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A NOTE ON EXTRAPOLATIVE EXPECTATIONS IN A DYNAMIC PRODUCER-CONSUMER MARKET†

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Abstract: The stability of dynamic producer-consumer markets is examined under extrapolative expectations with continuous time scale. We drop the simplifying assumption that the producers have the same speed of output adjustments.

1. INTRODUCTION

In our earlier paper, Szidarovszky and Molnàr (1994), we have proved the global asymptotical stability of a dynamic producer-consumer market under the assumption that the speeds of adjustment are the same for all firms. In this short note, we will prove the same result without that simplifying condition, extending it to the general case.

2. THE MODEL

Following Szidarovszky and Molnàr (1994), let N denote the number of producers supplying a service or a certain commodity, let $C_k(x_k)=B_kx_k^2+b_kx_k+c_k$ be the cost of firm k, where x_k is his output. Let p denote the price, then $Dp+d$ is the market demand function. Here $B_k>0$, $b_k\geq 0$, $c_k\geq 0$, $D<0$, and $d>0$ are given constants. It is also assumed that the expectation of firm k on the price is adjusted by equation

$$p_k^E(t)=p(t)+M_k\dot{p}(t)\,, \tag{1}$$

where $M_k\geq 0$ is a given constant. The case of $k=0$ corresponds to the market; that is, $p_0^E(t)$ is the expectation of the market on the price. At each time period $t\geq 0$, the expected profit maximizing output of firm k is given as

$$x_k^*(t)=-\frac{1}{2B_k}(b_k-p_k^E(t))\,, \tag{2}$$

and we assume that his output is adjusted proportionally to the difference of the profit maximizing and actual outputs:

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$$\dot{x}_k(t)=K_k(x_k^*(t)-x_k(t)), \qquad (k=1,2,\cdots,N) \tag{3}$$

where $K_k>0$ is a given constant. It is also assumed that the price moves as directed by the shortage, rising if the shortage is positive, decreasing if negative, and remaining stationary if zero. That is,

$$\dot{p}(t)=K\left(Dp_0^E(t)+d-\sum_{k=1}^{N} x_k(t)\right), \tag{4}$$

where $K>0$ is a given constant. Combine the above equations to derive the system equation

$$\dot{z}=\begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & -\frac{K_1M_1}{2B_1} \\ 0 & 1 & \cdots & 0 & 0 & -\frac{K_2M_2}{2B_2} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & -\frac{K_{N-1}M_{N-1}}{2B_{N-1}} \\ 0 & 0 & \cdots & 0 & 1 & -\frac{K_NM_N}{2B_N} \\ 0 & 0 & \cdots & 0 & 0 & 1-KDM_0 \end{pmatrix}^{-1} \cdot$$

$$\begin{pmatrix} -K_1 & 0 & \cdots & 0 & 0 & \frac{K_1}{2B_1} \\ 0 & -K_2 & \cdots & 0 & 0 & \frac{K_2}{2B_2} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -K_{N-1} & 0 & \frac{K_{N-1}}{2B_{N-1}} \\ 0 & 0 & \cdots & 0 & -K_N & \frac{K_N}{2B_N} \\ -K & -K & \cdots & -K & -K & KD \end{pmatrix} z+f, \tag{5}$$

where $z=(x_1, x_2, \cdots, x_N, p)^T$, and f is a given constant vector.

3. STABILITY ANALYSIS

We will now prove that for all positive values of K_k, M_k, B_k, K, and negative value of D, system (5) is globally asymptotically stable by verifying that all eigenvalues of the coefficient matrix have negative real parts. The eigenvalue equation can be written as

$$-K_k u_k + \frac{K_k}{2B_k} v = \lambda\left(u_k - \frac{K_k M_k}{2B_k} v\right) \qquad (k=1, 2, \cdots, N) \tag{6}$$

$$-K \sum_{k=1}^{N} u_k + KDv = \lambda(1 - KDM_0)v .$$

From the first equation we have

$$u_K = \frac{K_k(1+\lambda M_k)}{2B_k(K_k+\lambda)} v , \qquad (k=1, 2, \cdots, N) \tag{7}$$

where we assume that $\lambda \neq -K_k$. If $v=0$, then the eigenvector becomes zero. Therefore, we may assume that $v \neq 0$. Substituting this relation into the second equation, a single equation is obtained for λ:

$$-K \sum_{k=1}^{N} \frac{K_k(1+\lambda M_k)}{2B_k(K_k+\lambda)} + KD = \lambda(1-KDM_0) . \tag{8}$$

Let $\lambda = \alpha + i\beta$ be a real (or complex) root, then

$$-K \sum_{k=1}^{N} \frac{K_k(1+\alpha M_k + i\beta M_k)}{2B_k(K_k+\alpha+i\beta)} + KD = (\alpha+i\beta)(1-KDM_0) . \tag{9}$$

Compare the real parts of the left and right hand sides to see that

$$-K \sum_{k=1}^{N} \frac{K_k((1+\alpha M_k)(K_k+\alpha)+\beta^2 M_k)}{2B_k((K_k+\alpha)^2+\beta^2)} + KD = \alpha(1-KDM_0) . \tag{10}$$

If $\alpha \geq 0$, then the left hand side is negative and the right hand side is non-negative; therefore, only $\alpha < 0$ is possible.

Finally we note that the special case of $M_k = 0$ corresponds to Cournot expectations (see, for example, Okuguchi and Szidarovszky, 1990); therefore, we have also generalized the corresponding earlier results on Cournot expectations.

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