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SHORT-TERM AND LONG-TERM STRATEGIES IN DYNAMIC MARKETS

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Abstract: There are three major approaches to studying competition in dynamic markets: theoretical studies, field studies (case studies), and laboratory experiments (which include computer simulations). We believe any single approach cannot provide a full picture or a complete solution. Therefore, in this study we used a combination of all the three to attack the problem.

We first investigated how corporations competed in oligopolistic markets. We then used the theoretical approach to develop prediction schemes to predict the behavior of competitors. Finally we designed and implemented an artificial market with personal computers and local area networks to study how subjects learn, develop their strategies, and react to market changes.

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1. INTRODUCTION

1.1. *Background and Motivation*

Determining production level and price can be a very difficult and complicated task (Friedman, 1977; Okuguchi and Szidarovszky, 1990), especially when the interactions and competition among the producers are significant, for example, in the automobile and petroleum industries. Much information need to be considered, such as, strategies and past history of the competitors, available resources, forecasts, goals of the organization, uncertainties and risks (Friedman, 1981). Some of these factors are not easily quantifiable; however, in order to survive in marketplace, an understanding of all the above factors, especially of the behavior and strategies of the competitors, is extremely important.

The behavior and the mental models of real-world decision makers are

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complicated, subjective and inconsistent. Theoretical studies (e.g., Von Neumann and Morgenstern, 1944; Szidarovszky and Okuguchi, 1989a, 1989b and 1989c), field studies (e.g., D'Aveni, 1994), and laboratory experiments (e.g., Smith, 1962 and 1982), are the three major approaches. It is difficult and if not impossible for any model to include all the information used in the decision making process. A model can cover only a subset of the factors or the parameters of a real-world problem; therefore, it is only an approximation or a simplified version. But, the combination of any two or more approaches should do a better job. For example, using field studies or case studies to verify the theories developed.

Theoretical studies, case studies, and laboratory experiments complement each other. Without support or verification from the experiments or field studies, the theoretical models may be difficult to appreciate. Similarly, without mathematical models, the findings in case studies or experiments are difficult to be understood and verified. One question asked frequently, is: "To what extent can we trust a model and its solution?" We believe only field studies and/or experiments will yield a valid answer.

In this study we investigate the learning behavior (development of prediction scheme) and the selections of short-term and long-term strategies of the producers in dynamic markets. We define short-term strategy to be the prediction scheme and the rules that guide the decisions within each market transaction period, for example, one month. The long-term strategy is not subject to change caused by fluctuation of the market. In order to see the interactions and the competition among the producers, we chose an oligopoly market for our study. One important aspect of an oligopoly is the interactions among the producers are so significant that they cannot be ignored (Okuguchi and Szidarovszky, 1990). Under such conditions, in order to earn high profits, producers need not only to understand the behaviors of the market and of the competitors but also to develop good competition strategies.

1.2. Competitions in the Current Industries

Every industry in the U.S. to some extent can be considered an oligopoly; the telecommunications, automobile, and computer industries are some examples. Over the past few years, we have seen the giants, such as, AT & T, General Motors, and IBM, shocked by the smartness and agressiveness of their competitors.

The competitive advantages which they used to enjoy and be proud of have been shredded partially and torn into pieces in the hurricanes of competition. These companies used to have tremendous resources for R & D, new product advertisement, building new plants, hiring the best people, or setting up barriers to new market entries. However, times have changed. These competition advantages are no longer blessings. They may turn into burdens and cripple the operation of the entire enterprise.

To win in today's market, a corporation needs to discard its trational advantages and develop a new set. The new competition occurs in four arenas: (1) cost and quality, (2) timing and know-how, (3) creation and destruction of strongholds, and (4) accumulation and neutralization of deep pockets (D'Aveni, 1994). To win the competition in each arena, as Sun Tsu stated in a book written over 2500 years ago: "To obtain reliable information is the key for making plans" (Chan and Chen, 1989); or "knowing ourselves, knowing our opponents, knowing the weather, and knowing the earth are the keys" (Chan and Chen, 1989; Romm, 1991; Huang, 1993).

As we know, information plays an important role in management. If a corporation has collected enough information, then it can set up good strategies to beat its competitors. For example, sacrifice the short-term profits for a greater market share; Japanese automobile industry and the VLSI chip manufacturers have used this strategy. Alternatively, a company may use government subsidies to deepen its pockets as did Airbus Industrie, which received European government subsidies for over twenty years (D'Aveni, 1994). Some companies play psychological games with their competitors, for example, the long competition between Coca-Cola and PepsiCo, and the competition between the Windows NT and OS2. Such competitions are complicated and their strategies are difficult to explain with rules or theories. However, through experiments, we can observe the development and use of strategies, even in a very simple market system. Hopefully, from the experiment results we can develop guidelines and models for a better competition strategy.

1.3. Previous Research

Since the first theoretical study of dynamic markets by Cournot in 1838, the attention on dynamic markets has increased steadily. Among the models, the most simple prediction scheme assumes the following: When any producer determines its production level, he assumes all the other producers will not change their production levels and the price is simply determined by the total products sent into the market. That is, in an N-firm oligopoly

$$\mathbf{s}_n^E(t) = \sum_{l \neq n}^{N} \mathbf{x}_l(t-1)\,, \tag{1}$$

and

$$P = F(\mathbf{x}_1 + \mathbf{x}_2 + \cdots + \mathbf{x}_N) = F\left(\sum_{j=1}^{N} \mathbf{x}_j\right), \tag{2}$$

where $\mathbf{x}_l$ indicates the production vector of firm l, $\mathbf{s}_n^E(t)$ the n-th firm's expectation of the output of the rest of the industry, and P the price. However, such a model can not catch the multi-period phenomena, for example, the trend of the demand function and cyclic behavior. Also, only production level can be adjusted. The first critique was given by Bertrand in 1883; he argued that the assumption of Cournot's expectation is inappropriate. In a real market, besides production levels the producers also adjust their prices in order to reach optimal profits. Therefore, Cournot's model is always called the quantity setting and Bertrand's model the price setting.

Later other researchers also expressed concerns with the original Cournot model. More complicated models were developed in order to capture the behaviors of the real-world market (e.g., Edgeworth, 1925; Stackelberg, 1934; Sweezy, 1939). The main factors which are missing in the earlier models are the following: First, the real markets are dynamic; firms produce and sell goods repeatedly. Second, the decision makers are intelligent. They are able to observe, to learn, and to adjust the models and their parameters. After several time periods, the quality of their decisions should improve. In order to survive and earn higher profit, such learning capabilities are very important. Thus, the newer dynamic models that are able to learn from what has happened (for example, the trends and the other multi-period phenomena) and to deal with the changes of the real markets are more realistic.

In both economics and mathematics, more complicated models and solution methodologies have been developed every year. For example, Hotelling (1929) considered the effects of the distances between producers and buyers and transportation costs on the price setting. Chamberlin (1956) developed a model based on the concept that: if every producer is only concerned with his own profit, not the profits of the other producers, such competition cannot lead to the maximal profit for every produder. In order to reach maximal profits, understanding and compromise are necessary; such behavior is called a coalition.

In the fundamental work by von Neumann and Morgenstern (1944), the oligopoly was simply modelled as an N-person game. Based on the decisions given by all the players, the N-person game leads to a particular outcome in the set of possible outcomes, and each individual is assumed to have preference for the set of possible outcomes. Therefore, in an N-person oligopoly game, without knowing the decisions of the other players, every producer has to make a decision that he thinks will lead to his best outcome (best payoff); this is called a non-cooperative game. There are other types of games, for example, cooperative games (e.g., Harsanyi, 1959; Cyert and De Groot, 1973) and games with side payments (e.g., Aumann *et al.*, 1964). Game theories have also been studied and applied to solve many oligopoly problems (e.g., Nash, 1951; Shubik, 1959; Harsanyi, 1959; Szidarovszky, 1970; Friedman, 1977).

In oligopoly problems, how one producer interacts with the other producers is very important. However, economists are usually more concerned with the stability of a market as well as the existence and uniqueness of a solution. Studies of the existence and uniqueness of an equilibrium point for a single-product Cournot game date back to Frank and Quandt (1963). Later, Okuguchi (1976, 1983), Szidarovszky and Yokowitz (1977), Gabay and Moulin (1980), Kolstad and Mathiesen (1987), and Okuguchi and Szidarovszky (1990) further investigated similar problems.

Nonlinear complementarity problems were introduced by Karamardian (1969) and Okuguchi (1983). With results from the theory of nonlinear complementarity problems, Okuguchi and Szidarovszky (1990) analyzed the existence and uniqueness of the solutions of single product Cournot problems.

In the early days, the existence of equilibrium points for concave games as proven by the Nikaido–Isoda theorem (1955). Later, the existence of the equilibrium point of concave multiproduct oligopoly market games was proven (Okuguchi and Szidarovszky, 1985). For the monotone and strictly monotone problems similar results were introduced in Ortega and Rheinbolt (1970). For single product oligopoly with product differentiation, the result of existence was presented in Friedman (1986). More works on multiproduct oligopoly were done by Szidarovszky and Okuguchi (1989a, b, c), and a more comprehensive summary of related works was presented in Okuguchi and Szidarovszky (1990).

Beside the theoretical approaches, laboratory experiments and field studies are also important to the oligopoly research. The theoretical approaches have several limitations. For example, most economic problems in the real world are very complicated and time-dependent. In order to create a model for a problem, the complexity must be reduced and the uncertainties (i.e., its stochastic nature) must be ignored. It is possible that the model cannot accurately represent the problem. Laboratory experiments (which include computer simulations) and field studies not only provide the necessary bridge between the theoretical models and real-world problems, but also lead to the development of new models.

Many problems in economics and decision sciences, especially oligopolies, have been studied extensively by laboratory experimengs over the past forty years, and the contributions are significant (Smith, 1982).

The first oligopoly experiments were conducted in the late 1940's; they tested the two-sided markets with finite numbers of sellers and buyers. The participants are the students in the Columbia University and they acted as sellers and buyers (Smith, 1962). Such experiments proceed as follows: At the beginning of a trading period, the experiment conductor tells the sellers the unit cost and the buyers the retail price of the fictitious goods. Each buyer writes down the price and the quantity he is willing to buy, and each seller writes down the price and the quantity he is willing to sell. When all the offers are handed in, the experiment conductor determines the price and the quantity of goods sold. From these experiments scientists verify the models or the theories developed earlier or develop new models.

Results of the earlier oligopoly experiments (1959–1963) indicated that the competition in the demand side is always stronger. This interesting phenomenon reflects the nature of most markets in the 1950's, for example, some automobile manufacturers had backlogs of over six months. Experimental studies of oligopolies has become one of the main streams of research since the early 1960's (e.g., Sauerman and Selten, 1960; Friedman, 1963 and 1969). With the modern information technology, the efficiency, accuracy, complexity of the problems to be studied, and the analysis and dissemination of results have been much improved (McCabe *et al.*, 1991).

In Section 2, definitions of oligopoly markets will be provided. Section 3 will present a discussion of the hypotheses. Section 4 will discuss the design of the experiments. Section 5 provides the results of experiments, such as, the learning behavior, the uses of prediction schemes, and the development of decision strategies. Comments and conclusions are provided in Section 6.

2. DEFINITIONS OF OLIGOPOLY MARKETS

In this section, single-product markets will be introduced first; then multi-product markets will be defined.

2.1. *Single Product Dynamic Markets*

A single product dynamic market is very simple; it can be considered as a special case of multiproduct dynamic markets. We assume that the production cost, $C_n(x_n)$, for the n-th producer is a linear function of its production level, $x_n \in [0, L_n]$, where $L_n \in \mathbf{R}^+$ denotes the production capacity.

If we assume that there is no time lag between producing and selling the products, and also no inventory or backlog are allowed, the profit of each producer can be simply determined by the market price P, the production cost, and the production level. However, according to the supply-demand relation the market price P also depends on the total output of all producers. If the number of producers is small, the interactions among them should be significant and cannot be ignored.

Consider a market with N firms that produce a homogeneous good, where the market price, P, is determined by equation (2), and the revenue, R_n, of each firm is given as:

$$R_n = x_n \cdot F\left(\sum_{j=1}^{N} x_j\right) = R_n(x_1, x_2, \cdots, x_N). \tag{3}$$

Finally, the profit earned by firm n can be determined:

$$\begin{aligned} Q_n(x_1, x_2, \cdots, x_N) &= R_n(x_1, x_2, \cdots, x_N) - C_n(x_n) \\ &= x_n \cdot F\left(\sum_{j=1}^{N} x_j\right) - C_n(x_n). \end{aligned} \tag{4}$$

Analytically, a single-product market can be modelled as an N-person normal-form game, $\Gamma = \{N; X_1, \cdots, X_N; Q_1, \cdots, Q_N\}$, and it is defined by the set of strategies, $X_n = [0, L_n]$, and a set of payoff functions, Q_n.

DEFINITION 1. A vector $\mathbf{x}^* = (x_1^*, \cdots, x_n^*, \cdots, x_N^*)$ is called a Nash–Cournot equilibrium point of a single product oligopoly game, Γ, if for $n = 1, 2, \cdots, N$,

1. x_n^* is a strategy, i.e., $x_n^* \in [0, L_n]$, and

2. for any arbitrary $x_n \in [0, L_n]$,

$$Q_n(x_1^*, \cdots, x_n^*, \cdots, x_N^*) \geq Q_n(x_1^*, \cdots, x_{n-1}^*, x_n, x_{n+1}^*, \cdots, x_N^*) .$$

At the equilibrium point, $\mathbf{x}^*$, the strategy of each player, x_n^*, is optimal against the strategies, x_j^*, of any other player j, for all $j \neq n$. That is, no player can increase his own profit by unilaterally moving his strategy away from the Nash equilibrium point.

A multiproduct dynamic market is more complicated. The definition, the notations, and various expectation schemes will be discussed briefly next; however, we only test single-product markets in the experiments reported here.

2.2. *Multiproduct Oligopoly Models*

Consider a market with N producers where each producer produces M different products. If x_k^m ($1 \leq k \leq N$, $1 \leq m \leq M$) denotes the production level of firm k of product m, then the output of firm k is characterized by an output vector $\mathbf{x}_k = (x_k^1, \cdots, x_k^M)$. Let the production cost of firm k be denoted by $C_k(\mathbf{x}_k)$, and assume that the unit price P_m of product m depends on the total output vector

$$\mathbf{s} = \left(\sum_{k=1}^{N} x_k^1, \cdots, \sum_{k=1}^{N} x_k^M \right) \tag{5}$$

of the industry. By using this notation, the profit function of firm k is formulated as

$$\begin{aligned} Q_k(\mathbf{x}_1, \cdots, \mathbf{x}_N) &= \sum_{m=1}^{M} x_k^m \cdot P_m(\mathbf{s}) - C_k(\mathbf{x}_k) \\ &= \mathbf{x}_k^T \cdot \mathbf{P}(\mathbf{s}) - C_k(\mathbf{x}_k) , \end{aligned} \tag{6}$$

where $\mathbf{P} = (P_1, \cdots, P_M)^T$. If H_k denotes the set of all feasible output vectors for firm k ($k = 1, 2, \cdots, N$), then an N-person normal-form game $\Gamma = \{N; H_1, \cdots, H_N; Q_1, \cdots, Q_N\}$, can be defined to model such particular markets.

For this model, we assume that the unit price function $\mathbf{P}$ depends on the total output $\mathbf{s} = \sum_{k=1}^{N} \mathbf{x}_k$ of the industry, the demands for different products depend on each other, and the cost function C_k of firm k is assumed to depend only on the output $\mathbf{x}_k$ of firm k. Thus, in order to guarantee the existence of equilibrium, we assume that the conditions introduced in Okuguchi and Szidarovszky (1990) hold.

For the sake of simplicity assume that

$$\mathbf{P}(\mathbf{s}) = \mathbf{A}\mathbf{s} + \mathbf{b} ,$$

and for all k,

$$C_k(\mathbf{x}_k) = \mathbf{x}_k^T \mathbf{B} \mathbf{x}_k + \mathbf{b}_k^T X_k + c_k .$$

Therefore

$$Q_k(\mathbf{x}_1, \cdots, \mathbf{x}_N) = \mathbf{x}_k^T (\mathbf{A}\mathbf{s} + \mathbf{b}) - (\mathbf{x}_k^T \mathbf{B} \mathbf{x}_k + \mathbf{b}_k^T \mathbf{x}_k + c_k) . \tag{7}$$

It is also assumed that at each time period t, each firm maximizes its expected profit:

$$\begin{aligned} &\max \mathbf{x}_k^T(\mathbf{A}\cdot \mathbf{s}_k^E(t)+\mathbf{A}\cdot \mathbf{x}_k+\mathbf{b})-(\mathbf{x}_k^T\mathbf{B}\mathbf{x}_k+\mathbf{b}_k^T\mathbf{x}_k+c_k) \\ &\text{subject to} \quad \mathbf{x}_k\in H_k\,, \end{aligned} \tag{8}$$

where $\mathbf{s}_k^E(t)$ is the expectation of firm k on the output of the rest of the industry.

It is also assumed that the objective function is strictly concave; that is, matrix $(\mathbf{A}+\mathbf{A}^T-\mathbf{B}_k-\mathbf{B}_k^T)$ is negatively definite for all k, and the optimal solution $\mathbf{x}_k$ is an interior point of H_k. Therefore the first order optimality conditions imply that the gradient of the objective function with respect to $\mathbf{x}_k$ at the optimal solution equals zero. That is,

$$[(\mathbf{A}+\mathbf{A}^T)-(\mathbf{B}_k+\mathbf{B}_k^T)]\cdot \mathbf{x}_k+\mathbf{A}\cdot \mathbf{s}_k^E(t)+\mathbf{b}-\mathbf{b}_k=0\,. \tag{9}$$

Since matrix $(\mathbf{A}+\mathbf{A}^T)-(\mathbf{B}_k+\mathbf{B}_k^T)$ is invertible, we have the following:

$$\mathbf{x}_k(t)=-[(\mathbf{A}+\mathbf{A}^T)-(\mathbf{B}_k+\mathbf{B}_k^T)]^{-1}\cdot\text{-}\mathbf{A}\cdot \mathbf{s}_k^E(t)+\mathbf{b}-\mathbf{b}_k)\,. \tag{10}$$

From this equation we know that the expected profit-maximizing output of each producer depends only on its expectation, $\mathbf{s}_k^E(t)$. Hence different expectations lead to different systems dynamics.

2.3. *Expectations*

In this section, three different types of expectations will be briefly discussed and they will be used later in analyzing the experimental data. The detailed derivations of these expectation schemes are available in, for example, Okuguchi (1976) and Okuguchi and Szidarovszky (1990).

1) Cournot expectation:

$$\mathbf{s}_k^E(t)=\sum_{l\neq k}\mathbf{x}_l(t-1)\,,$$

which shows that each firm simply assumes that the other firms do not change their production levels from the previous time period.

2) Adaptive expectation:

$$\mathbf{s}_k^E(t)=\mathbf{s}_k^E(t-1)+\mathbf{D}_k\cdot\left(\sum_{l\neq k}\mathbf{x}_l(t-1)-\mathbf{s}_k^E(t-1)\right),$$

where $\mathbf{D}_k$ is a constant matrix. Adaptive expectations adjust the previous expectations with a portion of the errors of that estimation.

3) Extrapolative expectation:

$$\mathbf{s}_k^E(t)=\sum_{i=1}^{T}\mathbf{D}_i^{(k)}\cdot \mathbf{x}_k(t-i)\,,$$

where matrices $\mathbf{D}_i^{(k)}$ are given constant matrices with $\sum_{i=1}^T=\mathbf{I}$. We assume that this expectation depends on several previous observations. The simplest case is to consider only the observations of the previous two time periods, in which the above formula reduces to the following:

$$\mathbf{s}_k^E(t)=\mathbf{D}_k\cdot\sum_{l\neq k}\mathbf{x}_l(t-1)+(\mathbf{I}-\mathbf{D}_k)\cdot\sum_{l\neq k}\mathbf{x}_l(t-2),$$

by assuming again the linearity of the prediction function.

3. HYPOTHESES

As mentioned earlier, there are three primary purposes of this study: first, to identify different learning schemes and the uses of long-term and short-term strategies; second, to identify the interactions among the producers (for example, if some producers always make poor decisions, will that hurt the other producers? If so, how and to what extent?); third, to search for a model or a set of guidelines for producers in similar situations.

We conducted several experiments to test the following hypotheses:

HYPOTHESIS 1a. *If a market has linear demand and supply functions, then expectations discussed in the previous section (Cournot expectations, adaptive expectation, and extrapolative expectation) will be used by some producers.*

In other words, this hypothesis states that: If there is a linear demand and supply relation in a market, then some producers will develop schemes to predict the behavior of their rivals. Notice here we use "some" not "every".

HYPOTHESIS 1b. *Producers who consistently follow the recommendations of the expectation schemes will receive higher total profits.*

This hypothesis says—"A producer who develops a prediction scheme and follows its advice is always a winner".

HYPOTHESIS 2. *If a market continues for a significant length of time, a producer who is willing to sacrifice profits in the beginning for a greater market share will become a dominate player or leader later, and will surely have higher total profits.*

As we observed in the real world, achieving a greater market share was always the most important goal of producers. To occupy greater market share, additional costs are needed, for example, research and development, advertisement, and pricing incentives. Sometimes products have to be sold at prices that are lower than their costs. In such fierce competition, some producers cannot sustain such losses and have to leave the market.

Hypothesis 2 can be intepreted as: If a firm is aggressive and willing to absorb additional costs in the beginning for a greater market share, and if such aggressiveness can last long enough, other competitors will eventually back off.

Once the competitors back off, the producer becomes the leader, and it becomes increasingly difficult for the competitors to challenge the dominance. If the market lasts for a significant length of time, the leader is always able to earn enough profits to recover the additional costs incurred in the beginning.

HYPOTHESIS 3. *If some producers suddenly became very aggressive (try to change the state of the market and thus ignite a competition) or constantly make wild decisions (for example, suddenly raise production levels), then all producers suffer (the average total profits for all the producers drops).*

This is true as we saw in many industries, for example, airline services and telecommunications (D'Aveni, 1994). If a market has very few "wild" producers, it still can be stabilized by the modesty of the other producers. But if the ratio is high, then all the producers suffer (Szidarovszky and Yen, 1991; Wu and Yen, 1991).

4. EXPERIMENT DESIGN

In Section 2, several expectation schemes were discussed. They are the most straight-forward and can be easily identified. As we discussed earlier, these prediction schemes are very important because they help the producers to predict the rival's decisions at the production level.

Before we claim that prediction schemes are useful, it is important to know if they have been actually used and what benefits they have brought. Since theoretical aporoach cannot provide answers to such questions and the hypotheses in Section 3, we had to use laboratory experiments.

Five experiments have been conducted for this study, and they were carried out in the Economic Science Laboratory at the University of Arizona. A group of five subjects participated in each multi-period experiment as sellers. No buyers participated in these experiments, since the demand function $P(\sum_{i=1}^{5} x_i)$ governs the behavior of the market. The demand function determines the market price as a function of the total production by all the sellers in the market. It is assumed that all the products are sold in their production period and no backlog or inventory is allowed.

Each experiment consisted of fifty trading periods, and the length of the experiment was not revealed to the subjects. Subjects were undergraduate students of the University of Arizona. They were randomly recruited, and they were not required to have a major in either economics or business. All the subjects were promised $5.00 for showing up for an experiment, but only the first five were selected to participate.

In order to facilitate the calculations of production levels and the possible profits, subjects were allowed to use calculators. However, the software provided on-screen calculation for subjects, for example, his expected profit based on his production level and the assumed total productions of his rivals.

Once a group of five subjects was formed, instructions to the experiment were shown on the screens. One or more experiment monitors were available to answer any questions that related to the experiment in the instruction reading period. In order to avoid the formation of coalitions, subjects were told to remain silent, and absolutely no communication was allowed throughout the entire experiment.

Subjects were given an up-front capital payment. If the accumulated capital of any subject became negative, the subject was bankrupt and asked to leave the experiment. The first three periods were practice periods, and no monetary rewards given to the subjects for the profits earned.

The experiment subjects sat in separate carrels. In front of each subject was an IBM PC connected by local area network to a host computer (a RS 6000 server), from which each subject obtained the instructions of the experiment, the total production levels of the other subjects and the market prices. The decision on the production level of each subject was entered to the host computer through his terminal. When all the decisions had been entered and received by the host computer, the host computer summed up the productions, determined the market price, calculated the profit of each subject, and then, added the profits to the total capital accumulated by all subjects.

For all the demand function $P(t)$ was chosen to be:

$$P(t)=80-\sum_{k=1}^{5} x_k(t)\,. \tag{11}$$

The demand function remained the same for all trading periods. All the five subjects had the identical cost function C_k which depended only on her/his own production level:

$$C_k(x_k)=20\cdot x_k\,. \tag{12}$$

The price function and his/her own cost function were known to all participants. The graphs of these functions were even shown on the screen at all times. At each trading period each subject selected a production level. This single number was the only response from the participants at each trading period. They had a maximum of two minutes at each trading period to make their decisions.

The total profit of each subject was recorded and presented on the screen at all times. The number of time periods was not given to the subjects at the beginning of the experiment, since we did not want to have boundary induced effects. For the sake of simplicity we presented the list of information as shown on each subject's screen at all times:

— price function and the actual price of previous trading period,
— cost function and the actual cost of previous trading period,
— total production level divided into: his/her own last decision and the total output from the rest of the industry,
— profit earning at each trading period, and total earned up to that time.

The entire procedure can be easily duplicated, since the applied software

(COURNOT OLIGOPOLY) is now in public domain, and can be obtained from the Economic Science Laboratory of the University of Arizona by request.

5. DATA ANALYSIS

The first subsection provides a brief macro-analysis of the experiment outputs, with detailed discussion provided in the Appendix. The second subsection provides a micro-analysis of the behavior of some subjects.

5.1. Macro-Analysis Experiment Data

From the data sets collected we identified a wide range of variations in the subjects' behaviors, for example, in aggressiveness and the reactions to the changes of the market. Such variations are the most interesting and important findings in our experiment because they reflect the differences in the backgrounds and characteristics of these decision makers. However, the data also support part of the hypotheses as listed in Section 3. The following are the results.

1. Subjects who were aggressive in the beginning earned greater profits. In each experiment, the subject who was the most aggresive in the beginning (highest total productions in the first ten trading periods) received the highest total profits. That means Hypothesis 2 should be accepted. The conclusion we have is:—"A subject who is aggressive and willing to sacrifice profits in the beginning for a greater market share is able to make up the losses in the rest of the experiment and earn higher profits."

2. Some subjects have developed and used prediction schemes. Some subjects developed and used prediction schemes similar to the Cournot expectation (seven out of twenty five). We also found six subjects used adaptive expectations and seven subjects used extrapolative expectations in forming their decisions.

In summary, it is confirmed that some subjects (about one third) were able to develop and use prediction schemes either intentionally or unintentionlly. Note that all the subjects were undergraduate students and most of them did not know what expectations are (they were not required to have a business or economics major). However after playing few periods, they were able to realize that decisions must depend on what happened in the market and formulate some simple decision rules—"expectation schemes". This supports Hypothesis 1a.

However, Hypothesis 1b—"Subjects who developed and follow the prediction schemes had higher total profits." was not supported entirely by the experiment results. The reason seems to be that, to develop and follow prediction schemes is not enough; aggresiveness and cautiousness may also be more important. In addition, good prediction scheme helps; however bad prediction scheme is damaging. We will explain this below.

3. To earn higher total profits, subjects must be smart and aggresive. For detailed explanations, please see the Appendix.

4. Total profits of any subject depends not only on his performance but also on *the performances of the other competitors.* This finding appeared to contradict common sense because we expect those who do better than their competitors to win. However, in oligopolies, this is not entirely true. If some producers continuouslly make bad decisions, then everyone suffers, include those who try their best. However, if the competitors play smarter, it is possible that total profits might be even higher. This is one major characteristics of oligopolies and it can drive producers into forming coalitions.

5.2. Micro-analysis of Individuals' Behavior

Of all the players, subject 2 of group 1 (Subject A) and subject 3 of group 4 (Subject B) were chosen for more detailed analyses. The reason is they exhibited learning behavior and followed well-defined strategies.

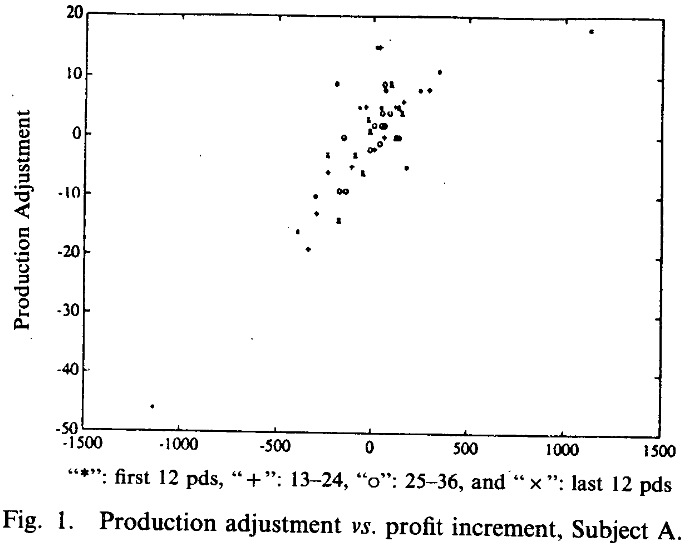
Prediction Schemes

The data set of the first experiment shows that subject A used the Cournot expectation. Figure 1 shows that the adjustments on the production level in most time periods depended on the profit Subject A earned in the previous-time period. The decisions made in the second half of the experiment provided even stronger support for this observation.

Figure 2 shows that the adjustments of production level made by Subject A also depended on the market price in the previous period.

Furthermore, it is also important to see that production-level decisions by Subject A had a strong correlation with the profit-maximizing outputs predicted by Cournot expectation as shown in Figure 3.

In order to identify the learning behavior of and what parameters were important to Subject A, we use regression analysis to analyze the experimental data. The first step is to identify the dependence of the predicted productions of the rivals, $S_A^*(t)$, on his own production level, $x_A(t-1)$, expected production levels, $S_A^*(t-1)$, and the actual production levels, $S_A(t-1)$, in the previous time period.



"*": first 12 pds, "+": 13–24, "o": 25–36, and "×": last 12 pds

Fig. 1. Production adjustment *vs.* profit increment, Subject A.

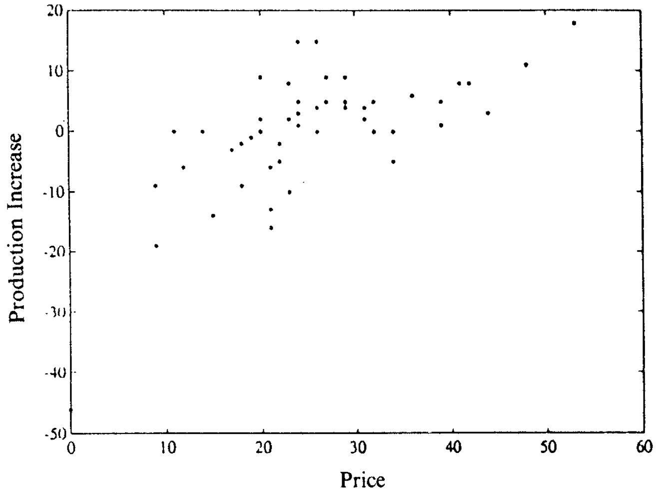


Fig. 2. Production adjustment *vs.* market price, Subject A.

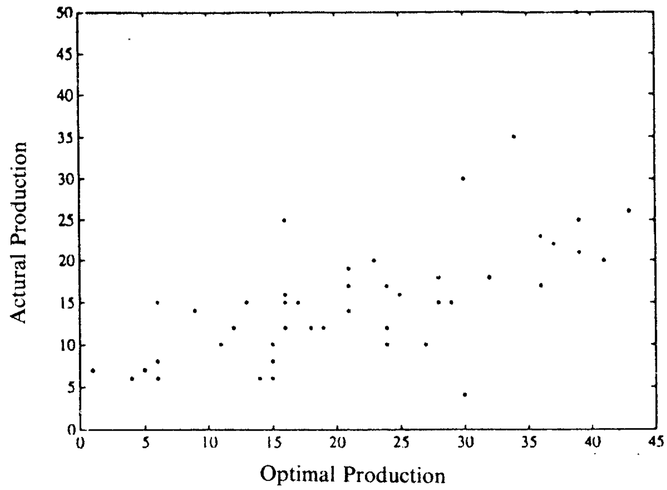


Fig. 3. Actual productions *vs.* productions predicted by Cournot Expectation Subject A.

We have the following linear relation:

$$S_A^*(t) \approx -0.282x_A(t-1) - 0.224S_A^*(t-1) + 1.023S_A(t-1)\,. \qquad (13)$$

The high value of $W_3 = 1.023$ indicates the decisions were dominated by the sum of the actual productions of the competitors in the previous time period, $S_A(t-1)$. The other two coefficients are small, so they are not important to Subject A.

We further calculate $\tilde{S}_A^*(t)$-the predicted production of the rivals by Subject A:

$$\tilde{S}_A^*(t) = -0.282x_A(t-1) - 0.224S_A^*(t-1) + 1.023S_A(t-1)\,, \qquad (14)$$

and plot both $\tilde{S}_A^*(t)$ and $S_A^*(t)$ together in Figure 4. It is clear that $\tilde{S}_A^*(t)$ follows closely $S_A^*(t)$, even if there were changes in the behavior of the competiors.

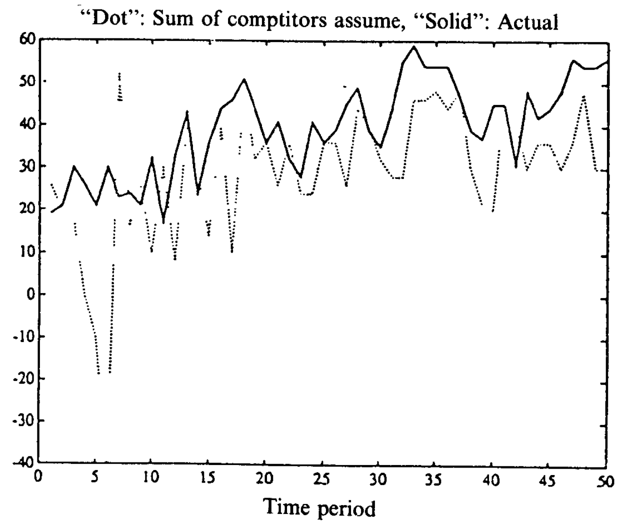


Fig. 4. Predictions *vs.* predictions fitted by regression, Subject A.

Such result was also supported by the t-test analysis. The t-values and the probabilities for parameters, $x_A(t-1)$, $S_A^*(t-1)$, and $S_A(t-1)$, were calculated as follows:

$$T_1 = -1.369\,, \qquad P_1 = 0.1777\,,$$
$$T_2 = 0.666\,, \qquad P_2 = 0.5888\,,$$
$$T_3 = 5.530\,, \quad \text{and} \quad P_3 = 0.0001\,.$$

Small $P_3 = 0.0001$ indicates the hypothesis that $S_A^*(t)$ depends on $S_A(t-1)$ should be accepted. High P_1 and P_2 values suggest the hypothesis that $S_i^*(t)$ depends on either $x_i(t-1)$ or $S_i^*(t-1)$ should be rejected. This confirms that Subject A used a pure Cournot expectation.

We next study the data set of Subject B. We have:

$$\tilde{S}_B^*(t) = 0.101x_B(t-1) + 0.546S_B^*(t-1) + 0.404S_B(t-1)\,,$$

The last two coefficients, $W_2 = 0.546$ and $W_3 = 0.404$, indicate that the expectation scheme depends on both $S_B^*(t-1)$ and $S_B(t-1)$. The dependence on the production level, $x_B(t)$, is difficult to estimate, since the ranges of $x_B(t-1)$, $S_B^*(t-1)$ and $S_B(t-1)$ are different. So, a small W_1 value does not necessarily mean that production level was not important to Subject B's decisions. Therefore, it is necessary to use a t-test to calculate the T_j values and the rejecting probabilities P_j:

$$T_1 = -8{,}201\,, \quad T_2 = -4.780\,, \quad \text{and} \quad T_3 = 4.889\,,$$

with very small P values. Low P_1, P_2 and P_3 values indicate the hypothesis that $S_B^*(t)$ depends on all the three variables should be accepted.

Based on these results, we conclude that Subject B used, not a pure Cournot

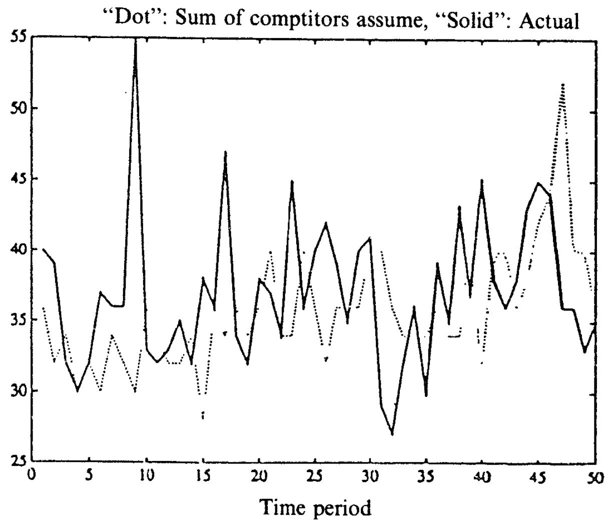


Fig. 5. Predictions *vs.* predictions fitted by regression, Subject B.

expectation, but a modified Cournot expectation that included more information. Such a complicated prediction scheme gave Subject B more accurate predictions on the behavior of the competitors. A more detailed discussion is provided later.

Similarly, we plot both $\tilde{S}_B^*(t)$ and $S_B^*(t)$ together in Figure 5. From the figure it is clear the approximated $\tilde{S}_B^*(t)$ follows the $S_B^*(t)$ very closely, especially during the periods when significant changes can be observed in the behavior of the competitors.

Learing Behaviors

It is important that we can identify the learning behavior in Figures 4 and 5. Such learning behavior and the development of the prediction schemes can be seen from the shrinking of the gaps between $\tilde{S}_i^*(t)$ and $S_i^*(t)$ as well as the adjustments to the changes. For Subject A, the data fit very well for the whole experiments except the first ten periods. This indicates that Subject A was learning and developing his/her prediction scheme during early periods.

For Subject B, the expectation scheme is more complicated and includes more information, therefore the experimental data set does not fit as well as for Subject A. Notice also that there is always a time lag. Some learning behavior can be seen as indicated by the shrinking of the gap between $\tilde{S}_i^*(t)$ and $S_i^*(t)$ and the quick adjustments adjustments to the changes of the market.

To see again what expectation schemes have been used we use regression analysis to study the dependences of the production decision, $x_i(t)$. We plot the production levels $x_i^*(t)$ predicted by Cournot expectation together with the true production levels $x_i(t)$ (See Figure 6 for Subject A and Figure 7 for Subject B). As the production levels of both subjects approximate the values predicted by Cournot expectation, these observations support Hypothesis la that some subjects developed and used prediction schemes. As mentioned earlier, it is remarkable that Subject

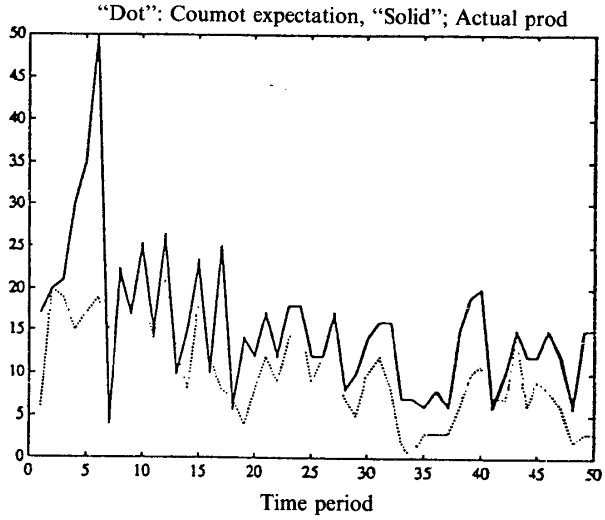


Fig. 6. Actual productions *vs.* productions determined by Cournot Expectation, Subject A.

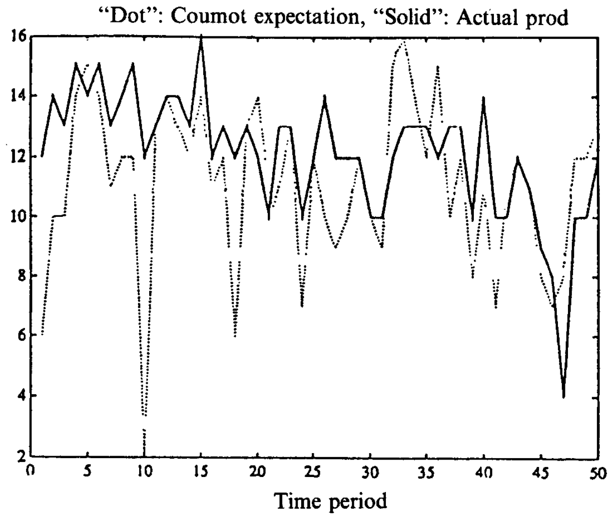


Fig. 7. Actual productions *vs.* productions determined by Cournot Expectation, Subject B.

B had only 98 units of total prediction erros, which is an average of less than two units per time period.

It is very difficult to tell whether Subject A or Subject B is better, because the total profits also depend on the performances of the rivals. However, we believe the more complicated prediction developed by Subject B was better than the pure Cournot expectation used by Subject A.

Time-dependent Memory and Extrapolative Expectations

Since a solid answer for the use of Extrapolative Expectations was not given in the previous subsection, we use the following analysis to explain how Extrapolative Expectations were developed and used.

Cournot expectation is very primitive since it depends only on the data of the previous period. We believe that the learning behavior (development of the prediction scheme and the strategy) should be a multi-period process. As we expect the decisions of some subjects depended on the data in earlier time periods, we use regression analysis and the t-test again. Fist we let

$$S_i^*(t) \approx \sum_{j=1}^{T} W_j \cdot S_i(t-j)\,. \tag{15}$$

For $T=1$, it renders a Cournot expectation. We first analyze the data set of Subject A. Starting with $T=3$ we have:

$$W_1 = 1.162\,, \quad T_1 = 4.561\,, \quad P_1 = 0.0001\,,$$
$$W_2 = -0.369\,, \quad T_2 = 1.291\,, \quad P_2 = 0.2034\,,$$
$$W_3 = 0.210\,, \quad T_3 = 0.835\,, \quad \text{and} \quad P_3 = 0.4081\,.$$

The decisions only affected by the data in the most recent single times period. It is interesting to see that W_2 is negative; this indicates that the expectation depended slightly on the adjustments $S_A(t-1) - S_A(t-2)$. We next let $T=4$, and we have

$$W_1 = 1.200\,, \quad T_1 = 4.547\,, \quad P_1 = 0.0001\,,$$
$$W_2 = -0.428\,, \quad T_2 = -1.501\,, \quad P_2 = 0.1410\,,$$
$$W_3 = 0.152\,, \quad T_3 = 0.536\,, \quad P_3 = 0.595\,,$$
$$W_4 = -0.004\,, \quad T_4 = -0.015\,, \quad \text{and} \quad P_4 = 0.9878\,.$$

These numbers provide the same message as $T=3$: The decisions only affected by the data in the most recent time period. However, $|T_j|$ decreases and P_j increases as j increases. This is a proof that the human memory decays.

By using the computed coefficients W_A we calculated $\tilde{S}_A^*(t)$, and we plot both $S^*(t)$ and $\tilde{S}_A^*(t)$ in Figure 8. From Figure 8 we see the gap between $S^*(t)$ and $\tilde{S}_A^*(t)$ is very small after the first five time periods. This tells us that Subject A developed a prediction scheme very quickly. Furthermore, and more importantly, his prediction scheme was accurate. Subject A was able to predict the transitions, and he followed what was recommended by the prediction scheme he developed.

Our conclusions here are: Airst, for Subject A, the data of the past one time period is the most important in his decisions. Second, the importance of the data in earlier time periods decreased as the distance in time increased.

We conducted a similar regression analysis and t-test for the data set of Subject B. First we let $T=3$ and have:

$$W_1 = 0.395\,, \quad T_1 = 4.083\,, \quad P_1 = 0.0002\,,$$
$$W_2 = 0.158, \quad T_2 = 1.607\,, \quad P_2 = 0.1154\,,$$
$$W_2 = 0.157\,, \quad T_3 = 1.618\,, \quad \text{and} \quad P_3 = 0.1130\,.$$

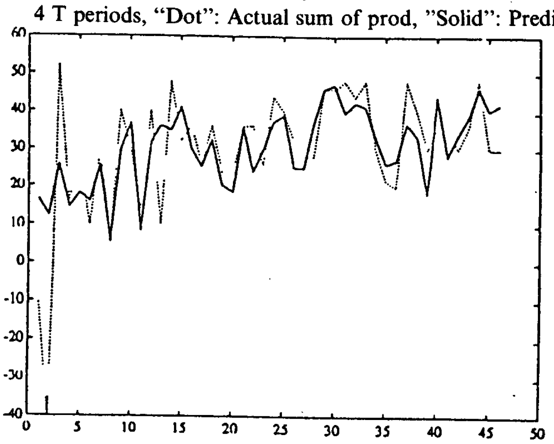


Fig. 8. Actual predictions $S^*(t)$ and predictions fitted by regression analysis $S_A^*(t)$ for $T=4$, Subject A.

It is clear that the decisions of subject B depended only on $S_B(t-1)$, but such dependence was not as strong as that of subject A; the reason has been discussed earlier. Influences of $S_B(t-2)$ and $S_B(t-3)$ on subject B's expectations were not very significant as indicated by the values of T_j and P_j. Next we let $T=4$ and we have:

$$W_1=0.384\,,\quad T_1=4.042\,,\qquad P_1=0.0002\,,$$
$$W_2=0.163\,,\quad T_2=1.717\,,\qquad P_2=0.0936\,,$$
$$W_3=0.145\,,\quad T_3=1.526\,,\qquad P_3=0.1348\,,$$
$$W_4=0.156\,,\quad T_4=1.652\,,\quad \text{and}\quad P_4=0.1062\,.$$

The results show again that data older than three times periods had little effect on Subject B's decisions. Such behavior was similar to that of Subject A, but the values of W_j tell us the decisions of subject B, unlike the decisions of Subject A which were dominated by $S_A(t-1)$ ($W_1=1.162$), were only affected partially by $S_B(t-1)$ ($W_1=0.384$).

The last analysis of the time-dependent memory is to study the dependence of expectation adjustments:

$$S_A^*(t)-S_A(t-1)\approx \sum_{j=1}^{T} W_j(S_A^*(t-j)-S_A^*(t-j-1))\,. \tag{16}$$

First, for $T=2$, we have $W_1=0.713$ and $W_2=0.332$. Next, for $T=3$, we have $W_1=0.738$, $W_2=0.437$, and $W_3=0.163$.

If we assume an exponential discount rate, the dependence of W_j can be approximated:

$$W_j\approx 0.723\cdot e^{-0.646\cdot(j-1)}\,,\qquad \text{for}\quad j\leq 3\,.$$

Such results is also supported by t-test. For example, for $T=4$,

$$W_1=0.608\,,\quad T_1=7.203\,,\qquad P_1=0.0001\,,$$
$$W_2=0.438\,,\quad T_2=4.476\,,\qquad P_2=0.0001\,,$$
$$W_3=0.287\,,\quad T_3=2.994\,,\qquad P_3=0.0047\,,$$
$$W_4=0.143\,,\quad T_4=1.694\,,\quad \text{and}\quad P_4=0.0981\,.$$

The results cannot be used to claim that Subject A or Subject B used Extrapolative Expectations; however, they do indicate that the decision makers' memories decayed.

6. CONCLUSIONS

In this study we used three approaches to studying competition in dynamic markets: theoretical studies, case studies, and laboratory experiments. We first investigated how corporations competed in dynamic markets. We then used the theoretical approach to develop prediction schemes for competition. Finally we designed and implemented an artificial market with personal computers and local area networks to study how subjects learn, develop their strategies, and react to market changes.

Such a combination led us to new insights impossible to reach by any single approach. In these experiments, we have observed the learning behavior, the development of various prediction schemes, and the selection of long-term strategy are very important. For example, subjects who have developed and used prediction schemes earned higher profits, also subjects who were more aggressive and more cautious became the leaders and earned higher profits. These observations are also supported by the cases in the real world, for example, the competitions in the automobile and telecommunications. In summary, pursuing a good long-term strategy is sometimes even more important than satisfying short-term goals.

Based on these findings, we propose guidelines for a decision model which can be used as the foundation for a decision support system to support daily operation and long-term strategic planning. In summary, in order to be a winner (earn high total profits), a producer should:

1. Choose a market in which it does not have too many "powerful" and "wild" competitors.
2. Develop an accurate and efficient scheme to predict the behavior or/and decisions of the competitors.
3. Based on the company's characteristics, adopt an appropriate long-term strategy.

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APPENDIX

Macro-Analysis of Experimental Results

The data sets collected show a wide range of variation in the subjects' behavior. Some of them developed good prediction schemes, made good decisions, and naturally, earned higher total profits, while others did not. Such variations in the subjects' behavior provide good insight into the behavior of decision makers in the real world.

To identify what behavior or strategies can lead to maximal total profits is also one of the goals of this subsection. However, since the profit earned by any one of the subjects depends not only on his decisions but also on the decisions of the other subjects, one strategy might work well in one experiment but may show poor performance in others. So, when we study the learning behavior of one subject, it is important to know the simultaneous behavior of the other subjects.

Data collected from the experiments are the market price, $P(t)$, the total input into the market, $\sum_{k=1}^{5} x_k(t)$, the production levels, $x_k(t)$, and the profits, $Q_k(t)$, of all the subjects, where $k \in \{1, 2, 3, 4, 5\}$ and $t = 1, 2, \cdots, 50$. Profit $Q_k(t)$ is determined as:

$$Q_k(t) = x_k(t) \cdot (P(t) - 20.0) .$$

The results of the experiments are provided in Table 1, where we use the following notation:

$Sx \quad = \sum_{t=1}^{50} x(t)$, sum of production,
$SE1 \quad = \sum_{t=1}^{50} |S^*(t) - S(t)|$, sum of prediction error,
$SE2 \quad = \sum_{t=1}^{50} |x(t) - x_C^*(t)|$, sum of differences between the actual production and the Cournot model's prediction,
$Sx10 = \sum_{t=1}^{10} x(t)$ sum of production of the first ten periods,
$Ind1 \quad = Sx - SE1$, first index of total profit,
$Ind2 \quad = Sx - SE2$, second index of total profit, and
$SQ \quad = \sum_{t=1}^{50} Q(t)$, total profit.

In the first column, the number in [ ] indicates the sum of $SE1$ (we call it the first-type errors), the number in ( ) is the sum of $SE2$ (the second-type errors), and the number in { } is the sum of total profits SQ from all the subjects in the same group.

From Table 1 we obtained the following results:

1. To be aggressive in the beginning is important. In each experiment, the subject who had the highest $Sx10$ value (highest total productions in the first ten trading periods) had the highest total profits. This is true for all the experiments: Subject 2 in group 1, subject 4 in group 2, subject 2 in group 3, subject 3 in group 4, and subject 3 in group 5. It is not a coincidence since we have observed similar behavior in earlier experiments, however, this is the first paper to discuss such behavior.

We also checked the hypothesis by t-test for all the 25 subjects. We chose the level of significance α to be 0.05. We assume a linear relation, which was computed by regression:

$$SQ_i = w_i \cdot Sx10_i + A_i ,$$

where $i = 1, \cdots, 25$. We obtained $T = 5.516$, and the rejection probability (P) was much lower than α. In all cases Hypothesis 2 should be accepted. The conclusion we have is:—"A subject who is aggressive and willing to sacrifice early profits for a greater market share can win."

2. Some subjects have developed and used prediction schemes. Some subjects have developed and used prediction schemes similar to the Cournot expectation. Every experiment had fifty trading periods, and some subjects had accumulated less than 150 units different from what the Cournot expectation suggested. In other words, the average difference was less than 3 units per trading period; this was

TABLE 1. Summary of Experiments

Exp. No.	Sub. No.	*Sx*	*SE*1	*SE*2	*Sx*10	*Ind*1	*Ind*2	*SQ*
1	1	419	526	268	39	−107	151	1221
[2984]	2	773	674	305	241	99	468	2702
(1449)	3	303	462	245	44	−159	58	1119
{8479}	4	381	460	205	61	−79	176	1437
	5	869	862	426	103	7	443	2000
2	1	747	585	303	177	162	444	3066
[2452]	2	688	546	279	103	142	309	2541
(1201)	3	231	289	135	32	−58	96	1193
{11608}	4	149	313	115	20	−164	34	669
	5	929	719	369	229	210	560	4139
3	1	447	297	147	95	150	300	4086
[1650]	2	668	458	216	130	210	452	5370
(794)	3	520	272	136	109	248	384	4890
{22248}	4	519	311	153	74	203	366	4692
	5	370	312	142	62	58	228	3110
4	1	452	283	147	87	169	305	4614
[1384]	2	360	287	117	74	73	243	3794
(630)	3	606	233	98	137	373	508	6350
{24479}	4	566	333	162	106	233	404	5344
	5	475	248	106	103	227	369	4977
5	1	735	665	330	60	70	405	3768
[2180]	2	209	355	132	59	−146	77	1532
(992)	3	658	442	188	150	216	470	4736
{17244}	4	390	326	154	78	64	236	2740
	5	626	392	188	120	234	438	4468

true for subjects 3 and 4 in group 2, subjects 1 and 5 in group 3, and almost every subject in group 4. Subject 3 in group 4 was the best in that his average difference was less than two units. His learning behavior and strategy is discussed in more detail in section 5.

We also ran the t-test for all the subjects. We first test the uses of Cournot Expectation; we let

$$S_i^*(t) = w_i \cdot S_i(t-1) + A_i ,$$

where $S_i^*(t)$ is the expected production levels of the rivals and $S_i(t-1)$ the actual production levels of the rivals in the previous time period. With the level of significance selected again to be 0.05, 7 subjects meet the requirement: subjects 3 and 4 in group 2, subject 3 in group 3, subjects 2, 3 and 5 in group 4, as well as subject 2 in group 5. The ratio, 7 out of 25, is very high.

We next test the uses of Adaptive Expectation. We let

$$S_i^*(t) = w_{i1} S_i^*(t-1) + w_{2i}\beta\left(\sum_{l \neq k} x_l(t-1) - S_i^*(t-1) \right).$$

Since $\beta \in (0, 1)$ and the selection of β can be time-dependent, we modifiy the equation slightly

$$S_i^*(t) = w_{i1} S_i^*(t-1) + w_{2i}\left(\sum_{l \neq k} x_l(t-1) - S_i^*(t-1) \right).$$

Six subjects meet this requirement. However, the β values were hard to obtain because they were not constants, but fluctuated or were time-dependent.

Lastly, we test the use of Extrapolative Expectation. We let

$$S_i^*(t) = \sum_{j=1}^{T} w_{ij} \cdot S_i(t-j) + A_i,$$

to test the Extrapolative Expectation up to T time periods earlier. If we let $T=3$, then we test the dependence on the rivals' production levels up to three earlier time periods. If $T=1$, it is a pure Cournot Expectation. So, again we found seven subjects' $P<0.05$, this confirms that seven subjects used Cournot Expectations. However, for $T=2$, the lowest value of P is 0.0863, and only two subjects' $P<0.1$. So our conclusion is: The hypothesis that some subjects used Extrapolative Expectation should be rejected. Which means that the subjects' decisions are based only on the data in the previous time period, not on earlier data.

In summary, in these experiments some subjects were able to develop and use prediction schemes either intentionally or unintentionally. All the subjects were undergraduate students, most of them never heard expectations (they were not required to have a business or economics major). However, after playing few periods, some subjects started to realize decisions must depend on what happened before, and formulated simple decision rules—expectation schemes. This supports Hypothesis 1a.

However, Hypothesis 1b—"Subjects who developed and follow the prediction schemes had higher total profits." was not supported entirely by experimental results. The reason was: To develop and follow prediction schemes was not enough; aggressiveness and cautiousness were also important factors. In addition, accurate prediction scheme helps, while an inaccurate prediction scheme is damaging. We will explain this below.

3. Both Ind1 and Ind2 are good measures of the total profits. Notice that $Ind1 = \sum_{t=1}^{50} x(t) - \sum_{t=1}^{50} |S^*(t) - S(t)|$ equals the total production minus the first-type error. The expectation $S^*(t)$ was computed from equation (10), where $s_k^E(t)$ was considered as the unknown. Prediction error is the difference between the total production a subject predicted his competitors would produce and what then actually produced. A high *Ind*1 means a subject was "aggresive" and also "smart", that is, able to keep a high production level and to predict the decisions of his competitors.

We ran a t-test to test the hypothesis that total profits SQ depends on $Ind1$; we have $T = 11.292$ and a very low P values. Thus, $Ind1$ is a very good indicator of the total profits.

The value $Ind2 = \sum_{t=1}^{50} x(t) - \sum_{t=1}^{50} |x(t) - x_C^*(t)|$ equals the total production minus the total second-type error. Similarly, we ran a t-test to test the hypothesis that SQ depends on $Ind2$. We have $T = 11.375$, again with a very small P value.

It provides almost the same accuracy as $Ind1$ in predicting total profits. If a subject is aggresive and follows what Cournot expectation suggested, he or she should be a winner.

4. Total profits of any subject depend not only on his performance but also on the performances of the other competitors. This statement seemed to be strange, because our common sense told us that, if the competitors did a poorer job than we did, then we are the winners. However, in oligopolistic markets, if some producers make bad decisions, then everyone suffers, including those who have done their best. This is an issue which concerns how to judge performance. If we compare ourselves with our competitors and we have better strategies, our total profits might be higher than that of others. However, if the competitors play smarter, it is possible that our total profits might be even higher. This is one major characteristic of oligopoly, and it sometimes drives producers into forming coalitions.

In Table 1, there are three numbers in the first column: $[\ \ ] = \sum_{i=1}^{5} SE1$, $(\ \ ) = \sum_{i=1}^{5} SE2$, and $\{\ \ \} = \sum_{i=1}^{5} SQ$. It is easy to see that the higher the total group error, the lower their total profit. Group 4 had very low values on both, so every subject received very high SQ. On the other hand, most of the subjects in group 1 did poor jobs and everyone suffered. For example, subject 2 of group 1 had higher $Ind1$ and $Ind2$ than subject 5 of group 3 and subject 2 of group 4, however, he received the lowest total profits.

We ran two sets of t-tests to conclude this section. We first tested the hypothesis that total group profits, $\{\ \ \}$, depend on first-type total errors, $[\ \ ]$. Then we tested if it depends on second-type total errors, $(\ \ )$. We have

$$[\ \ ]: T_1 = -29.316 \quad \text{and} \quad (\ \ ): T_2 = -36.861\,,$$

with very small P values. So, it is clear that the hypothesis should be accepted. Next we tested the hypothesis that individual's total profit, SQ, depends on the $[\ \ ]$, $(\ \ )$, $Ind1$, $Ind2$, and $SX10$. We have

$$\begin{aligned} [\ \ ]&: \quad T_1 = -8.167\,, \\ (\ \ )&: \quad T_2 = -7.692\,, \\ Ind1&: \quad T_3 = 11.429\,, \\ Ind2&: \quad T_4 = 12.315\,, \\ SX10&: \quad T_5 = 5.764\,, \end{aligned}$$

all with very low P values. So, it is clear that individual's total profit, SQ, depends on all five variables. This is a very important lesson. Although a company had very good strategy and was very aggressive, if competitors always made "bad" decisions, it will not earn high profits.