Title	MONOPOLY AND GENERAL EQUILIBRIUM: AN EXTENSION
Sub Title	
Author	D'AGATA, Antonio
Publisher	Keio Economic Society, Keio University
Publication year	1994
Jtitle	Keio economic studies Vol.31, No.2 (1994.) ,p.57- 62
JaLC DOI	
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Notes	Note
Genre	Journal Article
URL	https://koara.lib.keio.ac.jp/xoonips/modules/xoonips/detail.php?koara_id=AA00260492-19940002-0057

MONOPOLY AND GENERAL EQUILIBRIUM: AN EXTENSION

Antonio D'AGATA*

Abstract: The paper shows the existence of a monopolistic equilibrium in a general equilibrium model under conditions which are weaker than the usual ones adopted by the literature. More specifically, no convexity assumption of preferences of households and of production sets of firms is adopted; moreover, the assumption of possibility of inaction for the monopolist is also dropped.

JEL Classification Codes: D42, D51.

1. INTRODUCTION

In an article published on this journal, Itoh [5] provides, among other things, an existence theorem for a general equilibrium model with a monopolistic producer. To the best of our knowledge, before Itoh's work, Katz [6] and Cornwall [3] dealt with general equilibrium models with monopoly. All these three articles have the common feature of assuming that the origin must always be an element of the production set (if there exists one in the economy), and convexity of consumers' preferences and of the production sets of perfectly-competitive producers.

The aim of this paper is to provide a generalization of Itoh's existence result by trying to extend his analysis to the case in which the hypotesis of possibility of inaction is dropped for the monopolist, and the convexity condition of households' preferences and of firms' production sets is not necessarily satisfied.

2. THE MODEL

Consider a private ownership economy with l goods, m consumers and n producers: $\mathscr{E}(G, H, F, (X_h)_{h\in H}, (\geq_h)_{h\in H}, (\omega_h)_{h\in H}, (\theta_h)_{h\in H}, (Y_f)_{f\in F})$, where G (resp. H, resp. F) denote the index set of good (resp. households, resp. firms); for the remaining, quite standard, symbols the reader is referred to Itoh [5]. We assume that $F=\{0\}\cup F_c$, where the index 0 denotes the monopolist firm and set F_c the index set of perfectly competitive firms. Symbol p denotes the l-dimensional price

* I would like to thank an anonymous referee for useful comments. Financial support by MURST and CNR is gratefully acknowledged. Usual caveats apply.

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vector, p_i indicates the price of good i, π denotes the profit of firm 0. Finally, Δ is the l-1-dimensional open unit simplex.

The meaning of the following sets is obvious:

$$y_f(p)=\{y_f\in Y_f \mid py_f\geq py_f^{\dagger},\ y_f^{\dagger}\in Y_f\}\,,\qquad f\in F_c$$

$$B_h(p,\pi)=\left\{x_h\in X_h \mid px_h\leq p\omega_h+\sum_{f\in F_c}\theta_{hf}py_f+\theta_{h0}\pi,\ y_f\in y_f(p)\right\}$$

$$x_h(p,\pi)=\{x_h\in B_h(p,\pi)\mid x_h\geq_h x_h^{\dagger},\ x_h^{\dagger}\in B_h(p,\pi)\}$$

$$\xi(p,\pi)=\left\{z\in R^l \mid z=\sum_{h\in H}x_h-\sum_{f\in F_c}y_f-\sum_{h\in H}\omega_h,\ x_h\in x_h(p,\pi),\ y_f\in y_f(p)\right\}$$

$$H=\{(p,\pi)\in\Delta\times R_+ \mid \xi(p,\pi)\cap Y_0\neq\varnothing\}$$

$$Y_0(p,\pi)=\{y_0\in Y_0 \mid y_0\in\xi(p,\pi)\cap Y_0\neq\varnothing\}\,,\qquad (p,\pi)\in H\,.$$

The following definition is substantially that one adopted by Itoh [5].

DEFINITION. A *monopolistic equilibrium* for the economy $\mathscr{E}$ is a configuration $((x_h^*)_{h\in H},(y_f^*)_{f\in F},p^*,\pi^*)\in\prod_{h\in H}X_h\times\prod_{f\in F}Y_f\times\Delta\times R_+$ such that:

(i) $\forall h\in H$: $x_h^*\in x_h(p^*,\pi^*)$;

(ii) $\forall f\in F_c$: $y_f^*\in y_f(p^*)$

(iii) $p^*y_0^*\geq py_0$, $\forall p\in\Delta$, $y_0\in Y_0$, $y_0=\sum_{h\in H}x_h-\sum_{f\in F_c}y_f-\sum_{h\in H}\omega_h$, where $x_h\in x_h(p,\pi)$, $y_f\in y_f(p)$, and $\pi\geq 0$;

(iv) $p^*y_0^*=\pi^*$.

A.1. $\forall h\in H$:

(i) X_h is closed, convex and lower bounded; moreover, $\exists x_h^\circ\in X_h$: $x_h^\circ<\omega_h$;

(ii) preferences $\geq_h$ are complete, reflexive, monotonic, closed, and transitive;

(iii) $\theta_h\in R_+^{\#F}$, $\sum_{h\in H}\theta_{fh}=1$, $f\in F$.

A.2. One has:

(i) $\forall f\in F_c$: $0\in Y_f$,

(ii) $\forall f\in F$: Y_f is closed and compact.

A.3. $H\neq\varnothing$.

Assumptions A.1. and A.2. are standard, except for the fact that, unlike Cornwall [3], Itoh [5] and Kats [6], we do not assume convexity of preferences and of production sets. Notice also that the origin is not necessarily an element of set Y_0. Compactness of sets Y_f is adopted for the sake of simplicity; it can be easily replaced by weaker, more standard conditions. Assumption A.3. means that there exists a normalized price vector p and a non-negative profit level π such that the associated excess demand set has at least one element which is technologically feasible for the monopolist.

The following remark refers to the (more restrictive) case considered by Itoh:

REMARK 1. Under A.1. and A.2., if $0 \in Y_0$ and if preferences and production sets are convex, then A.3. holds true.

PROOF. Consider the perfectly-competitive economy associated with economy $\mathscr{E}$, i.e., consider economy $\mathscr{E}$ with $\pi = 0$ and $y_0 = 0$. Denote by P^w the set of Walrasian equilibria of this economy. By the assumptions adopted, $P^w \neq \varnothing$; therefore: $\xi(p^w, 0) = 0$, where $p^w \in P^w$. However, $0 \in Y_0$, thus $(p^w, 0) \in H$.

PROPOSITION. *Under* A.1.–A.3. *the economy* $\mathscr{E}$ *has a monopolistic equilibrium.*

LEMMA. *The following assertions hold true*:

(i) $(p, \pi) \in H \Rightarrow [\forall y_0 \in Y_0(p, \pi) : \pi = py_0]$.

(ii) *Set* H *is compact.*

PROOF OF LEMMA (i) Suppose that $(p, \pi) \in H$; then, by definition: $Y_0(p, \pi) \neq \varnothing$. Take any $y_0 \in Y_0(p, \pi)$; thus $y_0 \in \xi(p, \pi)$; i.e.: $y_0 = \sum_{h \in H} x_h^{\wedge} - \sum_{f \in F_c} y_f^{\wedge} - \sum_{h \in H} \omega_h$, for some $x_h^{\wedge} \in x_h(p, \pi)$, $y_f^{\wedge} \in y_f(p)$. Therefore, $py_0 = \sum_{h \in H} px_h^{\wedge} - \sum_{f \in F_c} py_f^{\wedge} - \sum_{h \in H} p\omega_h$. A.1(ii) implies that for any $h \in H$ and for any $x_h^{\wedge} \in x_h(p, \pi)$: $px_h^{\wedge} = p\omega_h + \sum_{f \in F_c} \theta_{hf} py_f^{\wedge} + \theta_{h0}\pi$. Therefore: $py_0 = \sum_{h \in H} p\omega_h + \sum_{h \in H} \sum_{f \in F_c} \theta_{hf} py_f^{\wedge} + \sum_{h \in H} \theta_{h0}\pi - \sum_{f \in F_c} \theta_{hf} py_f^{\wedge} - \sum_{h \in H} p\omega_h$. Because of A.1(iii), the last relation implies: $py_0 = \pi$.

(ii) Note first that under A.1. and A.2. the excess demand correspondence $\xi : \Delta \times R_+ \to R^l$ is upper hemi-continuous with compact values (see, for example, Berge [1, p. 116], Hildebrand and Kirman [4, p. 93]). Upper hemi-continuity and closed-valuedness of $\xi(\ )$ imply closedness of set H (see, for example Border [1, Exercise 11.18(b)]). Suppose now that H is not bounded; therefore, there exists a sequence in H, $(p^n, \pi^n) \to (p^\circ, \infty)$, where $(p^\circ, \infty) \in H$. Thus, by point (i) above there exists a sequence (y_0^n) such that $y_0^n \in \xi(p^n, \pi^n)$ and $\pi^n = p^n y_0^n$. Since $\pi^n \to \infty$ and vectors p^n are bounded one must conclude that at least one element of the vectors in the sequence (y_0^n) tends to infinity, contradicting the boundedness of set Y_0.

PROOF OF PROPOSITION. Set $\pi^* = \sup\{\mathrm{Proj}_{R_+} H\}$. Lemma (ii) ensures that π^* is well-defined and, moreover, $\pi^* \in \mathrm{Proj}_{R_+} H$. Choose $p^* \in \Delta$ in such a way that $(p^*, \pi^*) \in H$. Choose y_0^*, $x_h^*(h \in H)$, and y_f^* $(f \in F_c)$ in such a way that $y_0^* \in Y_0(p^*, \pi^*)$, $x_h^* \in x_h(p^*, \pi^*)$ $(h \in H)$, and $y_f^* \in y_f(p^*\ )$ $(f \in F_c)$. Taking into account that, from Lemma (i), $\pi^* = p^* y_0^*$, it is immediate to see that $((x_h^*)_{h \in H}, (y_f^*)_{f \in F}, p^*, \pi^*)$ is a monopolistic equilibrium.

3. AN EXAMPLE

In this section we shall provide a numerical example showing the logic of the result and the consistency of the model analysed in the previous section. Consider a private ownership economy with two households, one firm and two goods. The firm is the monopolistic agent of the economy and is indicated by 0. We assume that: $\theta_{10}=1$ and $\theta_{20}=0$, $X_h=R^2_+$, $\omega_h=(3, 1)$ $(h=1, 2)$ and households have the following utility function:

$$U_h(x_h)=\begin{cases}\min[x_{h1}, 0.5x_{h2}], & \text{if } x_{h1}<x_{h2}\\ (x_{1h}+x_{h2})/2 & \text{if } x_{h1}=x_{h2} \quad h=1, 2\\ \min[0.5x_{h1}, x_{h2}] & \text{if } x_{h1}>x_{h2}\end{cases}$$

Suppose that good 1 is an input and good 2 is an output; the production set of firm 0 is represented by the graph of the following correspondence:

$$y_{02}=\begin{cases}(-\infty, 0] & \text{if } -4<y_{01}\leq 0\\ (-\infty, 8] & \text{if } y_{01}=-4\\ (-\infty, 4-y_{01}] & \text{if } y_{01}<-4\end{cases}$$

Figure 1 illustrates the indifference curve of household h associated with a utility level equal to 1. Notice that the supply curve of household h must be a subset of the two rays R and R'. Its specific shape is determined by the monopolist's profit π. For example, if $\pi=0$, the supply curve of household h is represented by curve $(a, b]$ and $[c, d)$.

Normalize the price vector on the 1-dimensional unit simplex: $\Delta=(0, 1)$. The aggregate excess demand for goods 1 and 2 are the following: if $0<p_1\leq 0.5$, then $\xi_1(p, \pi)=(8p_1+4+2\pi)/(p_1+1)$ and $\xi_2(p, \pi)=(4p_1+2+\pi)/(p_1+1)$; if $0.5\leq$

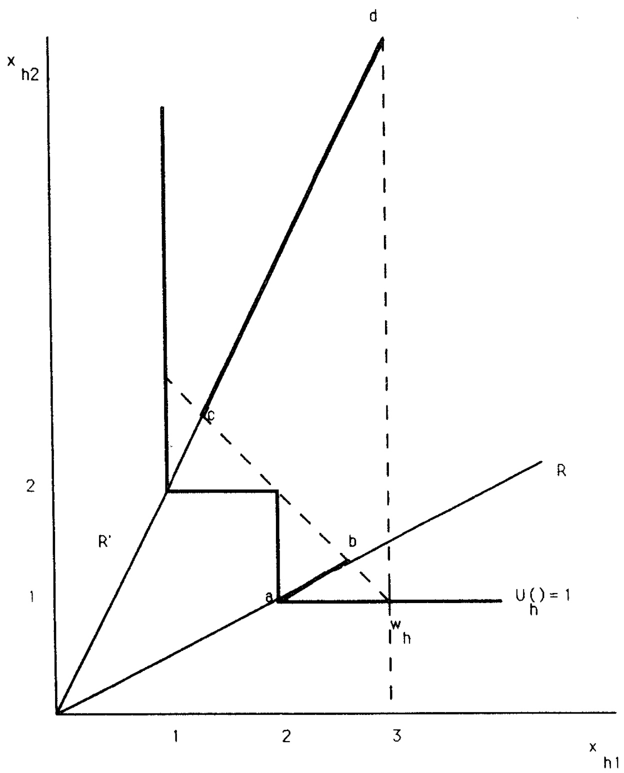

Figure 1.

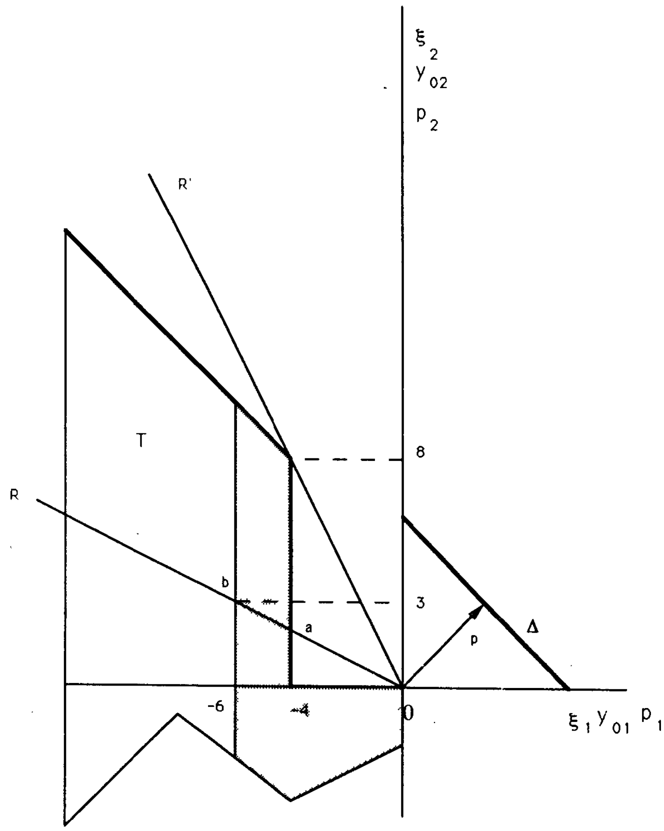

Figure 2

$p_2 < 1$, then $\xi_1(p, \pi) = (4p_1 + 2 + \pi)/(2 - p_1)$ and $\xi_2(p, \pi) = (8p_1 + 4 + 2\pi)/(2 - p_1)$.

In the first quadrant of Figure 2 the unit simplex is illustrated. The second quadrant of Figure 2 illustrates the production set of firm 0 (indicated by T). The shaded area is the feasible part of the production set. In this quadrant rays R and R' of Figure 1 are also represented. As pointed out before, the aggregate supply curve of households is always a subset of the set of points made up by rays R and R'.

It is immediate to see that neither preferences, nor the production set satisfy the traditional convexity assumption. Moreover, from Figure 1 it is easy to see that the economy with inactive producer has no Walrasian equilibrium.

REMARK 2. For this economy set H is non-empty. More specifically, $(p^\circ, \pi^\circ) \in H$, where $p^\circ = (1/2, 1/2)$ and $\pi^\circ = 2$. Moreover (employing notation of section 2) configuration $A = (((8/3, 16/3), (4/3, 8/3)), (4, 8), (1/2, 1/2), 2)$ is a monopolistic equilibrium.

PROOF. (Sketch) Notice that only production plan (4, 8) and the production plans in the set described by the segment $[a, b]$ in Figure 2 can potentially clear markets. In order to show that configuration A above is a monopolistic equilibrium, one has to show that (i) $(p^\circ, \pi^\circ) \in H$, and that $(4, 8) \in Y_0(p^\circ, \pi^\circ)$; (iia) there is no couple $(p, \pi) \in \Delta \times R_+$, $(p, \pi) \neq (p^\circ, \pi^\circ)$ such that $(4, 8) \in Y_0(p, \pi)$ and, at the same time, this couple yields profits at (4, 8) higher than the profits the same production vector yields at (p°, π°); moreover, (iib) there is no couple $(p, \pi) \in \Delta \times R_+$ which makes production plans in segment $[a, b]$ belonging to set $Y_0(p, \pi)$ and, at the same time, more profitable than point (4, 8) at (p°, π°). Proving facts (i), (iia) and (iib) is quite easy; therefore, for the sake of brevity, we leave it to the reader.

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