MARKETABLE SURPLUS, URBAN UNEMPLOYMENT AND DEVELOPMENT PLANNING

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First version received July 1993; final version accepted April 1994

Abstract: A time-minimisation problem of attaining a full-employment state is solved in a dual economy model where the rural-urban migration mechanism is of Harris–Todaro type. The optimum solution may appear as a policy of urban development at the most rapid rate.

1. INTRODUCTION AND SUMMARY

The purpose of this paper is to develop a dynamic planning model of a dual economy that can focus attention both on the problem of food shortage and on urban unemployment, and can examine the optimum investment allocation problem between the urban and rural sectors. The main conclusion obtained from the analysis of the model developed in this paper is that the optimum growth path is the balanced growth path, and that a policy of investment specialization to a particular sector may be optimal in the initial phase of development only if the economy is off the balanced growth path. Also there seems to be no conflict between a program of urban development and the solution to the urban unemployment problem. This result diametrically opposes the view of Todaro (1976) obtained from the analysis of a static model that there is no strict urban solution to the urban unemployment problem.

In the development process of the advanced urban sector of a less developed dual economy, one important problem is that the urban wage-rate is institutionally fixed in terms of food, produced only in the rural sector. Hence any program of urban development should give rise to the problem of shortage of food in the urban sector unless such a program is accompanied by a parallel program of rural development. The optimum allocation of investment between the rural and the urban sector is therefore of critical importance. Aspects of the problem have been analysed by Dixit (1969), Bardhan (1970) etc. in the dynamic models of capital accumulation. Dixit (1969) solved a time minimization problem of industrial

Acknowledgement. I am indebted to Prof. A. Bose of I.I.M., Calcutta, India, and a referee of this journal for their valuable comments. Remaining errors are solely mine.

development and came to the solution that the optimum growth path is the balanced growth path and the optimum agro-industry terms of trade is the competitive one. But these authors adopted the Lewis (1954) assumption of a perfectly elastic supply of labour from the rural to the urban sector at an institutionally fixed wage-rate; and hence failed to capture the urban unemployment problem resulting from rural-urban migration.

Rural-urban migration and urban unemployment problems have received considerable attention in the theoretical literature of development economics. The literature on this aspect starts with the models of Harris and Todaro (1970) and Todaro (1969, 1976). An institutionally given urban wage-rate and a wage-differential between the urban and the rural sectors form the basis of their framework. Migration from the rural sector to the urban sector results when actual rural wage-rate falls short of the expected urban wage rate defined as the actual urban wage rate multiplied by the ratio of urban employment to urban labour force. Migration equilibrium is established when expected urban wage-rate equals the actual rural wage rate, and the existence of urban unemployment is explained as a migration-equilibrium phenomenon. One of the implications of those models is that the urban unemployment problem can not be solved by a policy of urban development.[1]

The basic Harris–Todaro (1970) model has been re-analysed and extended by various authors in various directions.[2] However, the existing literature has remained essentially static in character, and hence has failed to make the allocation of investment and capital accumulation endogenous to the analysis. But clearly the nature of the optimum investment policy and the employment policy for the urban sector should be re-examined in terms of a dynamic planning model of a dual economy based on the Harris–Todaro (1970) migration-mechanism. One should look into the solution of a minimum-time problem of attaining full employment and to compare this with minimum-time solutions to a targeted urban development problem. The only dynamic model by Jha and Lachler (1981) examines the nature of optimum taxation and the production of public goods in the Harris–Todaro economy solving the Ramsey problem. But we do not find any time-minimization exercise of attaining a full employment state in their model. Also they do not give emphasis on the issue of investment-allocation between the agricultural sector and the industrial sector.

We consider a two sector dynamic planning model of a dual economy. In order to establish consistency between migration-equilibrium and urban unemployment, the Lewis (1954) hypothesis is replaced by the Harris–Todaro (1970) migration-mechanism. The present model is more general than that of Dixit (1969). On the one hand, it is based on the assumptions of specialization of production in the two sectors and the minimum urban wage rate, fixed in terms of rural sector's

[1] See Todaro (1976; Pages 211–212; 216).

[2] See, for example, Bhatia (1979), Calvo (1978), Bhagwati and Srinivasan (1974), Dutta-Chaudhuri (1983), Stiglitz (1974), edc.

product. On the other hand, it considers Harris–Todaro (1970) rural-urban migration-mechanism and hence can explain the time-behaviour of urban unemployment. A time minimization problem of attaining a full employment state starting from an initial urban unemployment state is solved; and the solutions obtained from this exercise are similar to those obtained in Dixit (1969) who solved a minimum-time problem of attaining a targeted industrial capital stock. So far as the optimal extraction of the surplus food is concerned, it should be left to a competitive food-market. The optimum growth-path is the balanced growth path.

The model is described in the section 2 of this paper. The time-minimization problem is solved in the section 3.

2. THE MODEL

The economy considered in this model is an internationally closed dual economy with an institutionally advanced urban sector and a backward rural sector. The urban sector produces capital good and the rural sector produces food, both with CRS Cobb–Douglas production functions using capital and labour as inputs. Capital used in both the sectors is of the same type; but once installed, they are non-shiftable. The allocation of investment between the two sectors is subject to the control of the planning authority. For the sake of simplicity, it is assumed that capital installed in either sector does not depreciate over time. The labour force in the advanced sector is supplied by the rural sector. There is no direct migration policy of the planning authority and the migration-mechanism from the rural to urban sectors is of Harris–Todaro (1970) type. Urban wage rate is fixed in terms of food. A fraction of the rural output is obtained by the planning authority imposing a proportional income tax in the rural sector, and the wage bill in the urban sector is met by the tax-revenue. Investment in the rural sector is made free of cost. The excess of food over the tax payment is consumed in the rural sector. The size of the labour force of the economy grows at a constant rate.

Let 1 and 2 stand for the urban and the rural sectors. Following notations are used in the model.

k_i = Capital stock in sector i as a ratio of total population.
l_i = Employment in sector i as a ratio of total population.
l_u = Urban unemployment as a ratio of total population.
x_i = Capital-labour ratio in sector i.
w = Urban wage rate.
v = Fraction of investment allocated to the urban sector.
n = Rate of population growth.
Y_i = Average productivity of labour in sector i.
r = Rate of proportional income tax in the rural sector.
t = Time-point.
O = Initial time-period.
T = Minimum Time.

f_i = Production function of sector i, normalized with respect to labour-use.
a_i = Capital-elasticity of output in sector i.
$i = 1, 2$.

Now we come to the equational-structure of the model. The Cobb-Douglas production function in sector i, normalized with respect to labour use, is given by

$$Y_i = f_i(x_i) = x_i^{a_i}\,. \tag{1}$$

The Harris–Todaro (1970) migration-equilibrium condition is given by

$$w \cdot l_1 = (1-r) \cdot f_2(x_2) \cdot (1-l_2)\,. \tag{2}$$

Here $(l_1/(1-l_2))$ is the probability of obtaining an urban job of the representative rural migrant and $(w \cdot l_1/(1-l_2))$ is his expected urban wage income. $((1-r) \cdot f_2(x_2))$ is the effective rural wage rate in terms of food, and migration from rural to urban sector continues so long as expected urban wage rate is higher than the actual rural wage rate. These two are equal in migration-equilibrium.

$$l_u = 1 - l_1 - l_2 \tag{3}$$

$$w \cdot l_1 = r \cdot f_2(x_2) \cdot l_2\,. \tag{4}$$

Equation (4) states that the wage-bill in urban sector is met by the tax-revenue.

The two equations of motion, governing the behaviour of k_1 and k_2 over time are the following:

$$\dot{k}_1 = v \cdot f_1(x_1) \cdot l_1 - n \cdot k_1\,, \tag{5}$$

and

$$\dot{k}_2 = (1-v) \cdot f_1(x_1) \cdot l_1 - n \cdot k_2\,. \tag{6}$$

We impose the condition

$$0 < r \leqslant a_2\,. \tag{7}$$

This implies that the tax rate on rural output should not exceed the competitive share of capital in rural output. Hence at least the competitive labour-share in rural output is guaranteed for the consumption of rural labour force.

Using equations (2) and (4), we have

$$l_2 = 1 - r\,. \tag{8}$$

So the employment-population ratio in the rural sector equals the consumption-income ratio in that sector.

From equations (4) and (8), we have,

$$l_1 = ((r \cdot f_2(k_2/(1-r)) \cdot (1-r)/w)\,. \tag{9}$$

So, given the urban wage rate, employment in the urban sector depends on the tax rate and per capita capital stock in the rural sector. Here l_1 is not a control variable; but is a state variable, being function of k_2 and r.

Using equations (2) and (3), we have,

$$l_u = ((w/(1-r)\cdot f_2(x_2)) - l)\cdot l_1\,. \tag{10}$$

Hence for $l_1 > 0$, this implies that $l_u \geqslant 0$ if $w \geqslant (1-r)\cdot f_2(x_2)$. The economy is in full-employment, when the actual urban wage rate equals the actual rural wage rate, i.e.,

$$w = (1-r)\cdot f_2(x_2)\,. \tag{11}$$

We define b_1 and b_2 as follows:

(i) $k_2 = b_1$ at $f_2(k_2) = w\,.$

(ii) $k_2 = b_2$ at $(1-a_2)\cdot f_2(k_2/(1-a_2)) = w\,.$

From equations (1), (8) and (11), we find,

$$w = k_2^{a_2}\cdot(1-r)^{(1-a_2)}\,. \tag{12}$$

As w is fixed, taking the total differential of (12), we find, $(dk_2/d(1-r)) = -(a_2 \cdot (1-r)/(1-a_2)\cdot k_2) < 0$.

So the higher the tax rate on rural output the larger is the rural per-capita capital stock consistent with the full-employment state at the given urban wage rate. Since $r \leqslant a_2$, one can establish the following proposition:

PROPOSITION 1. *$k_2 \geqslant b_2$ is a sufficient condition for full-employment.*

From (9), we find that $l_1 > 0$ only if $r > 0$; and for some $r > 0$, equation (11) is satisfied at some $k_2 \geqslant b_1$. Hence we find that the economy can not attain a full-employment state if $k_2 < b_1$.

3. THE DYNAMIC OPTIMIZATION

Because of the assumption of non-shiftability of capital in either sector, k_1 and k_2 change only over time, and this is regulated by the planner controlling the time-behaviour of v. So if the economy suffers from the urban unemployment problem in the initial stage of the plan, the full-employment state can be reached only at some future date. The time-minimization problem of attaining a full-employment state is equivalent to the minimum time problem of attaining a targeted per-capita capital stock in the rural sector because full-employment state is reached once the per-capita capital stock in the rural sector reaches the appropriate level depending on the tax rate. Since $k_2 \geqslant b_2$ is a sufficient condition for full-employment regardless of the tax rate, we solve the following minimum time problem:

Minimize $\int_0^T dt$ subject to equations and inequalities given by (1) to (7); $0 \leqslant v \leqslant 1$; $k_2(0) < b_1$; $k_2(T) \geqslant b_2$.

Using equations (1) and (9), we have,

$$f_1(x_1)\cdot l_1 = k_1^{a_1}\cdot k_2^{a_2\cdot(1-a_1)}\cdot((r/w)\cdot(1-r)^{(1-a_2)})^{(1-a_1)}\,. \tag{13}$$

So this minimum-time problem of attaining the full-employment state is given by: Minimize $\int_0^T dt$, subject to

$$\dot{k}_1 = v\cdot A\cdot k_a^{a_1}\cdot k_2^{a_2\cdot(1-a_1)} - n\cdot k_1\,, \tag{14}$$

$$\dot{k}_2 = (1-v)\cdot A\cdot k_1^{a_1}\cdot k_2^{a_2\cdot(1-a_1)} - n\cdot k_2\,, \tag{15}$$

$0\leqslant v\leqslant 1$; $0<r\leqslant a_2$; $k_2(0)$ given, and $k_2(T)\geqslant b_2$.

Here,
$$A = ((r/w)\cdot(1-r)^{(1-a_2)})^{(1-a_1)}\,. \tag{16}$$

Here k_1 and k_2 are the two state variables, and v and r are the two control variables. All these are functions of time.

The Hamiltonian is given by the following:

$$H = q\cdot A\cdot k_1^{a_1}\cdot k_2^{a_2\cdot(1-a_1)} - n\cdot q_1 k_1 - n\cdot q_2 k_2\,, \tag{17}$$

where $q = v\cdot q_1 + (1-v)\cdot q_2$; q_1 and q_2 being two co-state variables, functions of time. At each t, H is to be maximized through the choice of r and v. The necessary conditions for a program to be optimal are given by the followings (see Pontrjagin, 1962, page 298):

(i) H is maximized as a function of r and v subject to the equations of motions and other constraints.

(ii) There exist k_1, k_2, q_1 and q_2, continuous functions of time, satisfying (14), (15) and

$$q_i = -(dH/dk_i) \qquad \text{for} \quad i=1,2\,.$$

(iii) Transversality condition: $(q_1(T), q_2(T))$ is orthogonal to the plane, $k_2=b_2$; i.e., $q_1(T)=0$.

3.1 *Optimum Extraction of Rural Surplus*

First, we maximize the Hamiltonian with respect to r, keeping v fixed. We find,

$$(dH/dr) = q\cdot k_1^{a_1}\cdot k_2^{a_2\cdot(1-a_1)}\cdot(dA/dr)\,. \tag{18}$$

It can be easily shown from (16) that $(dA/dr)\geqslant 0$ for all $r\in[0, a_2]$; and hence from (18), we find that $(dH/dr)\geqslant 0$ for all r in the control region. Hence $r=a_2$ is optimal. So we have

PROPOSITION 2. *Optimal proportional tax on rural output should be equal to the competitive share of capital in rural output.*

Note that in this model, the planner supplies capital to the rural sector free of cost and shares a part of rural output. From the view point of analytical simplicity only we assume that the share goes to the urban sector in the form of an agricultural income tax. We never recommend it as a policy as it is not at all feasible to implement from the view point of political and administrative difficulties in a democratic country. The planner must leave the decision of consuming food or selling it in exchange for the industrial product to the free will of the peasants. Our analysis suggests that it is optimal to extract the capital's share in rural output as surplus. The peasants supply only labour input in food production and their optimal consumption equals their competitive share of labour. So if surplus food is extracted by selling the service of capital in exchange of food instead of supplying the service free of cost and imposing taxes, the optimal price of capital good should be equal to the marginal productivity of capital in rural sector. So far as the optimal extraction of the surplus food is concerned, it should be left to a competitive food market. If there is competitive rural-urban exchange, the optimal marketed surplus extraction policy suggests neither to subsidize the peasants nor to impose any additional tax on them.

Bhagwati and Srinivasan (1974), reanalysing the static Harris–Todaro (1970) model, have attempted to show that a wage subsidy to the urban sector plus a price subsidy to the rural sector yields the optimal solution. But this dynamic analysis of time-minimization exercise of reaching a full-employment state in a Harris–Todaro world, does not find the optimality of subsidizing the rural sector.

3.2. *Optimum Investment Allocation*

We take $A=A^*$ when $r=a_2$, i.e., r takes optimum value. Also $H=H^*$ when maximized with respect to r. Hence,

$$H^*=q\cdot A^*\cdot k_1^{a_1}\cdot k_2^{a_2\cdot(1-a_1)}-n\cdot q_1\cdot k_1-n\cdot q_2\cdot k_2\,;$$

and this is to be maximized by the choice of v. Note that, since $r=a_2$ is optimum, from definition (ii) and equation (12), it is clear that $k_2\geqslant b_2$ is a necessary and sufficient condition for full-employment.

The optimal v must satisfy the following property:

$$\text{Optimal } v \quad \begin{cases} =1 & \text{if} \quad q_1>q_2 \\ \in[0,1] & \text{if} \quad q_1=q_2 \\ =0 & \text{if} \quad q_1<q_2\,. \end{cases}$$

Also the co-state variables q_1 and q_2 satisfy the followings

$$\dot{q}_i=-(dH^*/dk_i) \qquad \text{for} \quad i=1,2\,. \tag{19}$$

First, we examine whether the policy of keeping v in the interior is ever optimal. This is so if $q_1=q_2$ for more than an instant, i.e., $\dot{q}_1=\dot{q}_2$. We find, from (19), that

$$\dot{q}_1=n\cdot q_1-A^*\cdot a_1\cdot k_1^{(a_1-1)}\cdot k_2^{a_2\cdot(1-a_1)}, \tag{20}$$

and

$$\dot{q}_2=n\cdot q_2-A^*\cdot a_2\cdot(1-a_1)\cdot k_1^{a_1}\cdot k_2^{(a_2\cdot(1-a_1)-1)}. \tag{21}$$

Now $q_1 = q_2$ and $\dot{q}_1 = \dot{q}_2$ implies that

$$(k_1/k_2) = (a_1/a_2 \cdot (1 - a_1)) \,. \tag{22}$$

If equation (22) is satisfied for more than an instant, obviously, we have $(\dot{k}_1/k_1) = (\dot{k}_2/k_2)$. So an interior investment allocation is optimal only along the balanced growth-path.

Again, $(\dot{k}_1/k_1) = (\dot{k}_2/k_2) \Rightarrow (k_1/k_2) = (v/(1-v))$ for all t. Now, from (22), we find that the optimal $v = (a_1/(a_1 + a_2 \cdot (1 - a_1))$.

Proposition 3. *If* $0 < v < 1$ *is optimal, Optimal* $v = (a_1/(a_1 + a_2 \cdot (1 - a_1))$ *along the balanced growth-path shown by* $(k_1/k_2) = (a_1/a_2 \cdot (1 - a_1))$.

Now we turn to examine the case of investment-specialization.[3] We define the following policies as follows:

POL (1) = [$v = 1$ for all $t \geqslant 0$.]

POL (2) = [$v = 0$ for all $t \geqslant 0$.]

POL (3) = [$v = (a_1/(a_1 + a_2 \cdot (1 - a_1)))$ for all $t \geqslant 0$.]

POL (4) = [$v = 1$ for all $t \in [0, t_0]$; $v = (a_1/(a_1 + a_2 \cdot (1 - a_1)))$ for all $t \geqslant t_0 > 0$]

POL (5) = [$v = 0$ for all $t \in [0, t_1]$; $v = (a_1/(a_1 + a_2 \cdot (1 - a_1)))$ for all $t \geqslant t_1 > 0$] .

If $v = 1$, $\dot{k}_2 = -n \cdot k_2 < 0$. So if $k_2(0) < b_2$, then with $v = 1$ for all $t \geqslant 0$, there does not exist any T, $0 < T < \infty$, such that $k_2(T) \geqslant b_2$. If there is urban unemployment in the beginning of the plan, the economy can not reach a full-employment state in any future date. Hence we have

Proposition 4. *If* $k_2(0) < b_2$, *the policy* POL (1) *is not optimal.*

If $v = 0$, then

$$(\dot{k}_2/k_2) = A^* \cdot k_1^{a_1} \cdot k_2^{(a_2 \cdot (1 - a_1) - 1)} - n \,.$$

Let,

$$a_2 \cdot (1 - a_1) - 1 = -m \cdot a_1 \,; \quad \text{with} \quad (1/a_1) > m > 1.$$

So

$$(\dot{k}_2/k_2) = A^* \cdot (k_1 k_2^m)^{a_1} - n \,; \quad \text{and} \quad (\dot{k}_2/k_2) \gtrless 0, \text{ for, } k_1 \lessgtr (n/A^*)^{(1/a_1)} \cdot k_2^m \,.$$

So, with $v = 0$, k_2 rises over time so long as

$$k_1 > (n/A^*)^{(1/a_1)} \cdot k_2^m \,;$$

reaches the maximum when

$$k_1 = (n/A^*)^{(1/a_1)} \cdot k_2^m \,;$$

[3] Mathematical details are given in the Appendix (I).

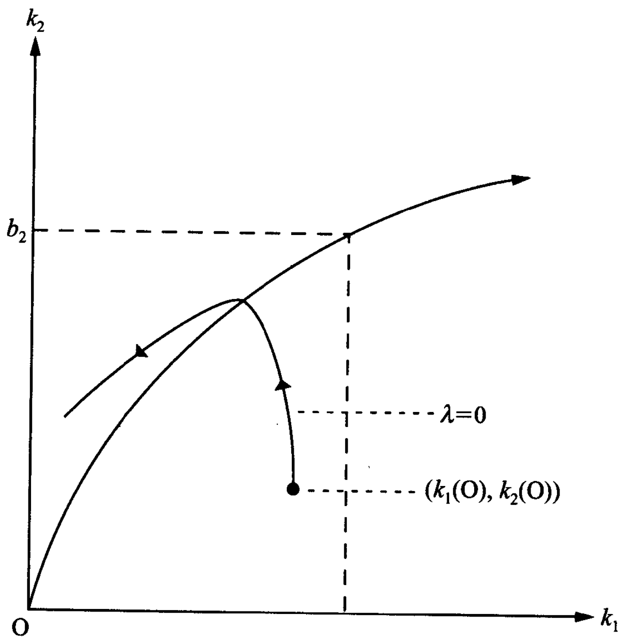

Fig. 1.

and falls when

$$k_1 < (n/A^*)^{(1/a_1)} \cdot k_2^m .$$

The behaviour of (k_1, k_2) over time, with $v=0$, depends on the initial combination $(k_1(0), k_2(0))$; and is shown by the trajectory in the Figure 1.

Since, with $v=0$, (k_1/k_2) falls over time; and $(k_1/k_2)\to 0$ as $t\to\infty$; one can easily prove the following:

THEOREM 1. *If* $k_1 > (n/A^*)^{(1/a_1)} \cdot k_2^m$ *at* $t=0$; *and if* $v=0$ *for all* $t\geqslant 0$; *then there exists a* $\bar{t}>0$, *such that* $k_1 = (n/A^*)^{(1/a_1)} \cdot k_2^m$ *at* $t=\bar{t}$.

Note that k_2 is maximum at $t=\bar{t}$; and hence $k_2(\bar{t})>k_2(t)$ for all $t\neq\bar{t}$. So if $k_2(\bar{t})<b_2$, then there does not exist any T, $0<T<\infty$, such that $k_2(T)>b_2$.

With

$$v=0,\ k_1(\bar{t})=k_1(0)\cdot e^{-n\cdot t} .$$

So $k_1(0)\cdot e^{-n\cdot\bar{t}}=(n/A^*)^{(1/a_1)}\cdot(k_2(\bar{t}))^m$ or, $(k_2(\bar{t}))^m=[k_1(0)\cdot e^{-n\cdot\bar{t}}]/[(n/A^*)^{(1/a_1)}]$. Now $k_2(\bar{t})<b_2 \geqq k_1(0)<(n/A^*)^{(1/a_1)}\cdot b_2^m\cdot e^{n\cdot\bar{t}}$.

So if $k_1(0)<(n/A^*)^{(1/a_1)}\cdot b_2^m\cdot e^{n\cdot\bar{t}}$ then with $v=0$, there does not exist any T, $0<T<\infty$, such that $k_2(T)\geqslant b_2$. Hence, we have

PROPOSITION 5. *If* $k_1(0)<(n/A^*)^{(1/a_1)}\cdot b_2^m\cdot e^{n\cdot\bar{t}}$, *and* $k_2(0)<b_2$, *then the policy* POL (2) *is not optimal.*

Note that it does not necessarily mean that the policy POL (2) is optimal if

$$k_1(0)>(n/A^*)^{(1/a_1)}\cdot b_2^m\cdot e^{n\cdot\bar{t}}.$$

So it is clear that if $k_2(0)<b_2$, and if

$$k_1(0)<(n/A^*)^{(1/a_1)}\cdot b_2^m\cdot e^{n\cdot\bar{t}},$$

then a policy of specialization of investment to either sector for ever can not be optimal as it fails to bring the full-employment state in any future date. The simultaneous allocation of investment to both the sectors is optimal only along a balanced growth path, given by the technological parameters of the production-functions of the two sectors. Can the interior investment-allocation policy along the balanced growth path lead the economy to full-employment in finite-time? More precisely, does any optimum solution to the problem exist at all?

Let us denote the mathematical expression $(a_1/(a_1+a_2\cdot(1-a_1)))$ by a new notation, z.
So,

$$(a_1/a_2\cdot(1-a_1))=(z/(1-z)).$$

Hence $v=z$ for all t along the balanced growth-path, given by $(k_1/k_2)=(z/(1-z))$.
Along the balanced growth path $(\dot{k}_1/k_1)=(\dot{k}_2/k_2)$.
Hence,

$$(\dot{k}_2/k_2)=(\dot{k}_1/k_1)$$

or,

$$(\dot{k}_2/k_2)=A^*\cdot(k_1/k_2)^{(a_1-1)}\cdot k_2^{(a_2\cdot(1-a_1)+a_1-1)}-n;$$

or,

$$(\dot{k}_2/k_2)=z\cdot A^*\cdot(z/(1-z))^{(a_1-1)}\cdot k_2^{(1-a_2)(a_1-1)}-n.$$

So $(\dot{k}_1/k_1)=(\dot{k}_2/k_2)\geqslant 0$, if

$$z\cdot A^*\cdot(z/(1-z))^{(a_1-1)}\cdot k_2^{-(1-a_2)\cdot(1-a_1)}\geqslant n,$$

or,

$$(z\cdot A^*/n)\cdot(z/(1-z))^{(a_1-1)}\geqslant k_2^{(1-a_2)\cdot(1-a_1)}.$$

Suppose, that, this is an equality at $k_2=\hat{k}_2$. As the economy moves along the path given by $(k_1/k_2)=(z/(1-z))$, then $(\hat{k}_1,\hat{k}_2)$ is the long-run equilibrium point of the system, where $\hat{k}_1=(z/(1-z))\cdot\hat{k}_2$. At this point, capital stock in both the sectors grow not only at the same rate, but also at a rate, equal to the natural rate of growth. With $v=z$ along $(k_1/k_2)=(z/(1-z))$, the economy can not go beyond the point $(\hat{k}_1,\hat{k}_2)$. Full-employment state can be reached only if $\hat{k}_2\geqslant b_2$. If $\hat{k}_2<b_2$, the economy enters into a low level equilibrium trap characterized by urban unemployment. So if $k_2(0)<b_2$, and $k_1(0)<(n/A^*)^{(1-a_1)}\cdot b_2^m\cdot e^{n\cdot\bar{t}}$, then the optimum solution to the time-minimization problem of attaining the full-employment

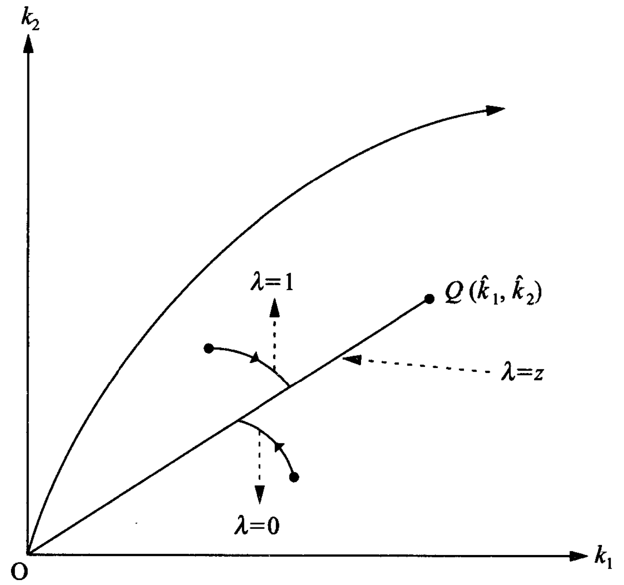

Fig. 2.

state exists only if $\hat{k}_2 \geqslant b_2$.
One can state

PROPOSITION 6. *If* $k_2(0) < b_2$; $k_1(0) < (n/A^*)^{(1/a_1)} \cdot b_2^m \cdot e^{n \cdot \bar{t}}$; and $\hat{k}_2 \geqslant b_2$; then the policy POL (3) is optimal when $(k_1(0)/k_2(0)) = (z/(1-z))$.

What happens if $(k_1(0)/k_2(0)) \neq (z/(1-z))$?

Using the Lemmas (5) to (8), we can prove the following proposition.[4]

PROPOSITION 7. *If* $k_2(0) < b_2$; $k_1(0) < (n/A^*)^{(1/a_1)} \cdot b_2^m \cdot e^{n \cdot \bar{t}}$; and $\hat{k}_2 \geqslant b_2$, then
(1) the policy POL (4) is optimal if $(k_1(0)/k_2(0)) < (z/(1-z))$
(2) the policy POL (5) is optimal if $(k_1(0)/k_2(0)) > (z/(1-z))$.

The different properties of optimum investment allocation policy mentioned in the different lemmas and propositions are now summarized. The interior investment allocation is optimal only along the balanced growth path, shown by the straight line $0Q$ in Figure 2. Q $(\hat{k}_1, \hat{k}_2)$ is the long-run equilibrium point to which the optimal growth-path converges. Off this path, only policies of specialization of investment are optimal for more than an instant. If the initial point $(k_1(0), k_2(0))$ lies above the $0Q$ straight line, the initial policy is one of specialization to industrial development till the balanced growth-path is reached. The policy then follows the optimal balanced growth turnpike. If the initial point is below the line, the optimum policy is one of initial rural development followed by a balanced development of both the sectors.

[4] Lemmas (5) to (8) are presented in the Appendix (2).

3. FINAL REMARKS

Obviously the model is abstract and fails to consider, among many other things, the role of different types of rural and urban institutions (like trade-unions, share tenancy, moneylenders, etc.) on the rural-urban migration. Hence one can of course, challenge the validity of the conclusions of such a restrictive model. But the static analysis of Harris and Todaro (1970) also shares the same limitations. On the other hand a dynamic analysis is better than a static one when the question raised makes reference to development policy and such dynamic extension is additionally interesting if it offers conclusions that would not be anticipated in the static counterpart.

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APPENDIX (1)

If $v=1$, then the equations of motion are $\dot{k}_1=A^*\cdot k_2^{a_1\cdot(1-a_1)}-n\cdot k_1$; and, $\dot{k}_2=-n\cdot k_2$.

Obviously $\dot{k}_2<0$ and hence k_2 falls as t rises. Also $k_2\to 0$ as $t\to\infty$.

$$(\dot{k}_1/k_1)-(\dot{k}_2/k_2)=A^*\cdot k_1^{(a_1-1)}\cdot k_2^{a_2\cdot(1-a_1)}>0\ .$$

So (k_1/k_2) rises as t rises; and $(k_1/k_2)\to\infty$ as $t\to\infty$. Hence we have, the followings:

LEMMA 1. *If $(k_1(0)/k_2(0))<(a_1/a_2\cdot(1-a_1))$, and if $v=1$ for all $t\geqslant 0$, there exists a $t_0>0$ such that $(k_1(t_0)/k_2(t_0))=(a_1/a_2\cdot(1-a_1))$.*

LEMMA 2. *If $(k_1(0)/k_2(0))>(a_1/a_2\cdot(1-a_1))$, and if $v=1$ for all $t\geqslant 0$, there does not exist any $t_0>0$ such that $(k_1(t_0)/k_2(t_0))=(a_1/a_2\cdot(1-a_1))$.*

If $v=0$, then

$$\dot{k}_1=-n\cdot k_1\ ,$$

and,

$$\dot{k}_2=A^*\cdot k_1^{a_1}\cdot k_2^{a_2\cdot(1-a_1)}-n\cdot k_2\ .$$

Obviously, $\dot{k}_1<0$ and hence k_1 falls as t rises. Also $k_1\to 0$ as $t\to\infty$.

$$(\dot{k}_1/k_1)-(\dot{k}_2/k_2)=-A^*\cdot k_1^{a_1}\cdot k_2^{(a_2\cdot(1-a_1)-1)}<0\ .$$

So (k_1/k_2) falls as t rises; and $(k_1/k_2)\to 0$ as $t\to\infty$. So we have the followings:

LEMMA 3. *If $(k_1(0)/k_1(0))>(a_1/a_2\cdot(1-a_1))$, and if $v=0$ for all $t\geqslant 0$, there exists a $t_1>0$ such that $(k_1(t_1)/k_2(t_1))=(a_1/a_2\cdot(1-a_1))$.*

LEMMA 4. *If $(k_1(0)/k_2(0))<(a_1/a_2\cdot(1-a_1))$, and if $v=0$ for all $t\geqslant 0$, then there does not exist any $t_1>0$ such that $(k_1(t_1)/k_2(t_1))=(a_1/a_2\cdot(1-a_1))$.*

APPENDIX (2)

We know that $\dot{q}_1\gtreqless\dot{q}_2$ when $(k_1/k_2)\gtreqless(z/(1-z))$.

Now using the lemmas (1)–(4), we can easily prove the followings:

LEMMA 5. *If* $v=1$ *is optimal for* $t=0$, *then* $v=1$ *is optimal for all* $t \in [0, t_0]$ *with* $(k_1/k_2) < (z/(1-z))$ *for all* $t \in [0, t_0]$.

LEMMA 6. *If* $v=0$ *is optimal for* $t=0$, *then* $v=0$ *is optimal for all* $t \in [0, t_1]$ *with* $(k_1/k_2) > (z/(1-z))$ *for all* $t \in [0, t_1]$.

LEMMA 7. *If* $v=1$ *is optimal for all* $t \in [0, t_0]$ *where* $(k_1(t_0)/k_2(t_0)) = (z/(1-z))$ *then* $v=1$ *is not optimal for any* $t > t_0$.

LEMMA 8. *If* $v=0$ *is optimal for all* $t \in [0, t_1]$ *where* $(k_1(t_1)/k_2(t_1)) = (z/(1-z))$ *then* $v=0$ *is not optimal for any* $t > t_1$.