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AD VALOREM AND SPECIFIC TAXES, AND OPTIMAL PIGOUVIAN TAX WITHIN COURNOT OLIGOPOLY*

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Abstract: First, we analyze the effects of a shift from specific to ad valorem taxation within Cournot oligopoly under general conditions on demand and cost functions. Second, we analyze the optimal Pigouvian tax rate for controlling emission of pollution within Cournot oligopoly also under general conditions. The second analysis is an adaptation of the analytical method adopted in the first analysis.

1. INTRODUCTION

Assuming *quadratic* cost functions, Dierickx, Matutes and Nevin (1988) have analyzed the effects of indirect taxation within Cournot oligopoly without product differentiation. Earlier Levin (1982, 1985), Katz and Rosen (1985), and later Besley (1989) and Okuguchi (1993a, 1993b) have taken up the same problem for specific tax. Their results show, among other things, that the Cournot equilibrium industry output and price decreases and increases, respectively, in the event of an increase in the rate of indirect tax, be it ad valorem or specific. Furthermore, Dierickx *et al.* have shown that outputs as well as profits may increase for some firms as a result of an increase in the rate of ad valorem tax if the demand function is concave and if, in addition, the firms' marginal costs are constant and identical. Delipalla and Keen (1992) have compared the effects of ad valorem and specific taxes within oligopoly model with conjectural variations and without product differentiation. They have assumed firms having identical cost functions and considered a tax reform called P-Shift from specific to ad valorem taxation, which is a local version of *matched pairs* of ad valorem and specific taxes in the sense of Suits and Musgrave (1955). According to Delipalla and Keen, the consumer price and aggregate profits decrease but the welfare increases in the events of a P-shift. Recently, Tanaka (1993) has analyzed the welfare effects of ad valorem tax within free entry Cournot oligopoly with identical firms and without product differentiation and shown that the tax burden brought about by ad valorem tax is smaller than the increase in the tax revenue. He has also shown under certain conditions that the

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tax burden due to valorem tax is smaller than that due to specific one *if firm' cost functions are identical and linear, and if, in addition, both taxes raise equal tax revenues.*

The main purpose of this paper is to analyze the effects of a shift from specific to ad valorem taxation within Cournot oligopoly without product differentiation. Our approach is quite general in that we assume neither identical cost functions nor linear ones. However, we assume, as in Tanaka, that the tax revenue is not affected by a shift from specific to ad valorem taxation. Our analysis is rendered possible by considering the basic relationships existing among individual firms' outputs on the one hand and the industry output, specific and ad valorem tax rates on the other. Levin (1982, 1985) has analyzed the effects of a Pigouvian tax policy within Cournot oligopoly model with externalities induced by firms' productive activities. His model can be interpreted as one where ad valorem tax is absent and individual firms are imposed specific taxes at non-uniform rates. Ebert (1992) has derived the optimal Pigouvian tax rate for externalities within Cournot oligopoly with identical firms which emit one unit of pollution per unit of output. Hence firms in his model can be interpreted to be imposed specific tax at uniform rate. However, we will be able to derive the optimal Pigouvian tax rate for more general conditions, adapting our analytical method in Section 2.

The organization of this paper is as follows. In Section 2 we will formulate our model and analyze the effects of a shift from specific to ad valorem taxation under most general conditions on the demand and cost functions. In Section 3, we will derive the optimal Pigouvian tax rate also under most general conditions. Section 4 concludes our paper.

2. THE MODEL AND ANALYSIS

Let there be n firms in Cournot oligopoly. The inverse demand function is $p=f(Q), f'<0$, where if x_i is firm i's output, $Q\equiv\sum_j x_j$ is the industry output and p is the price of the product. Let $C_i=C_i(x_i)$ be firm i's cost function. Then firm i's profit π_i is given by

$$(1)\qquad \pi_i\equiv x_i\{(1-t_v)f(Q)-t_s\}-C_i(x_i)\,,\qquad i=1,2,\cdots,n\,,$$

where t_v is the parameter for ad valorem tax, and t_s is the specific tax rate. Note that the ad valorem tax rate τ and t_v are related by $t_v=\tau/(1+\tau)$, where τ increases if and only if t_v increases. Given t_v and t_s, assuming an interior maximum, the first and second order conditions for firm i's profit maximization are given by (2) and (3), below, respectively.

$$(2)\qquad \partial\pi_i/\partial x_i=(1-t_v)f(Q)-t_s+x_i(1-t_v)f'(Q)-C_i'(x_i)=0\,,\qquad i=1,2,\cdots,n\,.$$

$$(3)\qquad \partial^2\pi_i/\partial x_i^2=(1-t_v)\{f'(Q)+x_if''(Q)\}+(1-t_v)f'(Q)-C_i''(x_i)<0\,,\qquad i=1,2,\cdots,n\,.$$

We now introduce the following two assumptions.

(A.1) $$f' + x_i f'' < 0 , \qquad i = 1, 2, \cdots, n .$$

(A.2) $$a_i \equiv (1 - t_v) f' - C_i'' < 0 , \qquad i = 1, 2, \cdots, n .$$

The assumption (A.1) has been widely used in analyzing the existence and stability of the Cournot oligopoly equilibrium in the absence of indirect taxes. The inequality $f' < C_i''$ which obtains when $t_v = 0$ in (A.2) has also been widely used in the same analysis. See Okuguchi (1976), and Okuguchi and Szidarovszky (1990). If (A.1) and (A.2) hold, the second order condition (3) is satisfied.

Expressing x_i in (2) as a function of O, t_v and t_s, we have

(4) $$x_i = \varphi^i(Q, t_v, t_s) , \qquad i = 1, 2, \cdots, n ,$$

where the partial derivatives have the following signs in the light of (A.1), (A.2) and (2).

(5) $$\begin{cases} \varphi_Q^i \equiv \partial x_i / \partial Q = -(1 - t_v)(f' + x_i f'')/a_i < 0 , \\ \varphi_{t_v}^i \equiv \partial x_i / \partial t_v = (f + x_i f')/a_i < 0 ,^1 \\ \varphi_{t_s}^i \equiv \partial x_i / \partial t_s = 1/a_i < 0 . \end{cases} \qquad i = 1, 2, \cdots, n .$$

Given t_v and t_s, the Cournot equilibrium industry output is characterized as a solution of

(6) $$Q = \sum_j \varphi^j(Q, t_v, t_s) \equiv \varphi(Q, t_v, t_s) ,$$

where (5) yields

(7) $$\varphi_Q < 0 , \quad \varphi_{t_v} < 0 , \quad \varphi_{t_s} < 0 .$$

The left-most expression in (6) is depicted as a 45 degree line and the right-most one as a downwards-sloping curve. Hence, if we assume $\varphi(0, t_v, t_s) > 0$, we can assert the existence of a unique solution of (6). Let the solution be

(8) $$Q^* \equiv Q^*(t_v, t_s) ,$$

where

(9) $$\begin{cases} \partial Q^* / \partial t_v = \varphi_{t_v} / (1 - \varphi_Q) < 0 , \\ \partial Q^* / \partial t_s = \varphi_{t_s} / (1 - \varphi_Q) < 0 . \end{cases}$$

Hence, the unique Cournot equilibrium industry output decreases and the corresponding equilibrium price increases in the event of an increase in both ad valorem and specific tax rates.

Taking into account the second and last expressions in (5), $\partial Q^* / \partial t_v$ and $\partial Q^* / \partial t_s$ are shown to be related in the following fashion.

[1] Rewrite (2) as $(1 - t_v)(f + x_i f') = t_s + C_i'$ and take into account $0 \le t_v < 1$, $t_s \ge 0$, $C_i' > 0$ to assert that $f + x_i f' > 0$ at the equilibrium.

(10) $$\partial Q^*/\partial t_v = f\partial Q^*/\partial t_s + f' \sum x_i \varphi^i_{t_s}/(1-\varphi_Q).$$

The equilibrium tax revenue T is given by

(11) $$T \equiv t_s Q^* + t_v f(Q^*)Q^*,$$

where

(12) $$\begin{cases} T_v \equiv \partial T/\partial t_v = f(Q^*)Q^* + \{t_s + t_v(f + Q^* f')\}\partial Q^*/\partial t_v, \\ T_s \equiv \partial T/\partial t_s = Q^* + \{t_s + t_v(f + Q^* f')\}\partial Q^*/\partial t_s. \end{cases}$$

Hence,

(13) $$\begin{cases} T_s|_{t_v = t_s = 0} = Q^*(0, 0) > 0, \\ T_v|_{t_v = t_s = 0} = f(Q^*)Q^* > 0. \end{cases}$$

Hence, $T_v > 0$ and $T_s > 0$ for (t_v, t_s) in the neighborhood of (0, 0). However, the signs of (12) are, in general, indeterminate. In order to avoid this indeterminacy, we assume as in Tanaka (1993) that

(A.3) $$T_v > 0, \quad T_s > 0.$$

Therefore, if T is held constant,

(14) $$dt_s/dt_v|_{dT=0} = -T_v/T_s < 0.$$

A small amount of calculation which takes into account (10) and (12) yields

(15) $$\begin{aligned} dQ^*/dt_v|_{dT=0} &= Q^*_v + Q^*_s dt_s/dt_v|_{dT=0} \\ &= \{f'(Q^*)Q^* \sum x_i \varphi^i_{t_s}/(1-\varphi_Q)\}/T_s > 0. \end{aligned}$$

Hence, the equilibrium industry output and price increases and decreases, respectively, in the event of a shift from specific to ad valorem taxation if the tax revenue is not affected by it.[2]

The social welfare at the equilibrium is defined by

(16) $$\begin{aligned} W &= \int_0^{Q^*} f(q)dq - f(Q^*)Q^* + \sum [x^*_i\{(1-t_v)f(Q^*) - t_s\} - C_i(x^*_i)] + T \\ &= \int_0^{Q^*} f(q)dq - \sum C_i(x^*_i), \end{aligned}$$

where x^*_i is firm i's equilibrium output. Hence,

(17) $$\begin{aligned} dW/dt_v|_{dT=0} &= dW/dQ^* \cdot dQ^*/dt_v|_{dT=0} \\ &= \{f(Q^*) - \sum C'_i(x^*_i)dx^*_i/dQ^*\}dQ^*/dt_v|_{dT=0}. \end{aligned}$$

The sign of dx^*_i/dQ^* is, in general, indeterminate. However, in the symmetric case

[2] However, if the inequalities in (A.3) are reversed, the equilibrium industry output decreases, as a consequence of which the equilibrium price increases in the event of the same shift.

where all firms' cost functions are identical, $dx_i^*/dQ^*=1/n>0$. Hence, taking into account $f(Q^*)-C_i'(x_i^*)>0$, which is a consequence of the first order condition (2), we have $dW/dt_v|_{dT=0}>0$ under (A.3). Even if firms' cost functions are not identical, the same conclusion holds provided $dx_i^*/dQ^*>0$ for all i, or more generally, provided that the expression between the brace in (17) is positive.

3. OPTIMAL PIGOUVIAN TAX

In this Section we will derive a formula for the optimal Pigouvian tax for controlling externalities within Cournot oligopoly. Let γ_i be emission of pollution per unit of firm i's output, and let t be the Pigouvian tax per unit of emission of pollution. Firm i's profit is

$$\pi_i \equiv px_i - C_i(x_i) - t\gamma_i x_i\,, \qquad i=1,2,\cdots,n\,. \tag{18}$$

The fixed order condition for firm i's profit maximization is

$$f(Q)+x_i f'(Q)-C_i'(x_i)-t\gamma_i=0\,, \qquad i=1,2,\cdots,n\,. \tag{19}$$

We assume that (A.1) in Section 2 and

$$f'<C_i''\,, \qquad i=1,2,\cdots,n \tag{A.4}$$

hold. Under (A.1) and (A.4), the second order condition is satisfied. Solving (19) with respect to x_i,

$$x_i \equiv \psi^i(Q,t,\gamma_i)\,, \qquad i=1,2,\cdots,n\,, \tag{20}$$

where

$$\left\{\begin{array}{l} \psi_Q^i \equiv \partial\psi^i/\partial Q = -(f'+x_i f'')/(f'-C_i'')<0\,, \\ \psi_t^i \equiv \partial\psi^i/\partial t = \gamma_i/(f'-C_i'')<0\,, \\ \psi_i^i \equiv \partial\psi^i/\partial\gamma_i = t/(f'-C_i'')<0\,, \end{array}\right. \qquad i=1,2,\cdots,n\,. \tag{21}$$

The Cournot equilibrium industry output $Q^*\equiv Q^*(t,\gamma_1,\cdots,\gamma_n)$ is a unique solution of

$$Q \equiv \sum_i \psi^i(Q,t,\gamma_i) \equiv \psi(Q,t,\gamma_1,\cdots,\gamma_n)\,. \tag{22}$$

The partial derivatives of Q^* are signed as follows.

$$\left\{\begin{array}{l} \partial Q^*/dt = \sum \psi_t^i/(1-\psi_Q)<0\,, \\ \partial Q^*/\partial\gamma_i = \psi_i^i/(1-\psi_Q)<0\,, \end{array}\right. \qquad i=1,2,\cdots,n\,. \tag{23}$$

The partial derivatives of the firms' outputs with respect to changes in t, γ_i and γ_k $(k\neq i)$ are as follows.

(24.1)
$$\partial x_i/\partial t=\psi^i_Q\partial Q/\partial t+\psi^i_t=-(f'+x_if'')/(f'-C''_i)$$
$$\times\left\{\sum_j\gamma_j/(f'-C''_j)/\left(1-\sum_j(f'+x_jf'')/(f'-C''_j)\right)\right\}$$
$$+\gamma_i/(f'-C''_i)\,,\qquad i=1,2,\cdots,n\,.$$

(24.2)
$$\partial x_i/\partial\gamma_i=\psi^i_Q\partial Q/\partial\gamma_i+\psi^i_i$$
$$=\psi^i_i\left(1-\sum_{j\neq i}\psi^j_Q\right)/(1-\psi_Q)<0\,,\qquad i=1,2,\cdots,n\,.$$

(24.3)
$$\partial x_i/\partial\gamma_k=\psi^i_Q\partial Q/\partial\gamma_k>0\,,\qquad i\neq k\,,\quad i,k=1,2,\cdots,n\,.$$

If the inverse demand function is linear and the firms have identical, constant marginal cost, we have

(24.1′)
$$\partial x_i/\partial t=\{\gamma_i+\sum\gamma_j/(n-1)\}/f'<0\,,\qquad i=1,2,\cdots,n\,.$$

Otherwise, the sign of $\partial x_i/\partial t$ is, in general, indeterminate. However, in the case of symmetric firms having identical cost functions, we have $\partial x_i/\partial t<0$.

Before proceeding further, let $D(\sum\gamma_ix_i)$ with $D'<0$ be the damage function. Then the social welfare is defined by

(25)
$$W(x(t))=\left\{\int_0^Q f(q)dq-f(Q)Q\right\}+\sum\{f(Q)x_i-C_i(x_i)-t\gamma_ix_i\}$$
$$-D(\sum\gamma_ix_i)+t\sum\gamma_ix_i\,,$$

where x is a vector of firms' outputs. The first order condition for maximization of W with respect to t yields

(26)
$$\sum\partial W/\partial x_i\partial x_i/\partial t=\sum\left\{f(Q)-f(Q)-f'(Q)Q+(f(Q)+x_if'(Q)-C'_i(x_i)-t\gamma_i)\right.$$
$$\left.+\sum_{j\neq i}x_jf'(Q)-D'\gamma_i+t\gamma_i\right\}\partial x_i/\partial t$$
$$=\sum(t\gamma_i-x_if'(Q)-D'\gamma_i)\partial x_i/\partial t$$
$$=0\,,$$

where we have made use of the first order condition (19). Hence, the optimal Pigouvian tax is determined by

(27)
$$t=D'+f'\sum x_i\partial x_i/\partial t/\sum\gamma_i\partial x_i/\partial t\,.$$

The sign of $\partial x_i/\partial t$ as well as that of the coefficient of f' is indeterminate. Okuguchi (1993a) has derived a necessary and sufficient condition for $\partial x_i/\partial t$ to be positive or negative. Equation (27) is a generalized version of Ebert (1992, Equation (8)).

Some remarks on special cases might be in order.

Case a: $\partial x_i/\partial t = \partial x_j/\partial t\,, \quad i \neq j\,, \quad \gamma_i = 1\,, \quad i, j = 1, 2, \cdots, n\,.$

In this case (27) reads

(28) $$t = D' + f'Q/n\,.$$

If all firms are symmetric, $\partial x_i/\partial t = \partial x_j/\partial t$ *for* $i \neq j$. In Ebert's model, firms are symmetric and $\gamma_i = 1$, for all i.

Case b: $\gamma_i = 1\,, \quad i = 1, 2, \cdots, n\,.$

In this case (27) reads

(29) $$t = D' + f' \sum x_i \partial x_i/\partial t/\partial Q/\partial t\,.$$

The coefficient of f' is a weighted average of x_is. If the inverse demand function is linear and if, in addition, the marginal costs *which may differ among firms* are constant, we have

$$\partial x_i/\partial t/\partial Q/\partial t = 1/n\,.$$

Hence, in this case (29) is identical to (28).

4. CONCLUSION

In Section 2 of this paper we have analyzed the effects of a shift from specific to ad valorem taxation on the assumption that the tax revenue is an increasing function of specific and ad valorem tax rates. Our analysis has been rendered possible by taking into account of the relationships existing among individual and industry outputs, specific and ad valorem tax rates. We have found that if the tax revenue is not affected by a shift from specific to ad valorem taxation, and if, in addition, firms are symmetric, the shift increases the equilibrium industry output and social welfare. However, if firms are not symmetric, the effects of the shift are indeterminate. In Section 3 we have derived the optimal Pigouvian tax formula for controlling emission of pollution within Cournot oligopoly and considered two special cases in relationship to Ebert's result for a symmetric case.

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