Title	PRICE AND QUANTITY COMPETITION: DO MIXED OLIGOPOLIES CONSTITUTE AN EQUILIBRIUM?
Sub Title	
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Publisher	Keio Economic Society, Keio University
Publication year	1995
Jtitle	Keio economic studies Vol.32, No.2 (1995.) ,p.15- 25
JaLC DOI	
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Notes	
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19950002-0015

PRICE AND QUANTITY COMPETITION: DO MIXED OLIGOPOLIES CONSTITUTE AN EQUILIBRIUM?

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First version received October 1994; final version accepted May 1995

Abstract: This paper explores market interaction by oligopolists who may choose different action spaces. It discusses a question of the possibility of equilibrium in oligopoly where some firms control prices and others quantities. More specifically, we consider a two-stage game in which three firms first decide simultaneously whether they take as a control variable price or quantity, and afterwards compete contingent on the chosen types of control variables. By adopting a numerical approach, we show that such mixed oligopolies may emerge as an equilibrium in the two-stage game, depending on the degree of substitutability and complementarity between goods.

1. INTRODUCTION

This paper is concerned with market interaction by oligopolists who choose different action spaces. There are two types of classical models in the theory of oligopoly: Cournot and Bertrand. In both models, the noncooperative equilibrium of Nash is employed as an equilibrium concept. While in the former model firms take quantities as control variables, in the latter prices are control variables. We investigate a question of why some firms prefer to be price players while others act as quantity players. More specifically, by making use of some numerical examples, we explore the endogenous equilibrium determination of oligopolists' choices of action spaces.

Looking at today's economies, many markets are characterized by a mixture of oligopolies that select different action spaces. A good example is provided by the auto industry in the U.S.: Daimler-Benz, the German manufacturer, produces a luxury car and its sales position is much smaller than General Motors, Ford and Chrysler. However, Daimler-Benz has been successful in targeting on a high-price, high-quality car. On the other hand, Honda and Toyota have focused

Acknowledgment. An earlier version of this paper was presented at Kobe University, Hosei University and University of Tsukuba. We are indebted to Koji Okuguchi, Takashi Sato, Masahiro Maruyama, Tatsuhiko Naryu, Tetsuya Shinkai and Mollick A. Vallera for helpful discussions. We are also grateful to an anonymous referee of this journal for very useful comments. We are solely responsible for any possible errors remaining in this paper, however. Financial support from the Japan Ministry of Education is gratefully acknowledged.

on the "second car" market by producing low-price, good-quality cars, and the performance of these Japanese auto manufacturers is very good. This shows the imperative necessity of analyzing market interaction by manufacturers who employ different control variables.

The oligopoly model in which some firms play Bertrand but others play Cournot was first studied Bylka and Komar (1975). The line of research on such mixed oligopolies was continued by later works including Singh and Vives (1984), Sakai (1991), Sato (1992) and Allen (1992). Although those papers recognized the importance of mixed oligopolies from both theoretical and policy perspective, they failed to answer the question of why a mixture of price and quantity competition simultaneously exists in oligopoly markets.

In this paper we consider a simple situation with three firms. The demand structure is linear and allows goods to be complements or substitutes. Firms have constant marginal costs and there are no fixed costs and no capacity limits. We assume that each firm can have two types of control variables at its disposal: price and quantity. Consider the two-stage game where the three firms first simultaneously select either type of action space and afterwards compete contingent on the chosen types of action spaces. We restrict our attention to subgame perfect equilibria of such a game. Then we can show that whereas all firms are either Bertrand or Cournot players in some circumstances, a variety of Bertrand–Cournot mixed oligopolies emerge in other circumstances. Mixed oligopolies may constitute an equilibrium in our two-stage game, depending on the degrees of substitutability between three pairs of goods. We thus see that there is nothing pathological about "mixed" oligopolies: they are really as normal as "pure Bertrand" or "pure Cournot" oligopolies.

This paper is organized as follows. Section 2 discusses the relationship between technical substitutability and mixed oligopolies. In Section 3, we present a model of three firms. In Section 4 we regard our oligopoly model as a two-stage game and find subgame perfect equilibria. Concluding remarks are given in Section 5.

2. THE RELATION BETWEEN TECHNICAL SUBSTITUTABILITY AND MIXED OLIGOPOLIES

In this section we discuss the relation between technical substitutability and mixed oligopolies. We will point out the possibility that mixed oligopolies may emerge only when the number of firms is more than two.

Let us begin our inquiry with a situation with two firms. As Table 1 shows, the two goods (goods 1 and 2) are either complements (called Type 2-M) or substitutes (called Type 2-S).

In their remarkable paper, Singh and Vives (1984) have adopted a two-stage approach to a duopoly model with product differentiation. At the first stage each firm determines whether it selects a price or a quantity as a control variable, and at the second stage both firms compete contingent on the chosen types of control variables. Singh and Vives have shown that it is a dominant strategy for each firm to pick a price (or a quantity) as a control variable if one good is a complement (or a substitute) for the other. As a result, whereas both firms play Bertrand (named BB in Table 1) in the case of complementary goods (Type 2-M), they play Cournot (named CC) in the case of substitute goods (Type 2-S). Given the Singh-Vives setup of the two-stage game, there is no room for mixed duopolies.

The question that might naturally occur is whether the Singh-Vives result aforementioned can be generalized to an oligopoly model in which the number of firms is not limited to two. Let us consider a situation with three firms. Then as Table 1 indicates, there exist essentially four types of technical complementarity or substitutability among goods.

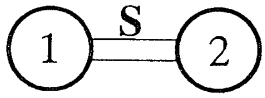
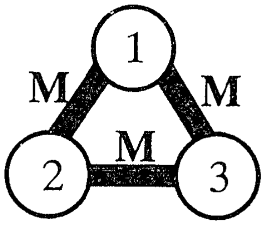
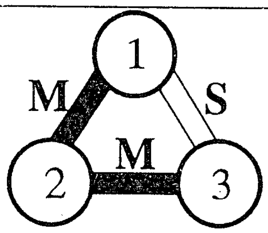
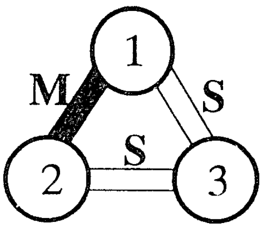
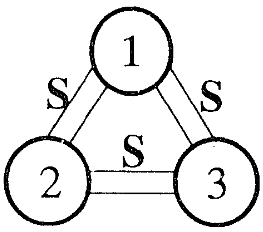
We must now take account of three pairs of goods, i.e., a pair of goods 1 and 2, another pair of goods 2 and 3, and a third pair of goods 3 and 1. One possibility is the symmetric situation that all the three goods are complementary (Type 3-MMM). By taking advantage of the Singh-Vives result, we can then conjecture that all the three firms are price-setting Bertrand players (BBB). By the same token, if all the three goods are substitutes (Type 3-SSS), then all the three firms are expected to be quantity-setting Cournot players (CCC).

The question of much interest would be what happens to the control variables of the three firms if some pairs of goods are complementary and other pairs substitutes. In these asymmetric situations (Types 3-MMS and 3-MSS), there would be so many possibilities regarding the choice of control variables by the three firms. Under some circumstances, there might be pure Bertrand or pure Cournot oligopolies. Under other circumstances, however, there might be the case in which some firms are Bertrand players and the others Cournot players.

The key to understand which action space firms choose in an equilibrium depends on what kind of competition firms wish to engage in. The choice of competition via price or via quantity depends on the complementarity or substitutability of goods they are producing. On the one hand, if a certain firm is producing a complement for what the rest of firms are producing, then this firm wishes to follow a sort of collusive behavior by taking up price as a control variable and raising prices together with other firms. On the other hand, if the firm is producing a substitute for what the rest of firms are producing, then it wants to avoid harmful price-cutting competition, thereby committing itself in selecting quantity as a control variable.

A more complicated yet more intriguing situation would occur when the product of a certain firm is a complement for the products of some of other firms and at the same time a substitute for the products of the rest. In such a mixed situation, whether this firm would like to be a price-setting Bertrand player or a quantity-setting Cournot player depends on the relative strength of complementarity and substitutability between products. As will be shown later, there are a variety of possibilities of "asymmetric equilibrium" with firms taking Bertrand behavior and Cournot behavior coexisting.

TABLE 1. TECHNICAL SUBSTITUTABILITY AND MIXED OLIGOPOLIES

<table>
<tr><th>Number of Firms</th><th colspan="2">Type: Technical Complementality or Substitutability</th><th>Bertrand and/or Cournot Oligopolies (Strategy Choice)</th></tr>
<tr><td rowspan="2">Two</td><td>2-M</td><td></td><td>Bertrand-Bertrand (BB)</td></tr>
<tr><td>2-S</td><td></td><td>Cournot-Cournot (CC)</td></tr>
<tr><td rowspan="4">Three</td><td>3-MMM</td><td></td><td>Pure Bertrand Oligopolies (BBB)</td></tr>
<tr><td>3-MMS</td><td></td><td>Possibility of Mixed Oligopolies</td></tr>
<tr><td>3-MSS</td><td></td><td>Possibility of Mixed Oligopolies</td></tr>
<tr><td>3-SSS</td><td></td><td>Pure Cournot Oligopolies (CCC)</td></tr>
</table>

As the saying goes, saying is one thing, but doing is another. As we have discussed above, the possibility of mixed oligopolies emerges only when the number of firms is more than two. Such a conjecture will be confirmed in the following sections.

3. A SIMPLE MODEL WITH THREE FIRMS

We analyze the following simple oligopoly model with product differentiation, which is based on the one presented in Sakai and Yamato (1989). On the production side, we have an oligopolistic sector with three firms, each one producing a differentiated good, and a competitive numéraire sector. Let x_0 be the output of the numéraire good, x_i be the output of the ith firm, and p_i be its unit price $(i=1, 2, 3)$.

On the consumption side, we have a continuum of consumers of the same type with utility functions which are linear and separable in the numéraire good. We assume that the utility function U of the representative consumer is quadratic:

$$U(x_0, x_1, x_2, x_3)=x_0+\sum_{i=1}^{3} \alpha_i x_i-(1/2)\left\{\sum_{i=1}^{3} \beta_i x_i^2+2(\gamma_1 x_1 x_2+\gamma_2 x_2 x_3+\gamma_3 x_3 x_1)\right\},$$

where α_i and β_i are all positive $(i=1, 2, 3)$. Note that γ_i stands for the degree of substitutability of a pair of two goods in the classical sense: for example, x_1 and x_2 are substitutes or complements according to whether γ_1 is positive or negative.[1]

If U is to be concave, the following matrix must be positive definite:

$$H=\begin{bmatrix} \beta_1 & \gamma_1 & \gamma_3 \\ \gamma_1 & \beta_2 & \gamma_2 \\ \gamma_3 & \gamma_2 & \beta_3 \end{bmatrix}.$$

This requires that the following conditions be satisfied:

(A1) $\beta_i>0$ $(i=1, 2, 3)$;

(A2) $\beta_1\beta_2-\gamma_1^2>0$, $\beta_2\beta_3-\gamma_2^2>0$, $\beta_3\beta_1-\gamma_3^2>0$;

(A3) $|H|\equiv\beta_1\beta_2\beta_3+2\gamma_1\gamma_2\gamma_3-\beta_1\gamma_2^2-\beta_2\gamma_3^2-\beta_3\gamma_1^2>0$.

The consumer is supposed to maximize U subject to his budget constraint. Inverse demand is then given by the system of linear equations:

$$p_1=\alpha_1-\beta_1 x_1-\gamma_1 x_2-\gamma_3 x_3,$$

$$p_2=\alpha_2-\gamma_1 x_1-\beta_2 x_2-\gamma_2 x_3,$$

$$p_3=\alpha_3-\gamma_3 x_1-\gamma_2 x_2-\beta_3 x_3.$$

By solving for x_i in the above system, we can write direct demand as

$$x_1=a_1-b_1 p_1+c_1 p_2+c_3 p_3,$$

$$x_2=a_2+c_1 p_1-b_2 p_2+c_2 p_3,$$

$$x_3=a_3+c_3 p_1+c_2 p_2-b_3 p_3,$$

where

$$a_1=\{\alpha_1(\beta_2\beta_3-\gamma_2^2)-\alpha_2(\beta_3\gamma_1-\gamma_2\gamma_3)-\alpha_3(\beta_2\gamma_3-\gamma_1\gamma_2)\}/|H|,$$

[1] Note that $\partial^2 U/\partial x_1\partial x_2=-\gamma_1$, $\partial^2 U/\partial x_2\partial x_3=-\gamma_2$ and $\partial^2 U/\partial x_3\partial x_1=-\gamma_3$.

$$a_2=\{\alpha_2(\beta_3\beta_1-\gamma_3^2)-\alpha_3(\beta_1\gamma_2-\gamma_3\gamma_1)-\alpha_1(\beta_3\gamma_1-\gamma_2\gamma_3)\}/|H|,$$
$$a_3=\{\alpha_3(\beta_1\beta_2-\gamma_1^2)-\alpha_1(\beta_2\gamma_3-\gamma_1\gamma_2)-\alpha_2(\beta_1\gamma_2-\gamma_3\gamma_1)\}/|H|,$$
$$b_1=(\beta_2\beta_3-\gamma_2^2)/|H|,\quad b_2=(\beta_3\beta_1-\gamma_3^2)/|H|,\quad b_3=(\beta_1\beta_2-\gamma_1^2)/|H|,$$
$$c_1=(\beta_3\gamma_1-\gamma_2\gamma_3)/|H|,\quad c_2=(\beta_1\gamma_2-\gamma_3\gamma_1)/|H|,\quad c_3=(\beta_2\gamma_3-\gamma_1\gamma_2)/|H|.$$

In addition to (A1)–(A3) aforementioned, let us assume that

$$\alpha_1(\beta_2\beta_3-\gamma_2^2)-\alpha_2(\beta_3\gamma_1-\gamma_2\gamma_3)-\alpha_3(\beta_2\gamma_3-\gamma_1\gamma_2)>0,$$
$$\alpha_2(\beta_3\beta_1-\gamma_3^2)-\alpha_3(\beta_1\gamma_2-\gamma_3\gamma_1)-\alpha_1(\beta_3\gamma_1-\gamma_2\gamma_3)>0,$$
$$\alpha_3(\beta_1\beta_2-\gamma_1^2)-\alpha_1(\beta_2\gamma_3-\gamma_1\gamma_2)-\alpha_2(\beta_1\gamma_2-\gamma_3\gamma_1)>0,$$

Then these newly introduced parameters are all positive except the parameters c_1, c_2 and c_3, which can take on positive or negative values. Clearly, the value of c_i measures the degree of substitutability of a pair of two goods à la J. R. Hicks and R. G. D. Allen. For instance, x_1 and x_2 are substitutes or complements in the sense of Hicks and Allen according to whether c_1 is positive or negative.[2] Firms are supposed to have constant marginal costs, k_i $(i=1,2,3)$. We consider from now on prices net of marginal costs. This is without loss of generality since we have only to replace α_i by α_i-k_i $(i=1,2,3)$, a_1 by $a_1-b_1k_1+c_1k_2+c_3k_3$, a_2 by $a_2-b_2k_2+c_2k_3+c_1k_1$ and a_3 by $a_3-b_3k_3+c_3k_1+c_2k_2$. The profit of firm i is then provided by $\Pi_i=p_ix_i$ $(i=1,2,3)$.

In this paper we deal with a variety of "pure" or "mixed" equilibria in which each firm may possibly pick either price or quantity as a control variable. Whereas a firm taking a Bertrand behavior sets its price so as to maximize its own profit, a firm taking a Cournot behavior chooses its output so as to maximize its profit. Since there are three fims in an industry, eight types of market structures are conceivable. They are: BBB, CBB, BCB, BBC, CCB, CBC, BCC and CCC, where B and C respectively denote a Bertrand-type firm and a Cournot-type firm. In all eight cases, the equilibrium concept we employ is the noncooperative Nash equilibrium.

In the BBB case firm 1 selects p_1 to maximize $p_1(a_1-b_1p_1+c_1p_2+c_3p_3)$, taking p_2 and p_3 as given; firm 2 chooses p_2 to maximize $p_2(a_2+c_1p_1-b_2p_2+c_2p_3)$, taking p_3 and p_1 as given; and firm 3 picks p_3 to maximize $p_3(a_3+c_3p_1+c_2p_2-b_3p_3)$, taking p_1 and p_2 as given. A set of reaction functions is then given by

$$a_1-2b_1p_1+c_1p_2+c_3p_3=0,$$
$$a_2+c_1p_1-2b_2p_2+c_2p_3=0,$$
$$a_3+c_3p_1+c_2p_2-2b_3p_3=0.$$

Let $D=2(4b_1b_2b_3-c_1c_2c_3-b_1c_2^2-b_2c_3^2-b_3c_1^2)$. Then it is straightforward to

[2] Note that $\partial x_1/\partial p_2=\partial x_2/\partial p_1=c_1$, $\partial x_2/\partial p_3=\partial x_3/\partial p_2=c_2$ and $\partial x_3/\partial p_1=\partial x_1/\partial p_3=c_3$.

compute equilibrium values of prices as follows:

$$p_1^{BBB}=\{a_1(4b_2b_3-c_2^2)+a_2(2b_3c_1+c_2c_3)+a_3(2b_2c_3-c_1c_2)\}/D\,,$$
$$p_2^{BBB}=\{a_1(2b_3c_1+c_2c_3)+a_2(4b_3b_1-c_3^2)+a_3(2b_1c_2+c_3c_1)\}/D\,,$$
$$p_3^{BBB}=\{a_1(2b_2c_3-c_1c_2)+a_2(2b_1c_2+c_3c_1)+a_3(4b_1c_2-c_1^2)\}/D\,.$$

At the BBB equilibrium, the profit of firm i is calculated by $\Pi_i^{BBB}=b_i(p_i^{BBB})^2$.

In a similar way, we are able to compute the equilibrium values for the remaining seven types of competition. For instance, in the "mixed" BCB case, firms 1 and 3 play Bertrand but firm 2 plays Cournot. In that case, firm 1 chooses p_1 to maximize p_1x_1, where x_1 is a function of p_1, x_2 and p_3, derived from the demand functions, namely, $x_1=(\alpha_1\beta_3-\alpha_3\gamma_3)/b_2|H|-(\beta_3/b_2|H|)p_1-(c_1/b_2)x_2+(\gamma_3/b_2|H|)p_3$. Likewise, firm 2 selects x_2 to maximize p_2x_2 and firm 3 picks p_3 to maximize p_3x_3, where both p_2 and x_3 are functions of p_1, x_2 and p_3. If we solve a set of reaction functions, then we can obtain a Nash equilibrium for the BCB case. The computation is straightforward yet lengthy, and may be omitted here.[3]

4. A TWO-STAGE GAME

We are ready to regard our oligopoly game with three firms as a two-stage game. At the first stage, each firm decides whether it chooses a price or a quantity as a control variable. And at the second stage, the three firms compete contingent on the choices of control variables. We limit our attention to subgame perfect equilibria of the two-stage game.

The payoff matrix these three firms face at the first stage is depicted in Figure 1. For each i, firm i is supposed to have two control variables, $s_i=\mathrm{B}$ (a price-setting Bertrand behavior) and $s_i=\mathrm{C}$ (a quantity-setting Cournot behavior). If all three firms are price-setting firms, then we have the "pure" Bertrand oligopoly case BBB, with the set of payoffs being $(\Pi_1^{BBB}, \Pi_2^{BBB}, \Pi_3^{BBB})$. When firms 1 and 3 are price-setting and firm 2 quantity-setting, there emerges the "mixed" Bertrand-Cournot-Bertrand case BCB, in which the set of payoffs is shown by $(\Pi_1^{BCB}$,

< s_3 = B >

	s_2 = B	s_2 = C
s_1 = B	BBB: $\Pi_1^{BBB}, \Pi_2^{BBB}, \Pi_3^{BBB}$	BCB: $\Pi_1^{BCB}, \Pi_2^{BCB}, \Pi_3^{BCB}$
s_1 = C	CBB: $\Pi_1^{CBB}, \Pi_2^{CBB}, \Pi_3^{CBB}$	CCB: $\Pi_1^{CCB}, \Pi_2^{CCB}, \Pi_3^{CCB}$

< s_3 = C >

	s_2 = B	s_2 = C
s_1 = B	BBC: $\Pi_1^{BBC}, \Pi_2^{BBC}, \Pi_3^{BBC}$	BCC: $\Pi_1^{BCC}, \Pi_2^{BCC}, \Pi_3^{BCC}$
s_1 = C	CBC: $\Pi_1^{CBC}, \Pi_2^{CBC}, \Pi_3^{CBC}$	CCC: $\Pi_1^{CCC}, \Pi_2^{CCC}, \Pi_3^{CCC}$

Fig. 1. The payoff matrix for three-firm game with two strategies: Bertrand and Cournot.

[3] We have derived a set of mathematical formulas for equilibrium profits for all eight types of market structures. We will send such formulas to any interested reader upon request.

Π_2^{BCB}, Π_3^{BCB}). Similar interpretations may be given to other combinations of Bertrand and Cournot strategies.

In general, subgame perfect equilibria of the two-stage game may or may not exist, depending on the demand and production structures. If we put a set of specific values for parameters such as α_i, β_i and γ_i ($i=1, 2, 3$), then we can compute the corresponding equilibrium. To this end, let us assume that $\alpha_i=10$ and $\beta_i=2$ for all i. In order to make our point sharp, we focus on the situation in which a pair of goods 1 and 2 are substitutes, but another pair of goods 2 and 3 and a third pair of goods 3 and 1 are both complements in the classical sense. In reality, such a combination of goods may arise if x_1, x_2 and x_3 represent a skirt, pants and a jacket, respectively.

Figure 2 shows four numerical examples. The uppermost chart (a) corresponds to the case that $\gamma_1=0.5$, $\gamma_2=-0.6$ and $\gamma_3=-0.6$. Interestingly, we find that

BBB: **44.30**, **44.30**, **92.13**	BCB: 40.89, **41.11**, 92.57	BBC: 46.23, 46.23, **84.90**	BCC: 41.62, 44.95, 81.58
CBB: **41.11**, 40.89, 92.57	CCB: 34.13, 34.13, 88.02	CBC: 44.95, 41.62, 81.58	CCC: 36.90, 36.90, 74.73

Chart (a) $\gamma_1 = 0.5, \gamma_2 = -0.6, \gamma_3 = -0.6$

BBB: 42.50, **18.09**, 67.63	BCB: **43.22**, **19.40**, **64.44**	BBC: 43.21, 20.52, 61.86	BCC: 41.83, 22.59, **57.37**
CBB: 38.04, 17.30, 67.12	CCB: **37.39**, 18.31, 63.46	CBC: 39.62, 19.48, 61.37	CCC: 37.28, 21.17, 56.58

Chart (b) $\gamma_1 = 0.5, \gamma_2 = -0.3, \gamma_3 = -0.6$

BBB: **18.09**, 42.50, 67.63	BCB: 17.30, 38.04, 67.12	BBC: 20.52, 43.21, 61.86	BCC: 19.48, 39.62, 61.37
CBB: **19.40**, **43.22**, **64.44**	CCB: 18.31, **37.39**, 63.46	CBC: 22.59, 41.83, **57.37**	CCC: 21.17, 37.28, 56.58

Chart (c) $\gamma_1 = 0.5, \gamma_2 = -0.6, \gamma_3 = -0.3$

BBB: 21.50, 21.50, 43.82	BCB: **21.43**, 22.62, 43.01	BBC: 21.73, 21.73, 43.57	BCC: 21.48, 23.20, 42.54
CBB: 22.62, **21.43**, 43.01	CCB: **22.35**, **22.35**, **42.08**	CBC: 23.20, 21.48, 42.54	CCC: 22.76, 22.76, **41.36**

Chart (d) $\gamma_1 = 0.5, \gamma_2 = -0.3, \gamma_3 = -0.3$

Fig. 2. Four numerical examples: $\alpha_1=\alpha_2=\alpha_3=10$ and $\beta_1=\beta_2=\beta_3=1$.

$c_1 = 14/39$, $c_2 = -10/13$ and $c_3 = -10/13$. Therefore, in the sense of Hicks and Allen also, a pair of goods 1 and 2 are substitutes, but another pair of goods 2 and 3 and a third pair of goods 3 and 1 are both complements. Note that this specific set of values on α_i, β_i and γ_i satisfy Assumptions (A1)–(A4) mentioned above. In this case, we see that the BBB case represents an equilibrium because the following relations hold:

$$\Pi_1^{BBB} = 44.30 > 41.11 = \Pi_1^{CBB},$$

$$\Pi_2^{BBB} = 44.30 > 41.11 = \Pi_2^{BCB},$$

$$\Pi_3^{BBB} = 92.13 > 84.90 = \Pi_3^{BBC}.$$

Starting with chart (a), let us replace $\gamma_2 = -0.6$ by $\gamma_2 = -0.3$, maintaining the same values for other parameters. Then we have chart (b). The outcome is clearly the BCB equilibrium since $\Pi_1^{BCB} > \Pi_1^{CCB}$, $\Pi_2^{BCB} > \Pi_2^{BBB}$ and $\Pi_3^{BCB} > \Pi_3^{BCC}$. Next, letting $\gamma_3 = -0.3$ and keeping the same values for other parameters as in chart (a), we have the payoff matrix depicted in chart (c). Then the CBB case emerges as an equilibrium because $\Pi_1^{CBB} > \Pi_1^{BBB}$, $\Pi_2^{CBB} > \Pi_2^{CCB}$ and $\Pi_3^{CBB} > \Pi_3^{CBC}$. Finally, if we let $\gamma_2 = -0.3$ and $\gamma_3 = -0.3$ and maintaining the same values for other parameters as in chart (a), then we are led to the payoff matrix shown in chart (d). It is an easy job to see that the CCB case constitutes an equilibrium since $\Pi_1^{CCB} > \Pi_1^{BCB}$, $\Pi_2^{CCB} > \Pi_2^{CBB}$ and $\Pi_3^{CCB} > \Pi_3^{CCC}$.

Figure 3 summarizes the feasibility of a variety of mixed oligopolies in a more visible and more comprehensive way. Note that γ_1 is fixed at 0.5. The horizontal axis measures γ_2 and the vertical axis γ_3. The whole (γ_2, γ_3) plane is divided into six areas. The outer blank area represents the non-feasible area, i.e., the one in which either the positive-definiteness condition of the Hessian matrix H or the nonnegativity condition of prices is not satisfied. It is of much interest to see how the remaining five dark areas are located contingent on the values of γ_2 and γ_3.

The CCC area indicates the pure Cournot situation and includes the positive orthant as a subset. Except for the non-feasible area, the negative orthant can be split into the four other oligopoly situations. They are: the BBB area, the BCB area, the CBB area and the CCB area. Points (a), (b), (c) and (d) correspond to charts (a), (b), (c) and (d) in Figure 2, respectively. Remarkably enough, the outcome of the two-stage game is quite sensitive to the degree of substitutability between any pair of goods. In fact, a slight change in the values of γ_2 and γ_3 could cause a drastic change in oligopoly situation from one type to another.

Let us attempt to give the reader some intuitions for the results.[4] Take a close look at Figure 3. Suppose that all γ_i's are positive. Then goods are all substitutes for each other, whence competing in price will be harmful to firms since it will bring all the prices down. As a result, all the firms will instead take up quantities as control variables in order to avoid harsh confrontation. Thus we have the CCC

[4] For this and the following interpretations, we are indebted to the referee.

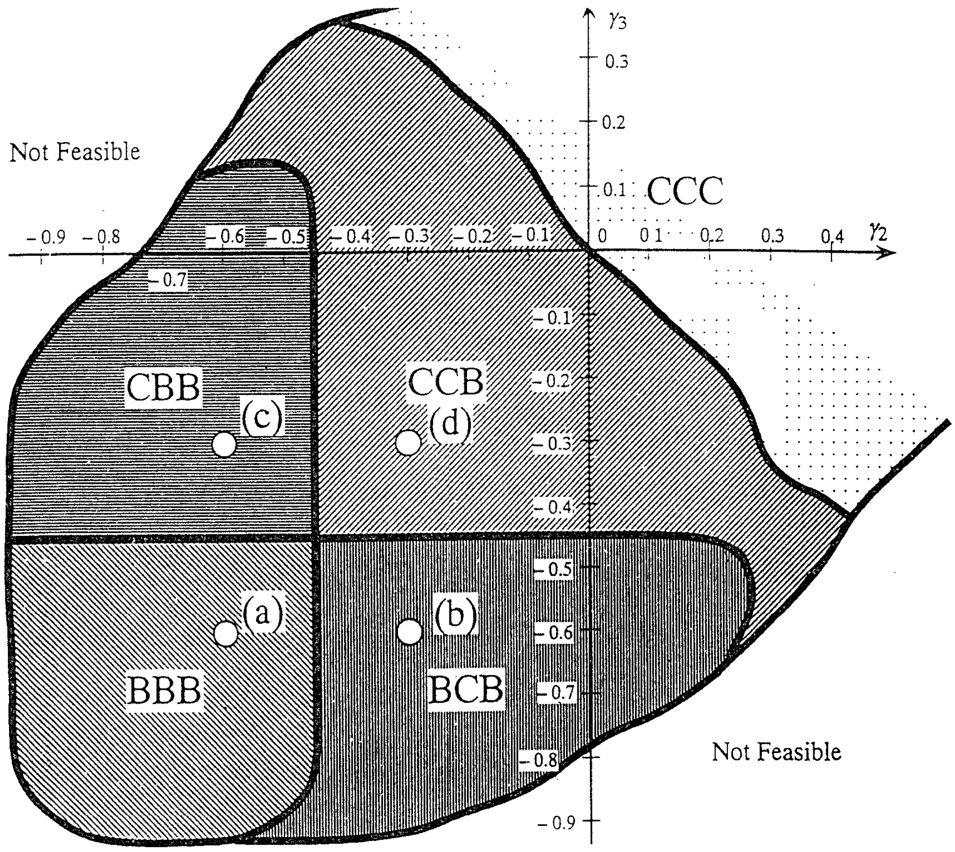


Fig. 3. The possibility of mixed oligopolies: $\alpha_1 = \alpha_2 = \alpha_3 = 10$ and $\beta_1 = \beta_2 = \beta_3 = 1$ and $\gamma_1 = 0.5$.

situation which agrees with common sense.

Now suppose that γ_2 and γ_3 become negative while γ_1 continues to be positive. This means that good 3 is a complement for both goods 1 and 2. If this degree of complementarity is quite strong, then in spite of the fact that goods 1 and 2 are substitutes, both firms 1 and 2 may take the advantage of collusive behavior with firm 3 in which the beneficial price-raising effect is more powerful than the harmful price-cutting effect. Consequently, this leads all the firms to take prices as control variables, which is the situation BBB. If the degree of complementarity aforementioned is rather weak, then the situation will change drastically. In this case, the price-cutting effect existing between firms 1 and 2 tends to dominate the price-raising effect between firms 2 and 3 and the one between firms 3 and 1. Indeed, the former effect becomes so powerful that firms 1 and 2 must take up quantities to mitigate the confrontation, with only firm 3 being unilaterally benefiting from raising its price. This is the situation CCB.

Let us turn to the situation in which the degree of complementarity between goods 2 and 3 is considerably weaker than the one between goods 3 and 1. This would approximately be the case where $-0.45 < \gamma_2 < 0$ and $-0.9 < \gamma_3 < -0.45$ in Figure 3. In this case, firm 1 could benefit more from price-raising with firm 3 than price-cutting against firm 2, so that firm 1 prefers price to quantity as a control variable. By contrast, firm 2 could hurt more from price-raising with firm 3 than price-cutting against firm 3, whence firm 2 wishes to take as a control variable quantity rather than price. This is the situation BCB. A similar interpretation can be given to the remaining situation CBB.

5. CONCLUDING REMARKS

In the above we have shown that mixed oligopolies in which some firms are of Bertrand-type but the others are of Cournot-type may constitute equilibrium outcomes in the two-stage game. Although in reality many markets are characterized by a mixture of price-setting and quantity-setting firms, such mixed oligopolies seem to have received less attention in the literature than they deserve.

We would like to emphasize the fact that there is nothing pathological about mixed Bertrand-Cournot oligopolies: they are really as normal as pure Bertrand or pure Cournot oligopolies. It is intriguing to see that the equilibrium position of the two-stage game is very sensitive to the values of γ_i's, the degrees of substitutability between any pair of goods. As shown above, a slight change in the values of γ_i's would possibly result in a drastic change in oligopoly situation from one type to another.

We note that our results have been obtained in a specific oligopoly model with explicit functional forms assumed in the demand function and the cost functions. We have especially worked with some specific examples to compute two-stage equilibrium values. Notwithstanding those specific assumptions, however, we believe that our results obtained in this paper are fundamentally robust for a more general framework. The question of how robust they really are will be left for further research.

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