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MODELS OF DYNAMIC EFFICIENCY IN THE PUBLIC SECTOR

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Abstract: Recently nonparametric techniques have been increasingly applied to measure efficiency of public sector organizations. These techniques are essentially based on the concept of Pareto efficiency. These are generalized here in several dynamic dimensions and their policy implications discussed briefly.

1. INTRODUCTION

Recently the nonparametric techniques known as 'data envelopment analysis' (DEA) have been increasingly applied in managerial economics to measure the efficiency of public sector organizations. Thus Ganley and Cubbin (1992), Fried et al. (1993) and Charnes et al. (1994) have discussed these recent developments in this field. However the DEA model and its current generalizations which are based on the concept of Pareto efficiency have been mostly static, since these are based exclusively on current inputs, thus biasing the efficiency comparisons against the capital-intensive processes. Capital inputs have their impact on output over time and technological innovations frequently change the marginal productivity of the inputs between years. Furthermore the production process is likely to be dynamic and time varying in the sense that the decision-making units (DMUs) may take more than one time period to adjust their decision variables to the desired or even optimal levels.

Farrell (1957) who first developed these nonparametric techniques later known as DEA models mentioned some of these dynamic issues when he developed the production frontier model and applied it to the cross-section data of agricultural farms in UK and USA. Farrell developed a firm-specific measure of technical efficiency by defining the production frontier as the maximum output obtained from a given set of inputs. One may mention two basic reasons why the behavior underlying the production frontier is likely to be dynamic. First of all, capital inputs have a multiperiod dimension, since they generate outputs in future periods. Yet the current DEA applications are based exclusively on current inputs, thus biasing efficiency comparisons against the capital-intensive processes. Secondly, the firms or other public sector DMUs which are compared in terms of relative efficiency by the Pareto-efficiency criterion may take more than one time period to learn and adapt to changing environments. This intertemporal adaptivity is ignored in current DEA models.

Our object here is to develop a class of models of dynamic efficiency, which provides efficiency comparisons over time. This also provides a link with the concepts of static efficiency. Two types of dynamic issues are analyzed here. One involves the use of incremental capital inputs leading to incremental outputs and the second introduces adjustment costs so as to modify the production function. Both these issues raise questions about intertemporal efficiency and how it is related to static efficiency.

2. MODELS OF DYNAMIC EFFICIENCY

Consider a sample of N DMUs each producing a single output (λ_j) with m inputs. The output vector λ has the input coefficient matrix A of dimension m by N which involves only the current inputs in the static framework. To compare the relative efficiency of the reference unit, i.e., the k-th DM with its current input vector X_k, we set up the following linear programming (LP) model:

$$\max z = \mathrm{e}'\lambda$$

$$\text{s.t.} \quad \mathrm{A}\lambda \leq X_k;\ \lambda \geq 0 \tag{1}$$

where e is a column vector with each element unity and the prime denotes a transpose. If the input matrix A and the input vector X_k of the reference DMU are available from the observed data, then the LP model (1) can be solved for the optimal λ^*. The dual of this model is:

$$\min g_k = \beta' X_k$$

$$\text{s.t.} \quad \beta'\mathrm{A} \geq \mathrm{e}';\ \beta \geq 0 \tag{2}$$

which yields the optimal dual vector β^* where $\mathrm{e}'\lambda^* = \beta^{*\prime} X_k$. The static efficiency model (2) may be presented in several other variations. For instance the constraint $\beta \geq \varepsilon$ e where ε is a small positive constant may be added so that the positivity of the optimal vector β^* is ensured. Also, the primal model (1) can be modified as

$$\min h_k = \theta$$

$$\text{s.t.} \quad \mathrm{A}\lambda \leq \theta X_k;\ \mathrm{e}'\lambda = 1,\ \lambda \geq 0 \tag{3}$$

where θ is a scalar representing the proportional reduction to all inputs of the reference unit DMU_k, the unit being evaluated, to improve efficiency. Here the reference DMU is efficient if and only if $\theta^* = 1.0$ and the slack variables for the constraint are zero.

In terms of the simpler models (1) and (2) one could easily characterize the production frontier as a best practice production function. For the LP model (2) the reference DMU_k is efficient if it holds

$$\sum_{i=1}^{m} \beta_i^* a_{ik} = 1\ , \quad \text{or} \quad \sum_{i=1}^{m} \beta_i^* x_{ik} = y_k \tag{4}$$

with a zero slack variable. Let $y_k^* = \sum_i \beta_i^* x_{ik}$ denote the optimal or potential output. The reference unit DMU_k is not efficient if its observed output (y_k) is less than the optimal output (y_k^*). A similar interpretation can be given of the output maximization model (1). The reference unit DMU_k is efficient if it holds that

$$\sum_{j=1}^{n} a_{ij}\lambda_j^* = x_{ik}, \qquad \text{all} \quad i = 1, 2, \cdots, m. \tag{5}$$

If $x_{ik}^* = \sum_{j=1}^{n} a_{ij}\lambda_j^*$ denotes the optimal inputs, then the reference DMU_k is inefficient if at least for one input i the inequality $x_{ik}^* < x_{ik}$ holds.

Now consider a dynamic generalization of the output maximization model (1). Assume that each input can be used either as current inputs or as incremental (capital) inputs. Then the dynamic model in one version can be specified as follows:

$$\max z(T-1) = \sum_{t=0}^{T-1} e'\lambda(t)$$

$$\text{s.t.} \qquad A\lambda(t) + B\Delta\lambda(t) \le X_k(t) \tag{6}$$

$$\lambda(t) \ge 0;\ t = 0, 1, 2, \cdots, T-1$$

where B is the incremental capital coefficient matrix and $\Delta\lambda(t) = \lambda(t+1) - \lambda(t)$ denotes the incremental outputs. A second version of the model uses two groups of inputs with $X_k(t)$ for the current inputs and $I_k(t)$ for the incremental capital inputs so that the constraints become

$$A\lambda(t) \le X_k(t);\ B\Delta\lambda(t) \le I_k(t) \tag{7}$$

$$\lambda(t) \ge 0;\ t = 0, 1, 2, \cdots, T-1.$$

In a more general set up the observed coefficient matrices A and B could be time-varying as A(t), B(t), and the objective function (6) could involve minimizing smoothing costs, e.g.,

$$\max z(T-1) = \sum_{t=0}^{T-1} [e'\lambda(t) - (\lambda(t) - \lambda(t-1))'(\lambda(t) - \lambda(t-1))].$$

Also the case of multiple outputs can be handled through an output vector $\lambda_s(t)$ for each DMUs as:

$$\max z(T-1) = \sum_{t=0}^{T-1} \sum_{s=1}^{N} p'(t)\lambda_s(t)$$

$$\text{s.t.} \qquad \sum_{s=1}^{N} [A_s\lambda_s(t) + B\Delta\lambda_s(t)] \le X_k(t) \tag{8}$$

$$\lambda_s(t) \ge 0;\ s = 1, 2, \cdots, N;\ t = 0, 1, \cdots, T-1$$

where the price vector $p(t)$ is provided either by the observed market prices or the subjective weights assumed to be common to all DMUs. In several applied studies by (Dyson and Thanassoulis (1988), and Sengupta (1990), the weights for the multiple outputs are either assumed to be equal or determined by the statistical criterion of canonical correlation theory.

A different type of dynamic generalization of the static model (1) may be suggested in terms of the adjustment cost approach. Holt et al. (1960) originally assumed that adjustment costs arise from production activities with changes in labor force. Treadway (1970), Lucas (1967) and Pindyck (1982) define adjustment costs as a function of investment and treat it as a deduction from total output or revenue. Following this latter approach one could define gross investment ($I_j(t)$) for each DMU$_j$ as

$$I_j(t) = \dot{K}_j + \delta_j K_j(t)$$

or

$$\dot{K}_j = v_j \lambda_j(t) - \delta_j K_j(t)$$

where $\dot{K}_j$ is the time rate of change of capital stock, δ_j is the fixed rate of depreciation and v_j is the investment-output coefficient. Assume a linear adjustment cost function $C_j(I_j) = \gamma_j v_j \lambda_j(t)$ and a linear capacity constraint for capital stock as $K_j(t) \leq \bar{K}_j$. Then one could specify the following intertemporal model of dynamic optimization:

$$\begin{aligned} \max z(T) = \int_0^T \sum_{j=1}^N [\lambda_j(t) - C_j(I_j)] \mathrm{d}t \\ \text{s.t.} \quad \sum_{j=1}^N a_{ij} \lambda_j(t) \leq x_{ik}(t) \\ \dot{K}_j(t) = v_j \lambda_j(t) - \delta_j K_j(t) \\ K_j \leq \bar{K}_j;\ \lambda_j(t) \geq 0, j = 1, 2, \cdots, N . \end{aligned} \tag{9}$$

This is an optimal control problem with inequality constraints for each t which are discrete and a continuous process of change of capital stock. The Hamiltonian for this problem is

$$H = \sum_{j=1}^N [(1 - \gamma_j v_j) \lambda_j(t) + \pi_j (v_j \lambda_j(t) - \delta_j K_j(t))] .$$

If an optimal program $\{\lambda(t); 0 \leq t \leq T\}$ exists, then by the standard results of control theory (Lewis, 1986) it must satisfy the dynamic adjoint equations for all $j = 1, 2, \cdots, N$:

$$\begin{aligned} \dot{K}_j = v_j \lambda_j(t) - \delta_j K_j(t);\ K_j(0) \text{ given} \\ \dot{\pi}_j = \delta_j \Pi_j(t) + s_j(t) \end{aligned} \tag{10a}$$

where $s_j(t)$ is the Lagrange multiplier for the capacity constraint on capital stock. Moreover at each moment of time t the control vector $\lambda(t)$ must maximize the

Hamiltonian—subject to the inequality constraints on control. This yields the LP model:

$$\max \sum_{j=1}^{N} [(1-\gamma_j v_j + v_j \pi_j(t))\lambda_j(t)]$$

$$\text{s.t.} \quad \sum_{j=1}^{N} a_{ij}\lambda_j(t) \leq x_{ik}(t);\ \lambda_j(t) \geq 0\,. \tag{10b}$$

The adjoint vector $\pi(t)$ must be continuous and also satisfy on the optimal trajectory a transversality condition

$$\lim_{t\to T} \pi^*(t) = 0 \tag{10c}$$

and a "jump condition" for continuity of $\pi^*(t)$:

$$\lim_{t\to T} [\pi^*(\tau + r) - \pi^*(\tau - r)] = 0 \tag{10d}$$

for any optimal path of $\pi(t)$ crossing a phase boundary at time $t=\tau$.

Several features of this dynamic efficiency model must be pointed out. First of all, the entire time profile of the efficiency path is characterized by the optimal program: $\{\lambda^*(t), \pi^*(t); 0 \leq t \leq T\}$. The two sets of adjoint equations (10a) describe the path of adjustment in the different regions of the phase space. Secondly, a static model of the form (10b) is embedded in every dynamic optimal program. This static model differs from the standard DEA models (1) and (2) in two respects. One is that both the dynamic shadow prices denoted by $\pi_j^*(t)$ and the marginal adjustment cost parameters γ_j enter into the objective function of (10b) but since $\pi_j^*(t)$ can be either positive, negative or zero, it may increase, decrease or keep unaltered the value of output, although the adjustment costs tend to reduce the objective function. Furthermore by the duality principle DMU_k will be efficient if it satisfies the following condition

$$\sum_{i=1}^{m} \beta_i^*(t) a_{ik} + v_k(\gamma_k - \pi_k^*(t)) = 1;\ \beta_i^*(t) \geq 0\,. \tag{11}$$

This condition will collapse into the static efficiency condition (4) if and only if $v_k(\gamma_k - \pi_k^*(t)) = 0$, which implies that either $\pi_k^*(t) = \gamma_k$ or, $v_k = 0$ or both. The most interesting case is the first one, i.e., $\pi_k^*(t) = \gamma_k$ which says that the dynamic shadow price of investment equals marginal adjustment cost at the optimum.

Thirdly, the optimality condition(11) is only a necessary part for total optimality and the sufficient condition is that

$$\dot{\pi}_k^* = \delta_k \pi_k^*(t) + s_k^*$$

along with (10c) and (10d). This is sometimes called the perfect foresight condition, which specifies the future evolution of the time path of $\pi^*(t)$. So long as the perfect foresight condition is not fulfilled, the investment path is not optimal and hence any forecast $\hat{\pi}_k(t)$ different from $\pi_k^*(t)$ when used in the constraint (11) would fail the dynamic efficiency test. Also, if the optimal value s_k^* of the capital constraint is zero, it implies another form of inefficiency due to less than full capacity utilization of capital stock.

Finally, one has to note the transversality condition (10c), which says that the policy of adding to capital in the long run has zero worth. If one lets $T \to \infty$ and adjoins a discount function $\exp(-\rho t)$ in the integrand of (9), the transversality condition becomes

$$\lim_{t\to\infty} e^{-\rho t}\pi^*(t) = 0 = \lim_{t\to\infty} = e^{-\rho t}\pi^*(t)K^*(t)$$

i.e., the long run value of optimal capital stock is zero.

A second way to introduce adjustment cost is to relate it to past investments. The adjustment const has been generally viewed in investment theory in a continuous form. Thus Treadway (1970) and more recently Pindyck (1982) derived the optimal profile of a firm's output vector, when its capital accumulation is subject to a continuous investment demand function. In the learning by doing model Lucas (1967) discussed technical progress in a continuous form by relating it to cumulative past investments. Following this version of the adjustment cost approach we may consider an example where the level of current input demand is dependent in part on the program of investment followed in the past:

$$\mathrm{A}\lambda(t) + \int_0^t G(s)\lambda(s)\,\mathrm{d}s \leq X_k(t)\,. \tag{12a}$$

Here $G(s)$ is the matrix of investment coefficients in the past and any input is assumed to be used either as a current input or as an investment input. In terms of continuous change this specification can be written as

$$\mathrm{A}\dot{\lambda}(t) + G(t)\lambda(t) + \dot{q}(t) = \dot{X}_k(t) \tag{12b}$$

where $q(t)$ is a nonnegative slack variable. A suitable objective function for this model is:

$$\max z(T) = \int_0^T [\mathrm{e}'\lambda(t) - C(\dot{\lambda}(t), \lambda(t))]\,\mathrm{d}t$$

where $C(\cdot)$ denotes a scalar objective function. Note that one can also incorporate the effect of cumulative investment as in (12a) to define a dynamic efficiency model as follows:

$$\max z(T) = \int_0^T \mathrm{e}'\lambda(t)\,\mathrm{d}t$$

$$\text{s.t.} \quad (12\text{a}) \text{ and } \lambda(t) \geq 0\,. \tag{13a}$$

This is a continuous LP problem and its dual can be derived as:

$$\min g_k(T) = \int_0^T \beta'(t)X_k(t)\mathrm{d}t$$

$$\text{s.t.} \quad \beta'(t)\mathrm{A} \geq \mathrm{e}' - \int_t^T \beta'(s)G(s)\mathrm{d}s \tag{13b}$$

$$\beta(t) \geq 0,\ 0 \leq t \leq T\,.$$

Let $\{\beta^*(t), 0 \leq t \leq T\}$ define the optimal path. This path will not satisfy the efficiency condition (4) of the static DEA model, unless for each t it holds:

$$\int_t^T \beta^{*\prime}(s)G(s)\mathrm{d}s = 0 \tag{13c}$$

i.e., the optimal value of future investments is close to zero. For the optimal program one must have

$$\int_0^T \mathrm{e}'\lambda^*(t)\mathrm{d}t = \int_0^T \beta^{*\prime}(t)X_k(t)\mathrm{d}t$$

and for some t it must hold for the efficient DMU_k:

$$\sum_{i=1}^m \beta_i^*(t)a_{ik} + \int_t^T \left(\sum_{i=1}^m \beta_i^*(s)g_{ik}(s)\right)\mathrm{d}s = 1 \tag{13d}$$

$$\beta_i^*(t) \geq 0;\ i = 1, 2, \cdots, m\,.$$

The second term on the left hand side represents the expected value of future investments, whereas the first part is the value of the current inputs.

On comparing the two efficiency conditions (11) and (13d) of the two adjustment cost models, it is apparent that they are very similar. Whereas condition (11) involves the dynamic shadow price variable $\pi_k^*(t)$ which must be correctly forecast for ensuring dynamic efficiency, the condition (13d) involves future prices $\beta^*(t)$, $\beta^*(t+1), \cdots, \beta^*(T)$ to be estimated.

3. AN EMPIRICAL APPLICATION

This section describes an empirical application of the incremental capital inputs model (6) in its dual form and analyzes the potential usefulness of the two forms of dynamic efficiency developed earlier.

The empirical application utilizes input-output data in logarithmic units for selected public elementary school districts in California for the years 1977–78, 1979–80 and soon up to 1987–88, with $T=6$. This data set was used in previous studies of static DEA models by Sengupta (1989), Sengupta and Sfeir (1990). Statistics of enrollment, average teacher salary, standardized test scores were all obtained from the published official statistics. Out of a larger set of 35 school districts, 25 were selected in three contiguous counties of Santa Barbara, Ventura and San Luis Obispo on the basis of separate homogeneity tests based on the Goldfeld and Quandt (1965) statistic. On the basis of the standardized test scores in reading, writing, spelling and mathematics, a composite output is defined as an average. As input variables we had a choice of eight variables, of which the following four were utilized in the dynamic LP model which is dual to (6): $x_1(t)=$ average instructional expenditures with teachers having experience of 3 years or more, $x_2(t)=$ diversity index measured by minority enrollment, $x_3(t)=$ average class size and $x_4(t)=$ average tax base of the district. Of these input variables the first is most important in the sense of knowledge-based capital stock and the second reflects a growing trend for the state economy due to increasing minority enrollment. The dual model corresponding to (6) solves the following LP model:

$$\min g_k(T-1)=\sum_{t=0}^{T-1}\beta'(t+1)X_k(t)+\beta'(0)\mathrm{B}\lambda(0)$$

$$\text{s.t.}\quad \beta'(t)\mathrm{B}+\beta'(t+1)(\mathrm{A}-\mathrm{B})\geq e'$$

$$\beta(t)\geq 0,\ t=0,1,2,\cdots,T-1\ .$$

The optimal values of the input coefficients $\beta^*(t)$ are reported in Table 1 along with the value of the average optimal output (y^*) and the average estimates for the whole period 1977–80. As expected the changes in $\beta_1^*(t)$ over time are most prominent and this is followed by $\beta_2^*(t)$ and $\beta_2^*(t)$. The estimates β_i^* for the whole period 1977–88 appear to be very near the average value for the $\bar{\beta}_i^*(t)=(1/6)\sum_{t=1}^{6}\beta_i^*(t)$. This suggests that the steady state approximation of the dynamic model would be appropriate in this case. Secondly, the last row of Table 1 shows the proportion of the efficient DMUs in each t. Although there is slight variation in this proportion with a slightly increasing trend, this appears to be very small. Since it is not possible to apply the standard statistical tests to these estimates $\beta_i^*(t)$ one cannot be definitive on this issue. Thirdly, when we average the input output data for a moving average of three years, i.e., 1977–82, 1979–84, 1981–86

TABLE 1. Estimates of the dynamic production frontier

	1977–78	79–80	81–82	83–84	85–86	87–88	77–88
$\beta_1^*(t)$	0.232	0.245	0.249	0.261	0.281	0.301	0.272
$\beta_2^*(t)$	0.524	0.521	0.517	0.510	0.504	0.498	0.519
$\beta_3^*(t)$	0.626	0.628	0.630	0.635	0.637	0.639	0.631
$\beta_4^*(t)$	0.180	0.178	0.171	0.164	0.161	0.151	0.169
$g_k^*(t)$	1.022	1.026	1.020	1.014	0.944	0.986	1.012
$y^*(t)$	4.243	4.261	4.296	4.383	4.411	0.413	0.430
$\mathrm{DMU}_k^*(\%)$	20	21	21	23	24	25	24

1. The estimate for the last column is based on the whole period 1977–88.
2. The optimal value (y^*) of the composite output is the mean of the optimal output level.
3. The last low denotes the proportion of the sample found to be efficient in the dynamic model.

and so on, the estimates of the coefficient vector $\beta^*(\tau)$ are observed to change more significantly, with the greatest change accounted for by the instructional expenditure. This suggests the need for treating dynamic efficiency quite differently from the static efficiency. This point is reinforced more strongly for the agricultural data, where changes in weather and risk attitudes of farmers have more pronounced impacts on the input-output coefficients. Similarly, for the industries where technological change has been more rapid, changes in optimal values $\beta^*(t)$ over $0 \leq t \leq T$ are more likely. Some empirical studies reported elsewhere by Sengupta (1992, 1994) provide support for such inference.

The dynamic efficiency models (6) and (7) have several potential applications in public sector decision-making framework involving both current and incremental inputs. First of all, the changes in the optimal output level $\Delta\lambda_k^*(t)$ of an efficient DMU_k would help to identify the impact of technological change or other structural factors. Thus the relative inefficiency of a static DEA model can be accounted for in terms of the increased output requirement planned before. Secondly, the effect of increasing T on the optimal trajectory $\{\lambda^*(t); 0 \leq t \leq T-1\}$ may be evaluated and utilized for policy purposes. If the trajectory tends to be stabilized as $T \to \infty$, then one could use the steady-state approximation of the dynamic model. Likewise the dual programs $\{\beta^*(t), \Delta\beta^*(t); 0 \leq t \leq T-1\}$ can be tested for their sensitivity. Thirdly, a notion of peer group efficiency can be developed by identifying the subset of DMUs which satisfy the dynamic efficiency conditions. The peer group can serve as a goal for the other DMUs which are relatively inefficient. In a static framework this has been analyzed by Beasley (1988) and Sengupta (1991). Finally, the sensitivity of the optimal programs $\{\lambda^*(t), \beta^*(t); 0 \leq t \leq T-1\}$ due to variations in the capital coefficients in matrix B would indicate the robustness or otherwise of a dynamic efficiency measure and hence any policy based on such measures. Clearly this aspect needs much more thorough analysis than so far attempted in the current DEA literature.

4. CONCLUSIONS

The relative inefficiency of input-output systems analyzed in the linear programming formulations of data envelopment analysis is mostly static, since it ignores the intertemporal effects of capital inputs and other dynamic factors. By way of extension of the static DEA approach two types of programming models are developed here for evaluating dynamic efficiency. One approach divides the inputs into two groups, the current and the capital inputs and then formulates a dynamic model involving incremental capital inputs. The second approach involves adjustment costs due to increased investment for future capacity expansion, or lagged investments in the past. An empirical application is also considered by way of illustration to show the estimation problems of a dynamic production frontier. Several areas of policy applications in production and resource planning are briefly pointed out.

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