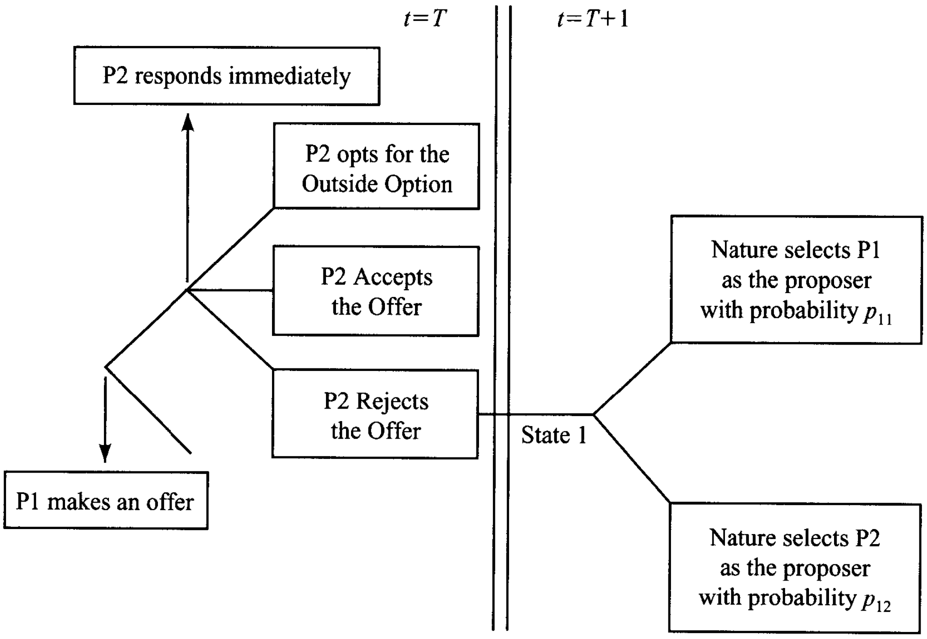

Fig. 1.

[6] One can consider the case where the discount factors of the two players are different. This will not affect the qualitative results in any way.

[7] A more general formulation would be where the payoff of the ith player also depends on who decides to opt out of the game. In this case one can interpret d_i as the outside option of the ith player when the ith player decides to leave the game. The subsequent analysis will not be affected in any way.

instantaneous. If player j accepts the offer then the game terminates with the implementation of that offer. If he rejects, then he can either opt out of the bargaining process, when the players receive their outside option payoffs, or he can stay in the game when at the next time period nature again selects one of the players as the proposer. The game continues in the above manner until an agreement is reached or one of theplayers opts for the outside option. The Rubinstein model with outside options is obtained in the limit as p_{11} tends towards zero and p_{21} tends towards one. This allows us to examine whether the established results depend on the deterministic structure of the game.

Notice that since in this game the players can stop the game at any period, by either accepting an offer or by opting for the outside option, the transition probabilities defined by us are slightly different from standard notions. These probabilities can be interpreted as arising from the interplay of bargaining skills and social conventions. First consider the case where the transition probabilities, though random, are state independent. In this case we can interpret these probabilities as arising from purely the interplay of bargaining skills of the two players.

We then discuss what we mean by the term bargaining skill. Clearly, under a game theoretic approach with rational players, there is no justification for introducing psychological factors like the ability in duping an opponent etc. (Nash (1953) adopts the same position.) Rather bargaining skill is interpreted as the ability of the agents in quickly formulating an offer. In our simplified framework, of course, the *formulation* of an offer requires very little skill. In reality, though, careful attention to various interlinked issues is required. Consider, for example, the bargaining process between two firms that are planning to form a joint venture. Typically we can expect the bargaining process to include issues like the financing of joint production and R & D facilities, the contribution of personnel to the joint venture firm by the parent firms, the organization of training and orientation programmes, providing access to retail outlets, etc. Observe that all such issues affect the final division of the pie among the two firms. Clearly, for a complex multi-product joint venture involving R & D, product development etc., formulation of offers is going to be quite complex.

It is the relative ability of the players in handling such issues that determine the comparative bargaining skills of the two agents. We now briefly discuss some of the factors that are likely to affect bargaining skills.

1. The quality of the support staff and the computing facility available to the two agents.

2. The size of the decision making unit. For example, if a firm has a single majority shareholder, then the decision making process is likely to be much faster compared to the case where there are several small shareholders of equal size.

3. The diversity of interests in the decision making unit. Lesser is the degree of such divergence, quicker is the decision making process. Thus if the decision making unit in a firm includes union representatives, then the speed of decision

making is likely to be less compared to the case where it does not.

Clearly, this point is related to point 2 above.

4. The flexibility of the formal rules of decision-making employed by the two contestants. For example, decision-making is likely to be faster if a simple majority is enough to implement a proposal, as compared to the case where (say) a two-thirds majority is required. Decision-making is also likely to be faster if there are well-established rules of thumb for handling routine decision problems. Finally, in case of firms decisions will be taken at a faster pace if the decisions need not be ratified in general body meetings.

Let us very quickly discuss the implications of the above discussion for the decision making speed of firms. At the initial stages of their development, firms are likely to be entrepreneurial or family-owned in nature, with a small number of relatively like minded decision-makers. As argued in points 2 and 3 above, this is conducive to faster decision-making. On the other hand, the quality of the support staff, as well as the computing facility commanded by such firms is likely to be inferior compared to those of older, more established firms. This suggests that if the negotiation is relatively less complex then younger firms have an advantage as far as the formulation of offers is concerned. This is because in such cases the absence of computing facilities etc is likely to be of less importance. For more complex negotiations, however, older firms are likely to have an advantage.

It is needless to say that the discussion above is quite rudimentary and just manages to scratch the surface of what is clearly a very complex issue. A proper discussion of such issues is, however, beyond the scope of the present paper. Hence, from now on we abstract from all such complexities and confine our attention to the probabilities p_{ij}.

In the general case, we can also expect social conventions to play some role in the determination of these probabilities. This is because the decision makers themselves are subject to these social conventions while formulating their offers. Where a society is concerned with the fairness of the bargaining structure, we can expect that the probability of any player being selected as the proposer would be higher if the other player was the proposer in the previous period. Clearly, this implies that the offer probabilities are state-dependent.

Another reason for state-dependence can be traced to our earlier argument. If agent i was the proposer in period $(t-1)$, then we can expect that it had used the $(t-2)$th period in formulating its offer. But then the other agent had also used the $(t-2)$th period in thinking about possible offers. Since agent i had already made its offer in period $(t-1)$, in the subsequent period agent j is more likely to make an offer, since it can draw on the groundwork it made in period $(t-2)$.

We solve for the (subgame) perfect equilibrium of this game. The proof generalises that in Shaked and Sutton (1984b).

We begin by introducing the following notations. Let $X_1^S(i)$ denote the supremum and $X_1^I(i)$ the infimum of P1's *expected* payoff in any perfect equilibrium of the game in state i. We can define, $X_2^S(i)$ and $X_2^I(i)$ for P2 in a similar manner. If

$X_i^S(j)=X_i^I(j)$, the common value is denoted by $X_i(j)$.

Now consider P1's offer whenever it is his turn to make an offer. Clearly, P1 must offer at least $\max\{\delta X_2^I(1), d_2\}$ if P2 is to accept. P1's payoff is therefore at most $[1-\max\{\delta X_2^I(1), d_2\}]$. Also P1 would offer at most $\max\{\delta X_2^S(1), d_2\}$, since P2 would accept this offer. Therefore P1's payoff is at least $[1-\max\{\delta X_2^S(1), d_2\}]$.

We then consider P2's offer whenever it is his turn to make an offer. He must offer at least $\max\{\delta X_1^I(2), d_1\}$ if P1 is to accept. P2's payoff is therefore at most $[1-\max\{\delta X_1^I(2), d_1\}]$. Again, P2 would offer at most $\max\{\delta X_1^S(2), d_1\}$, since P1 would accept this offer. Therefore his own payoff is at least $[1-\max\{\delta X_1^S(2), d_1\}]$.

Clearly the above implies that P1's payoff in state 2 is at most

$$p_{21}[1-\max\{\delta X_2^I(1), d_2\}]+p_{22}\max\{\delta X_1^S(2), d_1\}\,.$$

Therefore, it follows that

$$X_1^S(2)\leq p_{21}[1-\max\{\delta X_2^I(1), d_2\}]+p_{22}\max\{\delta X_1^S(2), \mathrm{d}_1\}\,. \tag{1}$$

Arguing similarly we obtain

$$X_1^I(2)\geq p_{21}[1-\max\{\delta X_2^S(1), d_2\}]+p_{22}\max\{\delta X_1^I(2), d_1\}\,, \tag{2}$$

$$X_2^S(1)\leq p_{11}\max\{\delta X_2^I(1), d_2\}+p_{12}[1-\max\{\delta X_1^S(2), d_1\}]\,, \tag{3}$$

$$X_2^I(1)\geq p_{11}\max\{\delta X_2^S(1), d_2\}+p_{12}[1-\max\{\delta X_1^I(2), d_1\}]\,, \tag{4}$$

$$X_1^S(1)\leq p_{11}[1-\max\{\delta X_2^I(1), d_2\}]+p_{12}\max\{\delta X_1^S(2), d_1\}\,, \tag{5}$$

$$X_1^I(1)\geq p_{11}[1-\max\{\delta X_2^S(1), d_2\}]+p_{12}\max\{\delta X_1^I(2), d_1\}\,, \tag{6}$$

$$X_2^S(2)\leq p_{21}\max\{\delta X_2^I(1), d_2\}+p_{22}[1-\max\{\delta X_1^S(2), d_1\}]\,, \tag{7}$$

$$X_2^I(2)\geq p_{21}\max\{\delta X_2^S(1), d_2\}+p_{22}[1-\max\{\delta X_1^I(2), d_1\}]\,. \tag{8}$$

Therefore, we have a set of eight inequalities in eight variables. Notice that, however, inequalities (1) to (4) form a sub-system of four inequalities in the four variables, $X_1^S(2)$, $X_1^I(2)$, $X_2^S(1)$ and $X_2^I(1)$. We use these four inequalities to demonstrate that $X_1^S(2)=X_1^I(2)$ and $X_2^S(1)=X_2^I(1)$. (See Appendix 1 for the proof.) Next it follows from inequalities (5) to (8) that, $X_1^S(1)=X_1^I(1)$ and $X_2^S(2)=X_2^I(2)$. This shows that the equilibrium is unique.

PROPOSITION 1. *There exists a unique (subgame) perfect equilibrium.*

Proposition 1 implies that we can simplify inequalities (1) to (4) to the following two equations

$$X_1(2)=p_{21}[1-\max\{\delta X_2(1), d_2\}]+p_{22}\max\{\delta\ X_1(2), d_1\}, \tag{9}$$

$$X_2(1)=p_{11}\max\{\delta X_2(1), d_2\}+p_{12}[1-\max\{\delta X_1(2), d_1\}]\,. \tag{10}$$

We next use the above two equations to completely characterise the set of equilibrium outcomes. The propositions are stated in terms of $X_1(2)$ and $X_2(1)$. It is easy to see that $X_1(1)=1-X_2(1)$ and $X_2(2)=1-X_1(2)$.

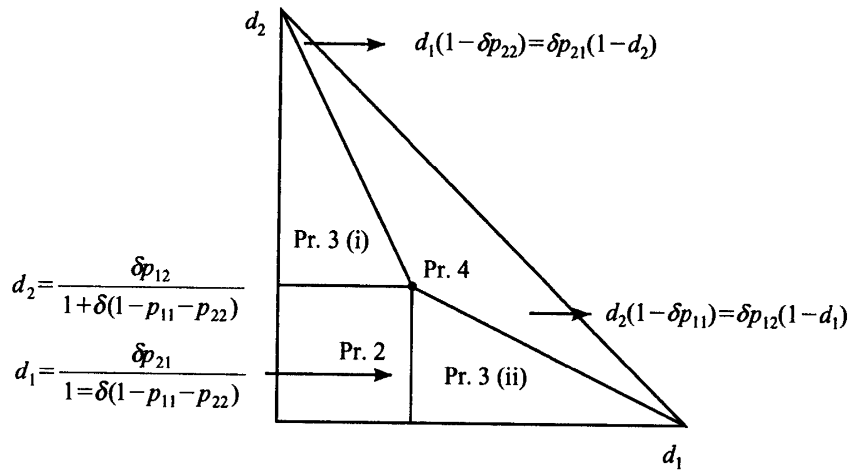

Fig. 2.

The proofs of the subsequent propositions involve manipulations of the above equations and have been relegated to Appendix 2. In Fig. 2 we depict the various parameter zones for which the propositions hold. For example, the zone Pr. 2 denotes the parameter zone for which Proposition 2 holds.

The next proposition examines the equilibrium outcome for low values of the outside option payoffs.

PROPOSITION 2. *If* $d_1<\delta p_{21}/[1+\delta(1-p_{11}-p_{22})]$ *and* $d_2<\delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$, *the outcome involves*

$$X_1(2)=\frac{p_{21}}{1+\delta(1-p_{11}-p_{22})}, \tag{11}$$

$$X_2(1)=\frac{p_{12}}{1+\delta(1-p_{11}-p_{22})}. \tag{12}$$

The strategies supporting the above equilibrium are as follows. P1 accepts any offer that yields him a payoff of at least $\delta p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and offers $\delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$ to P2 whenever it is his turn to make an offer. P2 accepts any offer that yields him at least $\delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$ and offers $\delta p_{21}/[1+\delta(1-p_{11}-p_{22})]$ to P1 whenever it is P2's turn to make an offer.

Observe that the outcome in Proposition 2 coincides with the Nash bargaining solution where the impasse point is taken to be the threat point. Notice that in this case the payoffs of the two agents are $X_1(2)$ and $X_2(2)$, where $X_1(2)$ is given by equation (11), and

$$X_2(2)=1-\frac{p_{21}}{1+\delta(1-p_{11}-p_{22})}. \tag{13}$$

Observe that $X_1(2)$ and $X_2(2)$ solves the Nash maximisation problem

$$\max_{X_1} (X_1)^{\frac{p_{21}}{1+\delta(1-p_{11}-p_{22})}}(1-X_1)^{1-\frac{p_{21}}{1+\delta(1-p_{11}-p_{22})}} .$$

Moreover, notice that the weights of the Nash bargaining solution, $p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $1-p_{21}/[1+\delta(1-p_{11}-p_{22})]$, are functions of the p_{ij}s, as well as δ. This suggests that bargaining skills, as well as patience plays a role in the determination of the bargaining weights. Furthermore, in the symmetric case where $p_{12}=p_{21}=1/2$, the payoff vector is (1/2, 1/2), and any difference in the size of the two outside option payoffs does not affect the outcome.

Thus Proposition 2 suggests that if the outside options of both the players are relatively small, then the threat point should be identified with the impasse payoffs.

In the next proposition we characterise the equilibrium for intermediate values of the outside option. We find that the outcome depends only on the outside option value of the player with the relatively higher value of the outside option. It does not depend on the outside option payoff of the other player. If the outcome depends on d_i, then the payoff of the ith player is increasing in d_i, and the payoff of the jth player is decreasing in d_i.

PROPOSITION 3. (i) *If* $d_2 \geq \delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$ and $d_1(1-\delta p_{22}) < \delta p_{21}(1-d_2)$, *the outcome involves*

$$X_1(2)=\frac{p_{21}(1-d_2)}{1-\delta p_{22}}, \tag{14}$$

$$X_2(1)=p_{11}d_2+p_{12}\left[1-\frac{\delta p_{21}(1-d_2)}{1-\delta p_{22}}\right]. \tag{15}$$

(ii) *If* $d_1 \geq \delta p_{21}/[1-\delta(1-p_{11}-p_{22})]$ *and* $d_2(1-\delta p_{11}) < \delta p_{12}(1-d_1)$, *the outcome involves*

$$X_1(2)=p_{21}\left[1-\frac{\delta p_{12}(1-d_1)}{1-\delta p_{11}}\right]+p_{22}d_1 , \tag{16}$$

$$X_2(1)=\frac{p_{12}(1-d_1)}{1-\delta p_{11}}. \tag{17}$$

The strategies supporting the equilibrium in Proposition 3 (i) are as follows. P1 accepts any offer that yields him at least $\delta p_{21}(1-d_2)/(1-\delta p_{22})$ and offers d_2 whenever it is his turn to make an offer. P2 accepts any offer that yields him at least d_2 and offers $\delta p_{21}(1-d_2)/(1-\delta p_{22})$ whenever it is his turn to make an offer.

The strategies in case of Proposition 3 (ii) are symmetric.

In this case the outcome coincides with the Nash bargaining solution where for the player with a relatively higher outside option, the threat point is identified with the outside option itself. In case of the other player, however, the threat point is identified with the impasse payoff. Consider Proposition 3 (i). Notice that in this case the equilibrium payoffs $X_1(2)$ and $X_2(2)$ are given by

$$X_1(2)=\frac{p_{21}(1-d_2)}{1-\delta p_{22}}, \tag{18}$$

$$X_2(2)=1-\frac{p_{21}(1-d_2)}{1-\delta p_{22}}. \tag{19}$$

Notice that the above outcome coincides with the solution of the Nash maximisation problem

$$\max_{X_1}(X_1)^{\frac{p_{21}}{1-\delta p_{22}}}(1-X_1-d_2)^{1-\frac{p_{21}}{1-\delta p_{22}}}.$$

Observe that as in Proposition 2 above, the bargaining weights depend on the p_{ij}s, as well as δ. Furthermore, notice that in the limit as $\delta\to 1$, the payoff vector reduces to $(1-d_2, d_2)$. This corresponds to the corner solution obtained under the Outside Option Principle.

Proposition 3 (ii) can be interpreted similarly. To begin with observe that in this case $X_1(2)$ can be re-written as follows:

$$X_1(2)=1-(1-d_1)\left[\frac{p_{22}+\delta(1-p_{11}-p_{22})}{1-\delta p_{11}}\right]. \tag{20}$$

It is now easy to see that $X_1(2)$ and $X_2(2)$ (where $X_2(2)=1-X_1(2)$) solves the Nash maximisation problem

$$\max_{X_1}(X_1-d_1)^{1-\frac{p_{22}+\delta(1-p_{11}-p_{22})}{1-\delta p_{11}}}(1-X_1)^{\frac{p_{22}+\delta(1-p_{11}-p_{22})}{1-\delta p_{11}}}.$$

Thus Proposition 3 represents an interesting intermediate case where neither of the standard interpretations of the threat point go through completely, but both hold to some extent.

Propositions (2) and (3) together demonstrate, that the critique offered by the Outside Option Principle is, for low values of the outside options, still valid.

Proposition 4 proves that for high values of the outside options, it is legitimate to identify the outside option vector with the threat point in the Nash bargaining solution.

PROPOSITION 4. *If* $d_1(1-\delta p_{22}))\geq\delta p_{21}(1-d_2)$ *and* $d_2(1-\delta p_{11})\geq\delta p_{12}(1-d_1)$, *the outcome involves*

$$X_1(2)=p_{21}-p_{21}d_2+p_{22}d_1, \tag{21}$$

$$X_2(1)=p_{12}-p_{12}d_1+p_{11}d_2. \tag{22}$$

The equilibrium strategies involve Pi offering d_j whenever it is his turn to make an offer and accepting any offer which yields him at least d_i.

It is easy to see that

$$X_1(2) = p_{21} - p_{21}d_2 + p_{22}d_1\,, \tag{23}$$

$$X_2(2) = p_{22} - p_{22}d_1 + p_{21}d_2\,. \tag{24}$$

It is obvious that this solution corresponds to the Nash bargaining solution where p_{21} is interpreted as the weight of P1 and p_{22} is interpreted as the weight of P2 in the Nash maximisation problem,

$$\mathrm{Max}_{\{X_1\}}(X_1 - d_1)^{p_{21}}(1 - X_1 - d_2)^{p_{22}}$$

For $p_{21} = 1/2$, the symmetric Nash bargaining solution is obtained. Therefore, when the outside options are large enough, the threat of adopting the outside option becomes credible and the Nash bargaining solution results, even though, in equilibrium, the outside option is not taken up.

It is easy to see that as $p_{11} \to 0$ and $p_{21} \to 1$ the area of the parameter zone for which the conditions in Proposition 4 hold becomes smaller. However, observe that in the limit the sufficient conditions reduce to $d_1 \geq \delta(1 - d_2)$ and $d_2 \geq \delta(1 - d_1)$. Clearly, the area of the parameter space is bounded away from zero, even in the limit as we approach the deterministic model. Therefore, for any non-degenerate transition rule, there always exist values of the outside option such that the expected payoffs coincide with the asymmetric Nash solution, where the threat point is identified with the outside option vector.

We then discuss the economic logic behind Propositions 2, 3 and 4. Taken together, these three propositions provide a theory about the modelling of the threat point. *Our analysis shows that the player with a low value of the outside option considers the impasse point to be his disagreement payoff, whereas the player with a high value of the outside option identifies the outside option itself with his threat point.*

Clearly, the above result is intuitively quite appealing. If the outside option is not too large, then the concerned agent would be unwilling to leave the bargaining table since the potential payoff from continuing the bargaining process is rather high. Thus he will continue to bargain even if no agreement is reached. Since his opponent also realises this fact, the outside option payoff of this agent have no strategic value as a threat point. Therefore it does not affect the outcome.

Suppose, however, that the outside option of the ith agent is quite large. Then the ith agent would prefer to opt for his outside option rather than continue to bargain fruitlessly. Thus his payoff should always be greater than or equal to his outside option, and hence the outside option acts as an effective threat point.

Finally, observe that while our analysis moderates the critique provided by the Outside Option Principle, it does not remove it completely. We find that the appropriate interpretation of the threat point depends on the difference between the size of the cake and an weighted average of the outside option payoffs. Identifying the threat point with the outside option vector is justified only when this weighted average is relatively large. Thus the interpretation of the disagreement point must be tailored to fit the economic application we have in mind. Our analysis does, however, provide some guidelines regarding the factors one needs to consider.

Let us illustrate this point in case of management-union bargaining. First consider the case where the strike-fund of the union is not too large, and the firm has to shut down production in the event of a strike. In this case the outside options of both the players are rather small, and the appropriate approach is to identify the threat point with the impasse vector. Next consider the case where the strike-fund is large, and even in the event of a strike the firm can earn a positive level of profit (perhaps by employing non-union workers). In this case the outside option vector is quite large and it is appropriate to identify the threat point with the outside option vector. As this example demonstrates, even within the same industry the proper interpretation of the threat point may vary from firm to firm, and hence empirical applications call for a great degree of circumspection.

3. CONCLUSION

In this paper we provide a unification of the two competing interpretations of the threat point in the Nash bargaining solution. Depending on parameter values we show that either one of the interpretations may be valid. Interestingly enough, there is an intermediate case where both the interpretations hold partially, though neither holds completely.

From an empirical point of view we provide a justification for treating the outside option vector as the threat point in the Nash bargaining solution. Unlike Dalmazzo (1992), our approach does not depend on a reduction in the size of the cake. Our analysis do suggest, however, that such an identification is legitimate only when the value of the outside option vector is relatively large. Whether, the sufficient condition on the outside option vector is satisfied, is a matter of empirical reality, and will vary from case to case.

Binmore, Shaked and Sutton (1989) perform a bargaining experiment which coroborates their thesis that outside options do not matter. The bargaining structure they use for their experiment is, however, deterministic. In the light of this essay, it would be of interest to perform an experiment with a probabilistic move structure, and compare the results with that of Binmore, Shaked and Sutton (1989).

4. APPENDIX 1

PROOF OF PROPOSITION 1. STEP 1. We begin by showing that $X_1^S(2)=X_1^I(2)\leftrightarrow X_2^S(1)=X_2^I(1)$. W.l.o.g. assume that $X_2^S(1)=X_2^I(1)=X_2(1)$. We have to show that $X_1^S(2)=X_1^I(2)$. Suppose to the contrary that $X_1^S(2)>X_1^I(2)$. First consider the case where $d_1\geq\delta X_1^S(2)>\delta X_1^I(2)$. It is easy to see that in this case inequalities

(1) and (2) reduce to

$$X_1^S(2) \le p_{21}[1-\max\{\delta X_2(1), d_2\}] + p_{22}d_1 , \tag{25}$$

$$X_1^I(2) \ge p_{21}[1-\max\{\delta X_2(1), d_2\}] + p_{22}d_1 . \tag{26}$$

Since by definition $X_1^S(2) \ge X_1^I(2)$, the above inequalities imply that

$$X_1^S(2) = X_1^I(2) = p_{21}[1-\max\{\delta X_2(1), d_2\}] + p_{22}d_1 . \tag{27}$$

One can therefore restrict attention to the following four cases,

(a) $\delta X_1^S(2) > d_1 \ge \delta X_1^I(2)$ and $d_2 \ge \delta X_2^S(1) = \delta X_2^I(1)$,
(b) $\delta X_1^S(2) > d_1 \ge \delta X_1^I(2)$ and $\delta X_2^S(1) = \delta X_2^I(1) > d_2$,
(c) $\delta X_1^S(2) > \delta X_1^I(2) > d_1$ and $d_2 \ge \delta X_2^S(1) = \delta X_2^I(1)$,
(d) $\delta X_1^S(2) > \delta X_1^I(2) > d_1$ and $\delta X_2^S(1) = \delta X_2^I(1) > d_2$.

We start by considering case (a). From inequalities (1) and (2) it follows that

$$X_1^S(2) \le p_{21}(1-d_2) + \delta p_{22} X_1^S(2) , \tag{28}$$

$$X_1^I(2) \ge p_{21}(1-d_2) + p_{22}d_1 . \tag{29}$$

Clearly, $X_1^S(2) \le p_{21}(1-d_2)/(1-\delta p_{22})$. Substituting for $X_1^S(2)$ and $X_1^I(2)$ in the inequality $\delta X_1^S(2) > d_1 \ge \delta X_1^I(2)$, we obtain

$$\frac{\delta p_{21}(1-d_2)}{1\text{-}\delta p_{22}} > d_1 \ge \delta p_{21}(1-d_2) + \delta p_{22} d_1 .$$

It is enough to observe that $\delta p_{21}(1-d_2)/(1-\delta p_{22}) > d_1$ implies that $\delta p_{21}(1-d_2) > d_1(1-\delta p_{22})$, whereas $d_1 \ge \delta p_{21}(1-d_2) + \delta p_{22} d_1$ implies $\delta p_{21}(1-d_2) \le d_1(1-\delta p_{22})$.

Next consider case (b). In this case inequalities (1) to (4) reduce to

$$X_1^S(2) \le p_{21}(1-\delta X_2^I(1)) + \delta p_{22} X_1^S(2) , \tag{30}$$

$$X_1^I(2) \ge p_{21}(1-\delta X_2^S(1)) + p_{22} d_1 , \tag{31}$$

$$X_2^S(1) \le \delta p_{11} X_2^S(1) + p_{12}(1-d_1) , \tag{32}$$

$$X_2^I(1) \ge \delta p_{11} X_2^I(1) + p_{12}(1-\delta X_1^S(2)) . \tag{33}$$

Straightforward substitutions yield

$$X_1^S(2) \le \frac{p_{21}}{1+\delta(1-p_{11}-p_{22})} , \tag{34}$$

$$X_1^I(2) \ge p_{21}\left[1 - \frac{\delta p_{12}(1-d_1)}{1-\delta p_{11}}\right] + p_{22} d_1 . \tag{35}$$

Since $\delta X_1^S(2) > d_1$, it follows that $\delta p_{21}/[1+\delta(1-p_{11}-p_{22})] > d_1$. However, from the condition that $d_1 \ge \delta X_1^I(2)$, one obtains the reverse inequality.

Next consider case (c). In this case it is clear that

$$X_1^S(2) \le p_{21}(1-d_2) + \delta p_{22} X_1^S(2)\,, \tag{36}$$

$$X_1^I(2) \ge p_{21}(1-d_2) + \delta p_{22} X_1^I(2)\,. \tag{37}$$

Obviously, $X_1^S(2) > X_1^I(2)$ implies $p_{21}(1-d_2)/(1-\delta p_{22}) \ge X_1^S(2) > X_1^I(2) \ge p_{21}(1-d_2)/(1-\delta p_{22})$, which is a contradiction.

Finally we examine case (d). It is easy to see that the inequalities (1) and (2) imply $X_1^S(2) \le p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $X_1^I(2) \ge p_{21}/[1+\delta(1-p_{11}-p_{22})]$, which in turn imply, since $X_1^S(2) \ge X_1^I(2)$, that $X_1^S(2) = X_1^I(2)$.

STEP 2. Consider the case where $X_1^S(2) > X_1^I(2)$, and $X_2^S(1) > X_2^I(1)$.
We distinguish 9 cases.

(a) $d_1 \ge \delta X_1^S(2) > \delta X_1^I(2)$ and $d_2 \ge \delta X_2^S(1) > \delta\ X_2^I(1)$,
(b) $\delta X_1^S(2) > \delta X_1^I(2) > d_1$ and $d_2 \ge \delta X_2^S(1) > \delta X_2^I(1)$,
(c) $\delta X_1^S(2) > d_1 \ge \delta X_1^I(2)$ and $d_2 \ge \delta X_2^S(1) > \delta X_2^I(1)$,
(d) $d_1 \ge \delta X_1^S(2) > \delta X_1^I(2)$ and $\delta X_2^S(1) > \delta X_2^I(1) > d_2$,
(e) $\delta X_1^S(2) > \delta X_1^I(2) > d_1$ and $\delta X_2^S(1) > \delta X_2^I(1) > d_2$,
(f) $\delta X_1^S(2) > d_1 \ge \delta X_1^I(2)$ and $\delta X_2^S(1) > \delta X_2^I(1) > d_2$,
(g) $d_1 \ge \delta X_1^S(2) > \delta X_1^I(2)$ and $\delta X_2^S(1) > d_2 \ge \delta X_2^I(1)$,
(h) $\delta X_1^S(2) > \delta X_1^I(2) > d_1$ and $\delta X_2^S(1) > d_2 \ge \delta X_2^I(1)$,
(i) $\delta X_1^S(2) > d_1 \ge \delta X_1^I(2)$ and $\delta X_2^S(1) > d_2 \ge \delta X_2^I(1)$.

We show that all 9 cases lead to contradictions.
First consider case (a). From inequalities (1) and (2) it follows that

$$X_1^S(2) \le p_{21}(1-d_2) + p_{22} d_1\,, \tag{38}$$

$$X_1^I(2) \ge p_{21}(1-d_2) + p_{22} d_1\,. \tag{39}$$

This implies that, since $X_1^S(2) \ge X_1^I(2)$,

$$X_1^S(2) = X_2^S(1) = p_{21}(1-d_2) + p_{22} d_1\,,$$

when the proof follows from Step 1.

We then consider case (b). (The argument would be similar in case (d)). Clearly in this case

$$X_1^S(2) \le p_{21}(1-d_2) + \delta p_{22} X_1^S(2)\,, \tag{40}$$

$$X_1^I(2) \ge p_{21}(1-d_2) + \delta p_{22} X_1^I(2)\,. \tag{41}$$

It is easy to see that, $X_1^S(2) > X_1^I(2)$ implies that $p_{21}(1-d_2)/(1-\delta p_{22}) \ge X_1^S(2) > X_1^I(2) \ge p_{21}(1-d_2)/(1-\delta p_{22})$, which is a contradiction.

Next we take up case (c). (Case (g) can be treated symmetrically). In this case inequalities (1) and (2) simplify to

$$X_1^S(2) \le p_{21}(1-d_2) + \delta p_{22} X_1^S(2)\,, \tag{42}$$

$$X_1^I(2) \ge p_{21}(1-d_2) + p_{22} d_1\,. \tag{43}$$

Solving, $X_1^S(2) \leq p_{21}(1-d_2)/(1-\delta p_{22})$. Substituting for $X_1^S(2)$ and $X_1^I(2)$ in the inequality $\delta X_1^S(2) > d_1 \geq \delta X_1^I(2)$, we obtain

$$\frac{\delta p_{21}(1-d_2)}{1-\delta p_{22}} > d_1 \geq \delta p_{21}(1-d_2) + \delta p_{22} d_1 \, .$$

It is enough to observe that $\delta p_{21}(1-d_2)/(1-\delta p_{22}) > d_1$ implies that $\delta p_{21}(1-d_2) > d_1(1-\delta p_{22})$, whereas $d_1 \geq \delta p_{11}(1-d_2) + \delta p_{21} d_1$ implies $\delta p_{21}(1-d_2) \leq d_1(1-\delta p_{22})$.

Next we examine the case where (e) holds. It is obvious, that the inequalities (1) and (2) imply $X_1^S(2) \leq p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $X_1^I(2) \geq p_{21}/[1+\delta(1-p_{11}-p_{22})]$, which in turn imply that $X_1^S(2) = X_1^I(2)$. Again the proof follows from Step 1.

Penultimately, consider case (f). (The proof for case (h) would be similar). In this case inequalities (1) to (4) reduce to

$$X_1^S(2) \leq p_{21}(1-\delta X_2^I(1)) + \delta p_{22} X_1^S(2) \, , \tag{44}$$

$$X_1^I(2) \geq p_{21}(1-\delta X_2^S(1)) + p_{22} d_1 \, , \tag{45}$$

$$X_2^S(1) \leq \delta p_{11} X_2^S(1) + p_{12}(1-d_1) \, , \tag{46}$$

$$X_2^I(1) \geq \delta p_{11} X_2^I(1) + p_{12}(1-\delta X_1^S(2)) \, . \tag{47}$$

Solving, we obtain

$$X_1^S(2) \leq \frac{p_{21}}{1+\delta(1-p_{11}-p_{22})} \, , \tag{48}$$

$$X_1^I(2) \geq p_{21}\left[1 - \frac{\delta p_{12}(1-d_1)}{1-\delta p_{11}}\right] + p_{22} d_1 \, , \tag{49}$$

$$X_2^S(1) \leq \frac{p_{12}(1-d_1)}{1-\delta p_{11}} \, , \tag{50}$$

$$X_2^I(1) \geq \frac{p_{12}}{1+\delta(1-p_{11}-p_{22})} \, . \tag{51}$$

Since $\delta X_1^S(2) > d_1$, it follows that $\delta p_{21}/[1+\delta(1-p_{11}-p_{22})] > d_1$. However $d_1 \geq \delta X_1^I(2)$ implies that $d_1 \geq \delta p_{21}/[1+\delta(1-p_{11}-p_{22})]$, which is a contradiction.

Finally, consider case (i). In this case, the inequalities simplify to

$$X_1^S(2) \leq p_{21}(1-d_2) + \delta p_{22} X_1^S(2) \, , \tag{52}$$

$$X_1^I(2) \geq p_{21}(1-\delta X_2^S(1)) + p_{22} d_1 \, , \tag{53}$$

$$X_2^S(1) \leq \delta p_{11} X_2^S(1) + p_{12}(1-d_1) \, , \tag{54}$$

$$X_2^I(1) \geq p_{11} d_2 + p_{12}(1-\delta X_1^S(2)) \, . \tag{55}$$

It is obvious that the solution involves

$$X_1^S \leq \frac{p_{21}(1-d_2)}{1-\delta p_{22}}, \tag{56}$$

$$X_1^I \geq p_{21}\left[1-\frac{\delta p_{12}(1-d_1)}{1-\delta p_{11}}\right]+p_{22}d_1, \tag{57}$$

$$X_2^S \leq \frac{p_{12}(1-d_1)}{1-\delta p_{11}}, \tag{58}$$

$$X_2^I \geq p_{11}d_2+p_{21}\left[1-\frac{\delta p_{21}(1-d_2)}{1-\delta p_{22}}\right]. \tag{59}$$

Since $\delta X_1^S > d_1$, it follows that $\delta p_{21}(1-d_2) > d_1(1-\delta p_{22})$. However, $d_1 \geq \delta X_1^I(2)$ implies that $d_1 \geq \delta p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $d_2 \geq \delta X_2^I$ implies that $d_2 \geq \delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$. Substituting the values into the previous inequalities yield a contradiction. ■

5. APPENDIX 2

We distinguish four cases,

(A) $d_1 \geq \delta X_1(2)$ and $d_2 \geq \delta X_2(1)$,
(B) $d_1 < \delta X_1(2)$ and $d_2 < \delta X_2(1)$,
(C) $d_1 < \delta X_1(2)$ and $d_2 \geq \delta X_2(1)$,
(D) $d_1 \geq \delta X_1(2)$ and $d_2 < \delta X_2(1)$.

In case (A), it follows from equations (9) and (10) that

$$X_1(2) = p_{21} - p_{21}d_2 + p_{22}d_1, \tag{60}$$

$$X_2(1) = p_{12} - p_{12}d_1 + p_{11}d_2. \tag{61}$$

In case (B), equations (9) and (10) simplify to

$$X_1(2) = p_{21}[1-\delta X_2(1)] + p_{22}\delta X_1(2), \tag{62}$$

$$X_2(1) = p_{11}\delta X_2(1) + p_{12}[1-\delta X_1(2)]. \tag{63}$$

Straightforward substitution yields

$$X_1(2) = \frac{p_{21}}{1+\delta(1-p_{11}-p_{22})}, \tag{64}$$

$$X_2(1) = \frac{p_{12}}{1+\delta(1-p_{11}-p_{22})}. \tag{65}$$

Next consider case (C). It is obvious that

$$X_1(2) = p_{21}(1-d_2) + \delta p_{22}X_1(2), \tag{66}$$

$$X_2(1)=p_{11}d_2+p_{12}(1-\delta X_1(2)) . \tag{67}$$

Solving, we obtain

$$X_1(2)=\frac{p_{21}(1-d_2)}{1-\delta p_{22}} , \tag{68}$$

$$X_2(1)=p_{11}d_2+p_{12}\left[1-\frac{\delta p_{21}(1-d_2)}{1-\delta p_{22}}\right] . \tag{69}$$

Case (D) is symmetric to (C). The outcome involves

$$X_1(2)=p_{21}d_1+p_{22}\left[1-\frac{\delta p_{12}(1-d_1)}{1-\delta p_{11}}\right] , \tag{70}$$

$$X_2(1)=\frac{p_{12}(1-d_1)}{1-\delta p_{11}} . \tag{71}$$

PROOF OF PROPOSITION 2. First consider case (A). Since $d_1 \geq \delta X_1(2)$, it follows that $d_1(1-\delta p_{22}) \geq \delta p_{21}(1-d_2)$. Substituting from the proposition hypothesis for the values of d_1 and d_2, we obtain $p_{12}>p_{12}$.

In case (B) there is nothing to prove since $X_1(2)=p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $X_2(1)=p_{12}/[1+\delta(1-p_{11}-p_{22})]$. Next consider case (C). (Case (D) can be treated symmetrically). In this case $X_2(1)=p_{11}d_2+p_{12}[1-\delta p_{21}(1-d_2)/(1-\delta p_{22})]$. Substituting for $X_2(1)$ in $d_2 \geq \delta X_2(1)$, we obtain that $d_2 \geq \delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$, which contradicts the hypothesis of the proposition. ∎

PROOF OF PROPOSITION 3. (i) In case (A), the condition that $d_1 \geq \delta X_1(2)$ implies $d_1(1-\delta p_{22}) \geq \delta p_{21}(1-d_2)$, which contradicts the hypothesis of the proposition.

Next consider case (B). In this case $X_1(2)=p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $X_2(1)=p_{12}/[1+\delta(1-p_{11}-p_{22})]$. But then $d_2<\delta X_2(1)$ implies that $d_2<\delta p_{12}/[1+\delta(1-p_{11}-p_{22})]$, which contradicts the hypothesis of the proposition. If (C) holds then there is nothing to prove. Lastly consider case (D). In this case $X_1(2)=p_{21}d_1+p_{22}[1-\delta p_{12}(1-d_1)/(1-\delta p_{11})]$. Substituting for X_1 in the condition $d_1 \geq \delta X_1(2)$, one obtains that $d_1 \geq \delta p_{21}/[1+\delta(1-p_{11}-p_{22})]$. The conditions of the proposition however yield the reverse inequality.

(ii) The proof in this case is similar to that of part (i). ∎

PROOF OF PROPOSITION 4. Clearly, if (A) holds, then there is nothing to prove. If (B) holds, then $X_1(2)=p_{21}/[1+\delta(1-p_{11}-p_{22})]$ and $X_2(1)=p_{12}/[1+\delta(1-p_{11}-p_{22})]$. Substituting the values of d_1 and d_2 in either of the proposition hypothesis one obtains a contradiction. Next consider case (C). (The proof in case (D) would be symmetrical). As before $X_1(2)=p_{21}(1-d_2)/(1-\delta p_{22})$. From condition (C) this implies that, $d_1(1-\delta p_{22})<\delta p_{21}(1-d_2)$. But this contradicts the hypothesis of the proposition. ∎

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