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BERTRAND AND HIERARCHICAL STACKELBERG OLIGOPOLIES WITH PRODUCT DIFFERENTIATION

Koji OKUGUCHI and Takeshi YAMAZAKI

Abstract: A hierarchical Stackelberg model where firms' entry is sequential is formulated for price-adjusting oligopoly with product differentiation. The firms' equilibrium prices, outputs and profits are derived and compared in relationship to the order of the firms' entry into the market. These equilibrium values are also compared with those for the non-hierarchical Bertrand oligopoly where all firms' decisions are simultaneously made.

1. INTRODUCTION

The equilibria have been compared by Anderson and Engers (1992) for the classical Cournot oligopoly where all firms choose outputs simultaneously and for a sequential Stackelberg oligopoly where firms choose outputs sequentially. It has been found, among other things, that the equilibrium price is lower and the total profits are lower for the hierarchical Stackelberg oligopoly than the Cournot oligopoly; that the first mover (entrant) does not necessary earn more than a Cournot oligopolist if there are more than two firms. Anderson and Engers have derived their results for the case where product differentiation is absent. In this paper we will analyze how the firms' equilibrium prices, outputs and profits are affected by the order of the firms' entry for a sequential Stackelberg price-adjusting oligopoly with product differentiation. We will also compare the equilibria for the non-sequential Bertrand and sequential Stackelberg oligopolies with product differentiation and with price strategies.

2. NON-SEQUENTIAL BERTRAND OLIGOPOLY

In this section each firm is assumed to determine its price simultaneously assuming that its rivals' prices are all given. Let there be n firms. If p_i, q_i and C_i are the price, demand and cost for the firm i, its demand and cost functions are given by (1) and (3) below, respectively.

$$q_i = a_0 + a_1 p_i + a_2 \sum_{j \neq i} p_j, \qquad i = 1, 2, \cdots, n, \tag{1}$$

where

$$a_1 < 0, \quad a_2 > 0, \quad -a_1 > (n-1)a_2. \tag{2}$$

$$C_i = c_0 + c_1 q_i\,, \qquad i = 1, 2, \cdots, n\,, \tag{3}$$

where

$$c_0 \geq 0\,, \quad c_1 > 0\,. \tag{4}$$

The firm i's profit π_i is defined in terms of prices as follows:

$$\pi_i \equiv \left(a_0 + a_1 p_i + a_2 \sum_{j \neq i} p_j \right)(p_i - c_i) - c_0\,, \qquad i = 1, 2, \cdots, n\,. \tag{5}$$

The identical Bertrand equilibrium price p_B is a solution of the following first order condition for profit maximization:

$$\partial \pi_i / \partial p_i = \left(a_0 + a_1 p_i + a_2 \sum_{j \neq i} p_j \right) + a_1 p_i - a_1 c_1 = 0\,, \qquad i = 1, 2, \cdots, n\,. \tag{6}$$

Hence

$$p_B \equiv \beta / (1 - (n-1)\alpha) > 0\,, \tag{7}$$

where

$$\alpha \equiv -a_2 / 2a_1\,, \quad \beta \equiv -a_0 / 2a_1 + c_1 / 2\,. \tag{8}$$

p_B is positive because of (2) and (4). On the other hand, the identical equilibrium output q_B corresponding to p_B is

$$q_B = a_0 + \{(a_1 - a_2) + n a_2\} \beta / (1 - (n-1)\alpha)\,. \tag{9}$$

The necessary and sufficient condition for $q_B > 0$ is

$$a_0 + (a_1 + (n-1) a_2) c_1 > 0\,. \tag{10}$$

Hence the demands must be positive for all firms when their prices are all equal to the identical marginal cost c_1.

3. HIERARCHICAL STACKELBERG OLIGOPOLY

In this section we consider a hierarchical Stackelberg price-adjusting oligopoly. If the entry is hierarchical, the i-th entrant's profit π_i is defined by

$$\pi_i(p_1, \cdots p_{i-1}, p_i) \equiv \left(a_0 + a_1 p_i + a_2 \sum_{j=1}^{i-1} p_j \right)(p_i - c_1) - c_0\,, \quad i = 1, 2, \cdots, n\,. \tag{11}$$

Taking into account the first order condition for maximization of (11) with respect to p_i, we have the equilibrium price for the sequential Stackelberg oligopoly as follows:

$$p_i^H = (1 + \alpha)^{i-1} \beta\,, \qquad i = 1, 2, \cdots, n\,, \tag{12}$$

where H refers to "hierarchical." Since p_1^H satisfies

$$p_1^H = \beta \equiv -a_0/2a_1 + c_1/2 > 0\,, \tag{13}$$

$p_i^H > 0$ for all i. The output q_i^H corresponding to (12) is shown by

$$q_i^H = a_0 + \beta\{(a_1 - a_2)(1+\alpha)^{i-1} + a_2((1+\alpha)^n - 1)/\alpha\}\,, \qquad i = 1, 2, \cdots, n\,. \tag{14}$$

From (12),

$$p_{i+1}^H > p_i^H\,, \qquad i = 1, 2, \cdots, n-1\,. \tag{15}$$

On the other hand, (14) coupled with (15) leads to

$$q_{i+1}^H - q_i^H = (a_1 - a_2)(p_{i+1}^H - p_i^H) < 0\,, \qquad i = 1, 2, \cdots, n-1\,. \tag{16}$$

A little calculation yields

$$\pi_{i+1}^H - \pi_i^H = (p_{i+1}^H - p_i^H)\left\{q_{i+1}^H - (a_1 - a_2)\left(a_0 + a_1 p_i^H + a_2 \sum_{j=1}^{i-1} p_j^H\right)\Big/ a_1\right\}, \quad i = 1, 2, \cdots, n\,, \tag{17}$$

where we have made use of the first order condition $\partial\pi_i/\partial p_i = 0$.

A further calculation, which is omitted and available upon request to the interested reader, leads to

$$\pi_n^H - \pi_{n-1}^H = -(p_n^H - p_{n-1}^H)(p_{n-1}^H - 2c_1)a_2/2\,. \tag{18}$$

Hence

$$\pi_n^H \lesseqqgtr \pi_{n-1}^H \quad \text{according as} \quad p_{n-1}^H \gtreqqless 2c_1\,. \tag{19}$$

Taking into account (12), the assertion (19) reads

$$\begin{aligned} &\pi_n^H \lesseqqgtr \pi_{n-1}^H \quad \text{according as} \\ &c_1 \gtreqqless -(1 - a_2/2a_1)^{n-2} a_0/2a_1/(2 - (1 - a_2/2a_1)^{n-2}/2)\,. \end{aligned} \tag{20}$$

In the case of duopoly, (20) is simplified as follows:

$$\pi_2^H \lesseqqgtr \pi_1^H \quad \text{according as} \quad c_1 \gtreqqless -a_0/3a_1\,. \tag{21}$$

As we have seen above, π_n^H may be larger or smaller than, or equal to π_{n-1}^H, but we have an unambiguous result

$$\pi_{n-1}^H > \pi_{n-2}^H > \cdots > \pi_2^H > \pi_1^H\,. \tag{22}$$

4. EQUILIBRIUM PRICES FOR BERTRAND AND HIERARCHICAL STACKELBERG OLIGOPOLIES

In this section we compare the equilibrium prices for Bertrand and hierarchical Stackelberg oligopolies. First we compare p_n^H and p_B. Taking into account (7) and (12), we can claim that $p_n^H < p_B$ is equivalent to

$$-(n-1)\alpha+\sum_{i=1}^{n-1}(n-1)!\{n(1-i)\}\alpha^i/i!(n-i)!-(n-1)\alpha^n<0\,. \tag{23}$$

Since α_i has a negative coefficient for $i=2, \cdots, n$ and zero for $i=1$, and since, in addition, $\alpha>0$, (23) holds unambiguously. Moreover, since p_i^H increases with i, we have

$$p_i^H<p_B\,, \qquad i=1,2,\cdots,n\,. \tag{24}$$

5. EQUILIBRIUM OUTPUTS COMPARED

In this section we compare the equilibrium outputs for Bertrand and hierarchical Stackelberg oligopolies. For i such that $p_i^H\leq\sum_j p_j^H/n$, we get in the light of (2) and (24),

$$q_i^H-q_B\geq(p_i^H-p_B)\{(a_1-a_2)+na_2\}>0\,.$$

Hence

$$q_i^H>q_B \quad \text{for } i \text{ such that} \quad p_i^H\leq\sum_j p_j^H/n\,. \tag{25}$$

Since $p_1^H, p_2^H, \cdots, p_n^H$ is an increasing geometric series, it follows that $q_i^H>q_B$ for i such that $i\leq n/2$. Furthermore, a little calculation yields $q_n^H<q_B$. The equilibrium industry output is unambiguously larger for the hierarchical Stackelberg oligopoly than for the Bertrand oligopoly as the following inequality holds in the light of (2) and (24).

$$\sum_i q_i^H-nq_B=(\sum p_i^H-np_B)\{(a_1-a_2)+na_2\}>0\,. \tag{26}$$

6. PROFITS COMPARED

In this section we compare the firms' equilibrium profits in the Bertrand and hierarchical Stackelberg oligopolies. First taking into accunt $q_n^H<q_B$ and $p_n^H<p_B$ as well as

$$\pi_i^H-\pi_B=(p_i^H-p_B)q_i^H+(p_B-c_1)(q_i^H-q_B)\,, \qquad i=1,2,\cdots,n\,, \tag{27}$$

we have

$$\pi_n^H<\pi_B\,. \tag{28}$$

If, in addition, $p_i\geq\sum_j p_j/n$, (27) leads to

$$\pi_i^H-\pi_B\leq(p_i^H-p_B)[q_i^H-\{(a_1-a_2)+na_2\}q_B/a_1]\,. \tag{29}$$

Since we have

$$p_{n-1} \geq \sum_j p_j/n \qquad \text{for} \quad n \geq 4\,, \tag{30}$$

(29) holds if $n \geq 4$. Furthermore, if $q^H_{n-1} < q_B$, we have from (27) $\pi^H_{n-1} < \pi_B$. On the other hand, if $q^H_{n-1} > q_B$, we have from (2) and (29) that $\pi^H_{n-1} < \pi_B$. Hence

$$\pi^H_{n-1} < \pi_B \qquad \text{for} \quad n \geq 4\,. \tag{31}$$

Combining (28) and (31), as well as taking into account (22), we obtain the following result:

$$\pi^H_i < \pi_B\,, \qquad i = 1, 2, \cdots, n \quad \text{if } n \geq 4\,. \tag{32}$$

This inequality does not necessarily hold for $n=2$ and $n=3$. However, since (28) is valid even for these cases, the monotonicity of the profits shown by (22) enables us to assert the validity of (32) also for these cases provided that

$$p_{n-1} \leq 2c\,. \tag{33}$$

By virtue of (8) and (12), the inequality (33) is rewritten for $n=2$ and $n=3$, respectively, as

$$c_1 \geq -a_0/3a_1 \tag{34}$$

and

$$c_1 \geq a_0/(8a_1/(-2a_1+a_2)+1)a_1\,. \tag{35}$$

7. CONCLUDING REMARKS

We have found that in the hierarchical Stackelberg oligopoly with product differentiation, the $(i+1)$st entrant's price is higher than that for the i-th entrant, but the output for the $(i+1)$st entrant is lower than that for the i-th entrant (see (15) and (16)); that the equilibrium prices for the hierarchical Stackelberg oligopoly are lower than those for the Bertrand oligopoly (see (24)); that in the hierarchical Stackelberg oligopoly the n-th (that is last) entrant' profit may be larger or smalller than, or equal to the $(n-1)$st entrant's one, but for $i=1, 2, \cdots, n-1$, the profit π^H_i is strictly increasing in i (see (19), (20) and (22)); that if $n \geq 4$, the firms' equilibrium profits for the hierarchical Stackelberg oligopoly are smaller that those for the Bertrand firms (see (32)), but if $n=2$ or 3, the similar relationships hold if the parameters in the demand and cost functions satisfy (34) and (35), respectively.

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